# MECHANICS

## FOR THE UPPER SCHOOL

MERCHANT AND CHANT

Walther, E.H.

TORONTO
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Ben Deacon

$$\left\{ v = u + at \right\}$$

$$s = vt$$

$$\left\{ \begin{aligned} s &= \tfrac{1}{2} at^2 \\ s &= ut + \tfrac{1}{2} at^2 \end{aligned} \right\}$$

$$v^2 = u^2 + 2as$$

# MECHANICS

## FOR THE UPPER SCHOOL

BY

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# PREFACE

In the present work the subject is approached from the experimental and observational side, and continual reference is made to things used in daily life.

Considerable space is devoted to practical applications. Throughout the book there are references to the automobile, the airplane, the base-ball curve and other familiar subjects. Indeed some may be disposed to criticise the amount of attention given to these matters, especially in the chapter on machines. But they are all interesting illustrations of mechanical principles and are of the greatest interest to our young people. For these reasons they are included, certainly not to supply material for written examinations.

Our sincere thanks are due to Professors A. L. Clark and J. K. Robertson, of Queen's University, who read most of the manuscript of the book, and offered many excellent suggestions. Valuable assistance was also given by Professor F. B. Kenrick, of the Department of Chemistry, and Professor G. A. Cornish, of the Faculty of Education, University of Toronto, in the preparation of the chapter on Surface Tension.

THE AUTHORS.

TORONTO, August, 1919.

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# CHAPTER I

## Measurement of Length and of Time

**1. The Need for Accurate Measurement.** It is often remarked that this is the age of science. The small beginnings of the railway and the steamboat can be traced back almost a century, but their great development has taken place during the last fifty years. The ordinary telephone, the wireless telegraph and telephone, the electric dynamo and transformer, the phonograph, the airplane, the automobile and many other mechanical conveniences which are common to-day, were entirely unknown to our grandfathers.

These are what we call *practical applications* of science and it is evident that we cannot have the application until the principles, or laws, of science on which it is based have been discovered and enunciated. Again, the discovery of these principles is made in the physical or chemical laboratory, usually by people who have no thought that their work will have practical application, though few scientific discoveries fail to be utilized at some time.

Now in making a scientific investigation into any problem we cannot make much progress unless we are able to measure accurately the various quantities with which we have to deal. In astronomy the methods of making accurate measurements of time and angle and length were devised at an early date, and that branch of science reached mature development long before any other branch. But in later times physics and chemistry have enormously increased their boundaries, through the development of accurate methods of measurement. We have learned to measure, often to many decimal places, the various effects produced by heat, or electricity or sound, and

have thus been able to state the precise laws according to which they act. Let us consider briefly some of the simpler measurements.

**2. Measurement of Length—The Metric System.** The commonest of all measurements is that of length. Whether we design a bridge, pile a cord of wood, purchase cloth or construct a watch, we must measure various lengths, sometimes with great accuracy. It is very necessary to have accurately fixed standards.

There are two systems of units in common use,—the Metric and the English. In the former the fundamental unit of length is the *metre.* This was intended to be one ten-millionth of the distance from the north pole to the equator, measured on the meridian through Paris, and years were consumed in trying to make a metal bar which should be of exactly this length. The task was completed in 1799. But since then further measurements of the earth have been made and it has been shown that the bar is a little shorter—perhaps a hair's-breadth—than it was intended to be. So now we define the metre without reference to the earth at all; it is the distance between two lines on a metal rod which is preserved in the International Bureau of Weights and Measures at Sêvres, near Paris. The measurements are to be taken when the rod is at the temperature of melting ice. Many copies of this standard have been made and supplied to various nations, though Canada has not yet received the standard which is due her as an autonomous nation. The bars are made of a hard and durable alloy composed of platinum 90 per cent. and iridium 10 per cent. and their form is shown in Fig. 1. As a result of the Great War the proposal

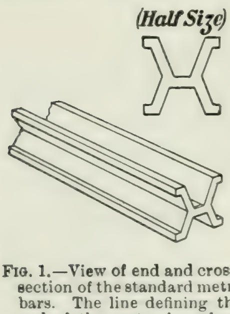



FIG. 1.—View of end and cross-section of the standard metre bars. The line defining the end of the metre is a short mark on the surface midway between the top and the bottom of the bar.

has again been seriously made that the British Empire and the United States both adopt the metric system for ordinary use.

**3. Divisions and Multiples of the Metre.** The metre is divided decimally, thus:

$\frac{1}{10}$ metre = 1 decimetre (dm.)
$\frac{1}{10}$ dm. = 1 centimetre (cm.)
$\frac{1}{10}$ cm. = 1 millimetre (mm.)
1 m. = 10 dm. = 100 cm. = 1000 mm.

For greater lengths, multiples of ten are used, thus:

10 metres = 1 decametre.
10 decametres = 1 hectometre.
10 hectometres = 1 kilometre (km.)
1 km. = 1000 m.

The *decametre* and the *hectometre* are not often used.

**4. The English System.** In this system the fundamental unit of length is the *yard*. It is said to have represented originally the length of the arm of King Henry I (1100-1135), but such a definition is not accurate enough for present-day requirements. The crude manner in which this unit was specified at that time, compared with the precise way in which it is fixed and reproduced now, may serve to illustrate the growth in the appreciation of science in the last 800 years.

The yard is now defined as the distance, at 62° F., between the centres of two transverse lines ruled on two gold plugs in a bronze bar, which is preserved in London, England, in the Standards Office of the Board of Trade of Great Britain. The bronze bar is 38 inches long and has a cross-section one inch square (Fig. 2). At $a$, $a$, wells are

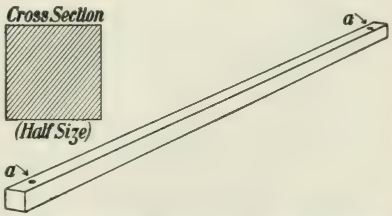



FIG. 2.—Bronze yard, 38 in. long, 1 in. sq. in section. $a$, $a$, are small wells in bar, sunk to mid-depth.

sunk to the mid-depth of the bar, and at the bottom of each well is the gold plug or pin, about $\frac{1}{10}$ inch in diameter, on which the line defining the yard is engraved.

The other units of length in ordinary use, such as the inch, the foot, the rod, the mile, are derived from the yard. Unfortunately, however, they are not obtained by dividing into tenths or by multiplying by tens, and so calculations in the English system are much longer and more tedious than in the metric system.

**5. Relations between the Two Systems.** In Great Britain the relation between the metre and the inch is officially stated to be,

1 metre = 39.370113 inches, or 1 yard = 0.914399 metre;

in the United States the metre is taken as the fundamental standard and other lengths are referred to it. By law,

1 metre = 39.37 inches, and hence 1 yard = 0.914402 metre.

Thus the U.S. yard differs from the Imperial yard by only 3 parts in 900,000; and they may be considered identical.

The following relations hold:

| | |
|---|---|
| 1 cm. = 0.3937 in. | 1 in. = 2.54 cm. |
| 1 m. = 39.37 in. = 1.094 yd. | 1 ft. = 30.48 cm. |
| 1 km. = 0.6214 ml. | 1 ml. = 1.609 km. |

Approximately 10 cm. = 4 in.
30 cm. = 1 ft.
8 km. = 5 ml.

In Fig. 3 is shown a comparison of centimetres and inches.

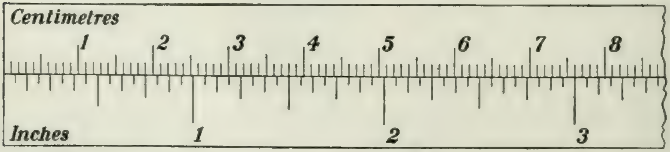



FIG. 3.—Comparison of inches and centimetres.

**6. Derived Units.** The ordinary units of surface and of volume are at once derived from the lineal units. The

imperial gallon is defined as the volume of 10 pounds of water at 62° F., or is equal to 277.274 cu. in. (The U.S. or Winchester gallon = 231 cu. in.). The litre contains 1000 c.c.

The following relations hold:

| | | | |
|---|---|---|---|
| 1 sq. yd. | = 0.836 sq. m. | 1 c.dm. | = 61.024 cu. in. |
| 1 sq. m. | = 10.764 sq. ft. | 1 gal. | = 4.546 l. |
| 1 cu. in. | = 16.387 c.c. | 1 l. | = 1.76 pt. |

**7. Measurement of Lengths.** Having fixed our units of length let us consider how to measure lengths with them and with what degree of accuracy we need to work. Suppose you wish to purchase a certain quantity of cloth at a dry-goods store. The clerk unrolls the cloth, and placing it alongside his yard-stick (which is a reproduction, more or less accurate, of the original standard yard) measures off the amount ordered. In this case the measurement is not very accurate, each yard of the cloth might easily be in error by half an inch. The skilful cabinet-maker must be much more accurate in the use of his 2-foot rule when he makes a piece of fine furniture. But in some mechanical operations a still greater degree of accuracy is demanded. In the manufacture of steel balls for ball bearings they should not differ by $\frac{1}{100000}$ inch and they should not vary from a perfect sphere by $\frac{1}{100000}$ inch. How shall we make these accurate measurements?

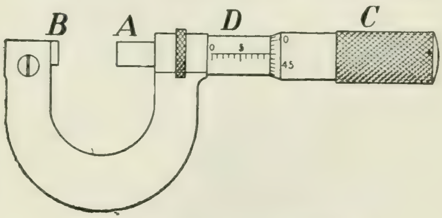



FIG. 4.—Micrometer wire gauge.

**8. Micrometer Screw Gauge.** Suppose we wish to determine accurately the thickness of a wire or of a metal plate. A suitable instrument to use is the screw gauge, illustrated in Fig. 4. $A$ is the end of an accurately made screw which works in a nut inside $D$, and can be moved back and forth by turning the cap or thimble $C$ which is

attached to it and which slips over $D$. Upon $D$ is a scale which counts the number of revolutions of $C$, while the bevelled end of $C$ is divided into a number of equal parts by which the fractions of a revolution are measured. By turning the cap the end $A$ moves forward until it reaches the stop $B$, and then the graduations on $D$ and $C$ should both read zero.

In order to measure the diameter of a wire we turn the screw back until the wire can just pass between $A$ and $B$, and then from the graduations on $D$ and $C$ we find the diameter required.

Suppose the pitch of the screw to be $\frac{1}{2}$ mm. Then with one revolution of $C$ the end $A$ moves through $\frac{1}{2}$ mm. Now if there are 50 divisions on the bevelled end of $C$ it is evident that when the screw turns through one division the end $A$ moves through $\frac{1}{50} \times \frac{1}{2} = \frac{1}{100}$ mm. Such an instrument will measure to $\frac{1}{100}$ mm.

Sometimes the pitch of the screw is $\frac{1}{40}$ inch and there are 25 divisions on the head $C$, in which case one division = $\frac{1}{25} \times \frac{1}{40} = \frac{1}{1000}$ inch.

**9. Vernier Calipers.** But it may happen that the object which we have to measure is too large for our gauge,—for

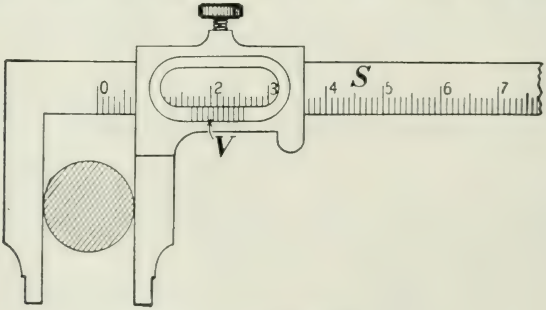



Fig. 5.—Vernier caliper.

example, a cylinder or a sphere an inch or more in diameter (though gauges have been made which will measure several

inches); or perhaps the degree of accuracy demanded is not so great. In this case we might use the vernier caliper, one pattern of which is illustrated in Fig. 5. As will be seen, there are two graduated scales,—one, $S$, on the bar of the instrument, and the other, $V$, on a jaw which slides upon the bar. The scale $V$ is called a *vernier* and its aim is to measure fractions of a division of the scale $S$.

The way to use it is as follows:—Push the vernier along until the cylinder or the sphere will just pass between the two jaws, and let the reading be that shown in the figure.

Usually the vernier is constructed so that $n$ of its divisions are equal to $n - 1$ divisions of the scale. Suppose 10 vernier divisions are equal to 9 scale divisions, and that the latter are millimetres. Then 1 division on the vernier is clearly 0.9 mm., and the difference between one scale division and one vernier division is 0.1 mm. In order to explain the action of

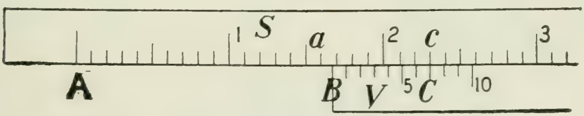



FIG. 6.—Scale and vernier.

the vernier consider the enlarged image of the scale and vernier (Fig. 6). Suppose the zero on the vernier occupies the position shown in Fig. 6. It is clear that the length $AB$ is equal to 16 mm. + a fraction of a millimetre. To find this fraction, look along the vernier and see where a line on it coincides with a line on the scale. It is seen that division 7 on the vernier coincides with the line $c$ on the scale. Then the fraction to be measured, namely the distance $aB$, is equal to the difference between the 7 divisions of the scale in the space $ac$ and the seven divisions of the vernier in the space $BC$. But the difference between one scale division and one vernier division is 0.1 mm. Hence the fractional part is $7 \times 0.1$ or

0.7 mm., and the entire space $AB$ is therefore 16.7 mm. or 1.67 cm.

For any other vernier the calculation is similar.

Of course there are other devices for the accurate measurement of lengths, each being designed for the special purpose in view, but in every case the screw or the scale or whatever is the essential part of the instrument must be carefully compared with a good standard before our measurements can be of real value.

**10. Unit of Time.** The earth is our great time-measurer, the period of a rotation being denoted a *day*. Imagine a plane to be drawn through the point where one stands and also through the axis of the earth. This is the observer's *meridian plane*, and as the earth turns on its axis this plane turns with it. During every rotation this meridian plane will come to the sun (and, in succession, to every other body in the sky) though to all appearances the sun comes to the meridian, not the meridian to the sun.

The interval from the moment when the centre of the sun is on the meridian until it next arrives there is called an *apparent solar day*. Unfortunately, however, these apparent solar days are not all of equal length, the reason why being fully given in works on astronomy. Taking the average of the lengths of all the apparent solar days, we obtain a *mean solar day*, and this is chosen as the fundamental unit of time.

It is divided into 24 equal parts, each being an hour; the hour is divided into 60 equal parts, each being a minute; the minute is divided into 60 equal parts, each being a second. Thus the day contains $24 \times 60 \times 60 = 86{,}400$ seconds.

Mean solar time is the kind which is measured off by our ordinary clocks and watches. In the chapters which follow the second will be more frequently used than the day.

## PROBLEMS AND EXERCISES

(For table of values see Section 5)

1. How many millimetres in $2\frac{1}{2}$ kilometres ?

2. Change 186,330 miles to kilometres.

3. Change 760 mm. into inches.

4. Lake Superior is 602 feet above sea level. Express this in metres.

5. Express, correct to a hundreth of a millimetre, the difference between 12 inches and 30 centimetres.

6. Find in inches, the diameter of the bore of the 42-centimetre gun.

7. The distance from Toronto to Montreal is 333 miles. Express this distance in kilometres.

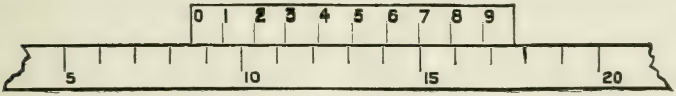



FIG. 7.

8. Give the reading indicated by the vernier in Fig. 7.

9. Give the reading of the barometer, illustrated in Fig. 8, in both English and metric scales.

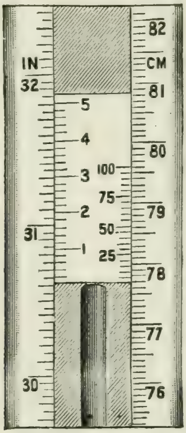



FIG. 8.

# CHAPTER II

## VELOCITY

**11. Bodies in Motion and at Rest.** Let us look out of the window as we travel in a railway train. The fences and the telegraph poles seem to be continually displaced backward, but our experience leads us to consider the earth, with these objects fixed in it, to be at rest while the train moves forward over it. Perhaps an automobile comes along upon a road parallel to the railway and keeps abreast with the train. We can see its wheels spinning around and the gas coming from its exhaust and we say that it, also, is moving over the earth. But let us fix our attention only on the upper part of the motor-car and not look at the ground at all; would we say that the motor-car is moving? No, it appears to be at rest. Or look at the other people travelling with you; are they at rest? You agree that both they and you are moving over the ground, but with regard to you they seem to be at rest. Thus an object may be in motion with respect to one body and, at the same time, at rest with respect to another.

**12. Definition of Motion.** When, then, is a body said to be in motion? Consider a line joining the rear of the train to a point on the track. As we travel forward this line continually increases in length, and so we may say that if the length of the line joining one body with another is changing, one body is moving with respect to the other. As to the motor-car, the line drawn from it to the train remains of constant length and, as far as the above definition is concerned, each is at rest with respect to the other.

Next, look at two children on a merry-go-round or "teetering" on a plank over a log. The line joining them is not changing its length, and yet each child will say that the other is in motion. In this case the length of the

line does not change but the *direction* does, and so we finally reach the following definition:

MOTION.—*One point is in motion with respect to another point when the line joining the two points changes in length or direction.*

We have spoken of the earth as being at rest, but a moment's thought assures us that it is not absolutely at rest. It rotates on its axis and so every particle of it is in motion with respect to the sun and the stars. In addition it revolves about the sun, and still further, the sun with the entire solar system, is moving through space, with respect to the stars. Indeed the motion of any particle of the earth is extremely complicated when we consider its motion with respect to the stars in the sky. It is quite evident that we cannot consider any point as absolutely at rest and so must consider the motion of one point with respect to another. Usually, however, in dealing with the motion of bodies we consider the earth to be at rest.

**13. Velocity,—Average, Uniform, Variable.** The road from Toronto to Hamilton is a very good one and a trip by automobile is very pleasant. Let us take one, and we can make a study of velocity on the way. Starting from Toronto at 10.00 a.m., we pass Port Credit (13 miles) at 10.45, Oakville (21 miles) at 11.10, and reach Hamilton (40 miles) at 12 noon. Thus we have passed over 40 miles in 2 hours, and we say our average velocity or speed during the entire trip was 20 miles per hour.

$$\text{Average velocity} = \frac{\text{Distance}}{\text{Time}}.$$

Of course we kept watching the speedometer all the way and we saw that its reading changed very often. Sometimes it said 5, then 10, 20, and perhaps 30 miles an hour, or, when the motor had to stop, it fell to 0. Now, while the reading on the speedometer was constant we realized that we were

travelling at a *uniform velocity*, by which we mean that we were *passing over equal distances in equal times.*

Suppose that over a 5-mile stretch of level road we kept the speedometer perfectly steady, and we found that it required 12 minutes to go this distance. Then, since the velocity was uniform, the rate was

5 miles in 12 minutes,
which = 25 " " 60 " or 1 hour.

But generally the speedometer did not remain constant for many seconds at a time, and we realized that we were travelling with *variable velocity,* in other words, the *distances passed over during successive seconds of time were not the same.*

**14. Velocity at a Point.** We have still more to learn from our motor trip. There was a hill which we decided to 'take' on high gear. Having rapidly descended the other hill we began the up-grade with the speedometer indicating 30 miles per hour, which gradually fell until at the top it indicated 10 miles per hour. The entire time required to go up was 20 seconds and the distance was 200 yards.

Now the speed was changing all the time and yet we know that at every point of the 200 yards the car had a definite rate of motion or velocity. Also, to each point of the 200 yards corresponded a definite moment of time in the 20 seconds. Hence, we can say that at every point of the course and at every moment of the time the body had a definite velocity. It is difficult to explain this statement in any simpler terms, but the meaning becomes more definite when we discuss how to measure the velocity at a point.

**15. Measure of Velocity at a Point.** Let $AB$ (Fig. 9)

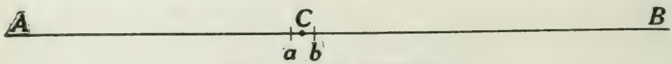


Fig. 9.—Finding velocity at $C$.

represent the 200 yards traversed at varying speed. We wish to measure the velocity at the point $C$.

We can arrange two electrical contacts 1 foot apart, one 6 inches in front of $C$, the other 6 inches back of $C$, such that as the car passes over them it will make, by means of an electrical spark, a record on a moving strip of smoked paper upon which a tuning fork is writing a wavy line as shown in Fig. 10. Let the fork make 100 complete vibrations per second, that is, each complete wave represents $\frac{1}{100}$ sec., and let $c$, $d$ be the marks recorded as the car passed over $a$, $b$ (Fig. 9) respectively. These are just 3 full waves apart, and consequently the car passed over 1 foot from $a$ to $b$ in $\frac{3}{100}$ sec. We easily calculate that 1 foot in $\frac{3}{100}$ sec. is equivalent to $22\frac{8}{11}$ miles per hour.

Fig. 10.—Measuring a short interval of time.

This is the average velocity during the $\frac{3}{100}$ sec. and does not express exactly the velocity at $C$. If next we take a space of $\frac{1}{2}$ foot with $C$ in the middle of it and determine the average velocity when passing over it, we shall be still nearer the desired result. It is evident that the shorter the space we take, the nearer we approach the velocity desired. Hence, we say that if $s$ is an *infinitely small* length containing $C$ and $t$ is the *infinitely short* time taken to pass over $s$, the

$$\text{velocity at } C = \frac{s}{t} \cdot$$

Some writers make a distinction between velocity and speed, the latter term being used to mean simply *rate of motion*, the former including direction as well as rate. Thus two velocities would not be considered equal unless they were equal in absolute amount and the motion was in the same direction. This distinction is very useful in a full treatment of motion, but in this book we shall deal almost entirely with motion in a straight line and no sharp distinction between the terms will be made.

### PROBLEMS

1. Find the equivalent, in feet per second, of a speed of 60 miles per hour.

2. An eagle flies at the rate of 30 metres per second; find the speed in kilometres per hour.

3. Express in miles per hour a velocity of (1) 40 feet per second, (2) 100 yards per minute.

4. A point moves at the rate of 50 miles in $1\frac{1}{2}$ hours. What is its velocity in feet per second?

5. Find the ratio of velocities of (1) 60 miles per hour and 44 feet per second, (2) 5 miles per 6 minutes and 10 feet per $\frac{1}{4}$ second.

6. One body moves over 30 yards in 7 minutes, and another over 12 feet in 5 seconds. If their velocities are uniform, compare them.

7. A velocity of 20 miles per hour is $v$ times a velocity of 30 feet per second. What is $v$?

8. A body has a uniform velocity of 8 feet per second. What is its displacement in 11 hours?

9. A body is moving with a uniform velocity of 20 cm. per second. What is its displacement in metres in 10 hours?

10. A body moves uniformly in a straight line at the rate of $a$ feet per second. What is its displacement in miles in $b$ hours?

11. A body is moving uniformly at the rate of $c$ cm. in $s$ seconds. How far does it go in $h$ hours?

12. The velocity of a train is 15 miles per hour. Find (1) how many minutes it will take to go 50 yards, (2) how many seconds it will take to go 25 feet.

13. The velocity of a point is $a$ feet per $b$ seconds. How long does it take it to go $c$ miles?

14. A sledge party in the Arctic regions travels northward, for ten successive days, 10, 12, 9, 16, 4, 15, 8, 16, 13, 7 miles, respectively. Find the average velocity.

15. If at the same time the ice is drifting southward at the rate of 10 yards per minute, find the average velocity northward.

16. A point has displacements of 9 cm., 10 cm., 11 cm., and 12 cm. in four consecutive seconds. Find its average velocity (1) for four seconds, (2) for the first three seconds, (3) for the last three seconds.

17. A point is displaced 5 cm., 3 cm., 1 cm., $-1$ cm., $-3$ cm. in five consecutive seconds. What is its average velocity (1) for the five seconds, (2) for the first three seconds, (3) for the last three seconds, (4) for the middle three seconds?

## CHAPTER III

### Inertia and Force

**16. Rest and Uniform Motion.** Walking along the road day after day, we see a stone beside the path, but one morning it is gone! Now if some person should tell you that the stone *of itself* moved away you would consider him not in his right mind. Such an occurrence is entirely contrary to all our experience.

Sometimes we hear stories of how, in a dimly-lighted room, when several people had laid their hands upon a table, it began to move and to give certain mysterious "rappings." Whatever truth there may be in some of these reports, we may be sure that the table did not *of itself* get up on its legs and walk about. We know that such things do not happen!

Lifeless bodies at rest, when left to themselves, remain at rest.

Again, in playing base-ball or cricket, when the batter strikes a 'hot grounder' the ball rolls for a long distance before it comes to rest, and the smoother the field the farther it rolls. If the ball is driven along a cement-paved street it goes still farther; and if we try the experiment on a long stretch of smooth ice it seems almost as if the ball will never stop. The friction of the surface brings it to rest at last, but it is easy to believe that if we could construct a flat level surface which would be entirely without friction, a body started upon it would go on in a straight line at the same rate for ever.

If one could travel far out into space, away beyond the influence of any celestial body, and could there launch an object, large or small, it would continue to move in a straight line with the velocity initially given to it for all time—unless it should come under the influence of some other body.

**17. Newton's First Law of Motion.** In the preceding section statements were made which everyone recognizes to be

in accordance with his experience, and which may be taken as axioms, that is, self-evident truths. Now, in 1687, Sir Isaac Newton published his great book entitled, "The Mathematical Principles of Natural Philosophy," which is generally considered to be the greatest scientific book ever published. At the beginning of this he states his famous three laws of motion, the first of which is as follows:

*Every body continues in its state of rest, or of uniform motion in a straight line, unless it be compelled by external* FORCE *to change that state.*

This is at once seen to be a concise summing-up of the universal experience of man.

**18. Force.** In the above examples the stone and the table moved; we conclude, then, that a force entirely external to them acted upon them. Similarly with the rolling ball; it gradually travelled more slowly, and the change in its motion was due to the force of *friction*, which was quite external to it.

If a body is allowed to fall freely, we know that it moves in a straight line but with increasing velocity, and we say therefore that a force acts upon it. In this case it is the force of gravity or the attraction of the earth. If a body is projected outwards there is a change both in the velocity and in the direction of motion—both produced by the force of gravity.

It is well, then, to have clearly in mind:

(i) if a body changes from being at rest to being in motion;

(ii) if the speed of a body is changed; or

(iii) if the direction of motion is changed, even without change in speed; then *force* is acting on the body.

**19. Inertia.** It used to be a familiar trick on April Fool's day to place an old hat near the path to tempt the unsuspecting passer-by to kick it. Now a vigorous kick will change the hat's condition of rest into motion with a considerable velocity; but if there happens to be a brick under the hat,—well, it is

quite a different matter! It is much more difficult, or requires a much greater muscular effort, to set in motion the hat-and-brick than the hat alone. We say the former has much greater *inertia.*

A body possessing great inertia requires a great effort, that is, a great force, to put it in motion; and an equally great force is needed to stop the body if it is in motion.

There is no danger in stopping a football going down the field, but a cannon ball (cannon balls were formerly spherical) of the same size would simply plough through all the players on a field and would do great damage.

An empty barrel has little inertia, and when rolling down an incline can easily be stopped, but look out for it if it is filled with flour or oil or other heavy substance.

**20. Resistance to Change of Motion.** Many simple experiments illustrate the inertia of bodies.

Lay a book on a sheet of paper on a table. By a quick jerk the paper can be pulled out, leaving the book practically where it was before.

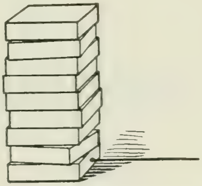


FIG. 11.—Illustrating inertia.

Pile a number of blocks as in Fig. 11 and attach a cord to one near the bottom. A vigorous pull on the string will remove the block to which it is attached and leave the others in the pile as before. It required a considerable force to pull out the block, on account of its inertia; and the other blocks remained behind on account of their inertia.

These experiments illustrate the inertia shown by a body when it is at rest. It might be remarked that inertia is not confined to inanimate objects. Human beings very frequently exhibit it, but in this case it is usually called 'laziness.' Indeed 'inertia' is a Latin word, and 'laziness' is its English translation. But the inertia of a lifeless body differs from the laziness of a living person in the fact that it requires as great

a force to stop the former when it is in motion as to start it from rest, but not so in the case of the latter!

As illustrations of the inertia of a body in motion the following may be mentioned:

When a locomotive leaves the rails and is quickly brought to rest the cars behind still continue their motion forward and usually do great damage.

If one is standing up in a street car when it is turning a corner it is well to hold to a strap, as one's body tends to continue in the original direction of motion.

In jumping over a ditch you take a run, leap into the air, and the inertia of your body carries it forward.

When shovelling coal or snow you start its motion and its inertia causes it to continue until it reaches where you want it to go.

We see then that *the* INERTIA *of a body is that property by which the body opposes any change in its condition of rest or of uniform motion in a straight line.*

**EXERCISES**

1. Suspend a small bag filled with sand (Fig. 12) by a thread not much stronger than will sustain the load. By means of a similar thread, attach a small bar or handle to the lower part of the bag. Grasp the bar and pull steadily downwards until one of the threads breaks.

FIG. 12.

Which thread breaks? Why should this thread rather than the other break?

Now suspend the bag and bar as before. Again grasp the bar, and, with a quick jerk, pull suddenly downward.

Which thread now breaks?

2. Suspend a heavy weight,* say 10 pounds, by a stout cord 15 or 20 inches long. Tie a fine thread around the middle of the weight and give it a sudden pull sideways.

* The iron balls used as safety-valve weights answer well for many such purposes in the laboratory. They can be had at most foundries.

What change takes place in the condition of (*a*) the thread, (*b*) the weight?

Tie the thread again around the weight, and, by means of a series of well-timed, gentle pulls, set the weight swinging to-and-fro. When it is going through a fairly wide arc, try to stop the weight at its lowest position by suddenly tightening the thread when it reaches this point.

Describe the action of both weight and thread.

3. Lay a card over the mouth of a bottle (Fig. 13), and place a small

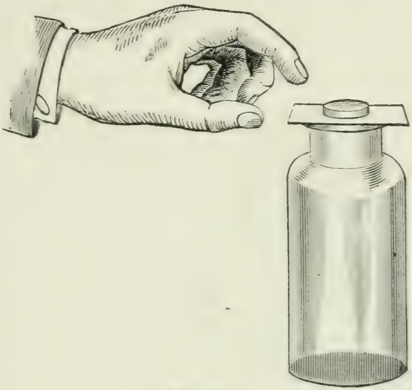


FIG. 13.

coin on the card above the opening. Suddenly drive the card off by striking it with the finger.

What becomes of the coin? Explain its behaviour.

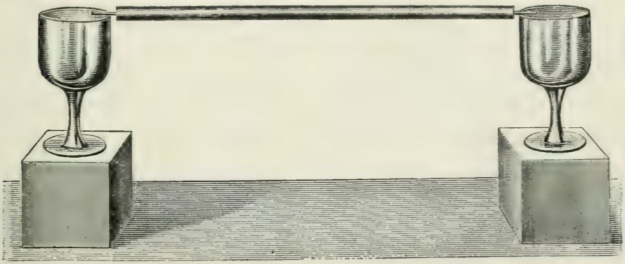


FIG. 14.

Explain each of the following:

(*a*) If a needle is driven into each end of a broomstick and the stick supported by resting the needles on glass goblets, as shown in Fig. 14,

the stick may be broken by striking it at the middle, a quick, strong blow with a heavy iron rod without breaking the needles or goblets. Try the experiment.

(*b*) A rider is liable to be unhorsed if the horse shies or stops suddenly.

(*c*) A person who steps from a rapidly moving car is in danger of being thrown to the ground. It is less dangerous to step out in the direction in which the car is moving than in the opposite direction.

(*d*) A circus rider can pass over a rope extended across the ring and regain his footing on his horse by leaping straight up when he comes to the rope.

(*e*) The outside bank is worn away when a river takes a sharp turn.

(*f*) "So suddenly did the motor-car stop that one of the occupants of the front seat was pitched through the windshield and those in the rear seat were propelled over into the front seat."—(From a newspaper).

(*g*) A well-loaded automobile, or a steamship with a full cargo, rides more smoothly than if it is without load.

**21. Mass.** What is there in a body which gives to it this characteristic property known as inertia? We are accustomed to say that it is its *mass*, though it is impossible to define or explain what mass is. Frequently the mass of a body is said to be the *quantity of matter* in it, but that does not really *explain* it, as we do not know what matter is. Such a definition would not supply a method of measuring the mass of a body.

The mass of a body is proportional to its inertia. Let us try the following experiment. *A*, *B*, *C*, *D* are cubes of the same size, made of cork, cement, iron and lead, respectively, and all painted precisely alike (Fig. 15). From their outward appearance one cannot judge their relative masses, but let us take a light ruler and strike them in succession with equal blows. The cork

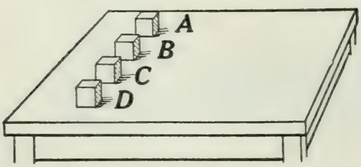



FIG. 15.—Comparison of masses.

cube starts off with considerable speed and probably goes off the table. Its resistance to being moved is slight, or its inertia is small and so is its mass. The cement block may move a couple of feet, the iron possibly a foot but the lead one only a few inches. In this way we can arrange the masses in order and, indeed, get some idea of one in terms of the others. It is, indeed, by comparing the effect of a force on various masses that we can compare them. As we shall see later, the force we usually employ is the attraction of the earth.

**22. Units of Mass.** There are two units of mass in common use. In the metric system the fundamental unit is the kilogram. The world's standard kilogram is a cylinder of platinum-iridium alloy almost exactly $1\frac{1}{2}$ inches in diameter and in height. It is preserved at Sêvres, France. A large number of equal standard masses have been made and distributed to different nations.

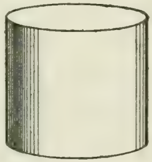


FIG. 16. — Standard kilogram, made of an alloy of platinum and iridium. Height and diameter each 1.5 inches.

The kilogram is divided decimally as follows:

$$\frac{1}{1000} \text{ kilogram} = 1 \text{ gram.}$$
$$= 10 \text{ decigrams.}$$
$$= 100 \text{ centigrams.}$$
$$= 1000 \text{ milligrams.}$$

The original kilogram was intended to represent the mass of 1000 c.c. (1 litre) of water when at its maximum density (at 4° C.)

Hence 1 c.c. water = 1 gram-mass.

In the English system the pound is the fundamental unit. The standard pound is a certain piece of platinum, which

is preserved in the Standards Office in London, England. Its form is shown in Fig. 17.


FIG. 17.—Imperial Standard Pound Avoirdupois. Made of platinum. Height 1.35 inches; diameter 1.15 inches. "P.S." stands for *parliamentary standard.*

Unfortunately the pound is not divided decimally, and the calculations which involve the pound are more complicated than those in the metric system.

In the English system

$$1 \text{ grain} = \frac{1}{7000} \text{ pound (avoirdupois).}$$

$$1 \text{ ounce} = \frac{1}{16} \text{ pound} = 437.5 \text{ grains.}$$

A grain of wheat was taken from the middle of the ear, and being well dried, was used as a standard *grain.**

The relation of the pound to the kilogram is officially stated by the British Government to be

1 kilogram = 2.2046223 pounds avoirdupois.
1 gram = 15.4323564 grains.
1 ounce avoir. = 28.349527 grams.

Approximately 1 kg. = $2\frac{1}{5}$ lb.
1 oz. = $28\frac{1}{3}$ grams.

**EXERCISES**

1. Determine by means of a balance and weights the masses of several small pieces of rock or metal.

2. By weighing find the metric equivalent of an ounce weight.

3. Measure off 50 cm. of iron stove-pipe wire, weigh it and calculate the weight in grams per centimetre in length.

4. Take another piece of the same wire, of unknown length, weigh it, and from the weight per centimetre determined in the last experiment, calculate the length of the wire. Verify your result by measuring its length with a metre scale.

What is your percentage of error?

*In addition we have two other sets of weights. *Troy* weight is used in weighing gold, silver, and precious stones. 24 grains = 1 pennyweight (dwt.), 20 dwt. = 1 ounce (oz.). 12 oz. = 1 lb. Thus 1 lb. troy = 5760 grains.

*Apothecaries'* weight is used in mixing medicines. 20 grains = 1 scruple (sc.), 3 sc. = 1 dram (dr.), 8 drs. = 1 oz., 12 oz. = 1 lb. Apothecaries' pound = Troy pound.

5. Counterpoise a beaker on a balance, run into it from a burette 100 c.c. of water. Weigh the water.

What is the mass of the water?

What is the mass of one cubic centimetre of it?

6. Weigh one cubic centimetre of water by running it from a burette into a counterpoised watch-glass.

(1) How does your result compare with that obtained in Experiment 5?

(2) What would be the mass of one litre of the same water?

**23. Gravitation Units of Force.** It was noted in Section 18 that, whenever the motion of a body is being changed, a force is acting on it. But force may be developed without motion being produced. For example, if we exert a muscular effort and pull one end of a spring rigidly fastened at the other end, we stretch it but there is no motion of the spring as a whole. If we pull with a greater force, we stretch it still more. The stretch is proportional to the force.

Now take a standard pound-mass and hang it from the end of the spring. The spring is stretched a certain amount, and we know therefore that a force must be developed in the spring. This force is due to the pull or attraction of the earth on the pound-mass, and is called the weight of the mass. Next suspend two standard pound-masses; the spring is stretched twice as much, the pull of the earth on them, (that is, their weight) being twice as great. If, further, we have a pointer attached to the spring which moves past a fixed scale (Fig. 18) by adding a succession of masses, we can calibrate the scale, and, in this way, construct a means of measuring the magnitude of forces similar to the familiar spring-balance. The unit force we make use of is, therefore, the pull of the earth on a mass of 1

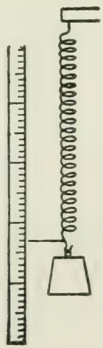


Fig. 18.—The weight of the body stretches the spring.

pound, or briefly stated, 1 pd. wt. This is called the gravitational unit of force. In the metric system, the corresponding unit of force is, of course, 1 gram-force or 1 gram-wt., which is defined to be equal to the attraction of the earth on a mass of 1 gram, or briefly, it is equal to the weight of 1 gram.

**24. Mass and Weight.** Suppose we take the spring with the standard pound-mass to various places on the surface of the earth or perhaps have it taken up several miles in a balloon; will the spring always be stretched the same amount? With a sufficiently sensitive spring, it would be found that the amount of stretch changed with the height of the ascent. Now the mass of the standard pound obviously does not change,—it is the same body which is carried about. Therefore, since the stretch of the spring varies, the pull on the spring, in other words, the weight of the standard mass, must change. A clear distinction must, therefore, be made between mass and weight. A gram-mass is a certain quantity of matter which will remain the same wherever it may be taken; while a gram-wt. or a gram-force varies with one's position on the earth's surface and would change continuously if one should go off into space.

**25. Absolute Unit of Force.** Although the variation in the weight of a gram or of a pound at various places on the surface of the earth is not great, it would never do to choose as an absolute standard a force which has not the same value at all places. Our absolute standard unit of force is defined in terms of the ability of a force to change the motion of a body. Before discussing this further we must consider other matters which form the subjects of the next two chapters.

**26. Comparison of Masses.** This is done

(*a*) By the spring-balance, at the same place. If two bodies produce the same stretch in the spring, their masses are equal as explained in Sec. 23 above.

(*b*) By the common balance.

A good balance should have arms precisely equal in length, and the pans should also be precisely the same.

Upon pan *A* (Fig. 19) place a standard kilogram. That pan at once descends. This is due to the fact that the earth exerts an attractive force on the kilogram, and this force on a body is proportional to its mass.

By careful filing one can make another body which when put on *B* will just balance the body on *A*, and their masses will be equal.

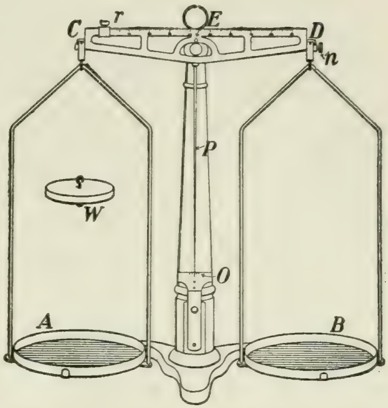



Fig. 19.—A simple and convenient balance. When in equilibrium the pointer *P* stands at zero on the scale *O*. The nut *n* is for adjusting the balance and the small weights, fractions of a gram, are obtained by sliding the rider *r* along the beam which is graduated. The weight *W*, if substituted for the pan *A*, will balance the pan *B*.

With care one can make two bodies which will have equal masses and each equal to one-half the original kilogram. Continuing this process, we can make a set of "weights" having any fractional masses we desire.

Next put any mass on *A*, and by adding our set of weights to *B* we can balance *A* and thus determine its mass.

**Exercise**:—How could you duplicate a 'weight' (1) if the arms are not equal, (2) without using a balance of above type at all?

# CHAPTER IV

## ACCELERATION

**27. Accelerated Motion.** One is not afraid to jump from a verandah to the ground, but hesitates to do so from the top of a high fence, and he would simply refuse to leap from an upstairs window unless it were done to save his life. The reason is obvious enough. The greater the distance a body falls through the air, the faster it moves, and in falling only a few feet a person may acquire a velocity great enough to injure him when he strikes the ground.

On going down a grade, even though the engineer shuts off the steam, the train continually gains in speed and the brakes may have to be set in order to observe the instruction "safety first." If a stone is thrown upwards its velocity gradually diminishes until the stone stops and it then comes downwards with continually increasing velocity.

When the velocity of a body is changing, the motion is said to be *accelerated*. If the velocity is diminishing we more often say that the motion is retarded, but a retardation may be considered a negative acceleration.

**28. Measuring the Velocity of a Body.** In Fig. 20 is shown a small car or trolley mounted on light wheels which

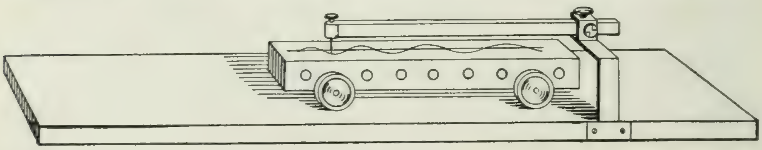


FIG. 20.—Measuring the velocity of the car.

turn with very little friction. It is about 2 feet long and $2\frac{1}{2}$ inches wide. When given a smart push on a level roadway the car runs for a considerable distance with almost uniform

speed. A metal bridge is fastened to the board and a flat steel spring is attached by one end to the bridge. The other end of the spring carries a soft brush which can be filled with ink. A long strip of paper is tacked on the top of the car and upon this the brush traces a record of the motion of the car.

First push the car along under the brush when the spring is at rest. The tracing on the paper is a long straight line. Next start the brush vibrating and give the car a quick push. It moves along approximately uniformly and the tracing on the paper has a wavy form like that shown in Fig. 21.

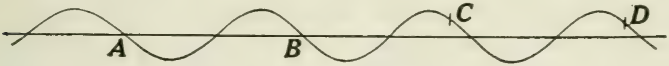



FIG. 21.—Uniform velocity.

(Before pushing the car it is generally best to raise one end of the board slightly so that the car will not stop if started but will not start of itself. In this way allowance is made for the friction unavoidably present).

It is evident that while the car moved through a distance $AB$ or $CD$ the spring made a complete vibration. If, then, we know the period of the spring, that is, the time required for a complete vibration, we can determine the speed of the car. For example, if the period is $\frac{1}{5}$ sec. and $AB$ is 8.4 cm., the velocity (supposed uniform) is 42 cm. per second.

**29. Study of Acceleration.** Next raise one end of the board and allow the car to run down. Sometimes a trigger arrangement allows one to start the car moving and the brush vibrating at the same time, but in what follows the brush and car were started simply by hand.

In Fig. 22 is shown an actual trace obtained when the inclination of the board to the horizon was about 3°. The period of the spring was $\frac{1}{5}$ sec., and it is clear that the car moved through the distances $AB$, $BC$, $CD$ . . . . . . during

successive vibrations. By means of a metre stick it was found that the

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| distances | $AB$ | $AC$ | $AD$ | $AE$ | $AF$ | $AG$ | $AH$ | $AK$ | |
| were | .90 | 3.53 | 7.61 | 13.15 | 20.18 | 28.71 | 38.60 | 49.92 | cm. |

These are shown in column I. From these, by subtraction, we obtain

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| the lengths | $AB$ | $BC$ | $CD$ | $DE$ | $EF$ | $FG$ | $GH$ | $HK$ | |
| to be | .90 | 2.63 | 4.08 | 5.54 | 7.03 | 8.53 | 9.89 | 11.32 | cm. |

These are given in column II.

Now during the first vibration, while the car moves from $A$ to $B$ its velocity is increasing, and since it passes over .90 cm. in $\frac{1}{5}$ sec. its *average* velocity $= .90 \times 5 = 4.50$ cm. per sec. If the velocity of the car is increasing uniformly this average velocity will be precisely the velocity at $a$, the mid-point of the vibration or $\frac{1}{10}$ sec. from the beginning. If the increase in the velocity is not perfectly uniform the velocity at $a$ will be approximately 4.50 cm. per sec.

During the second vibration the car travels from $B$ to $C$, a distance of 2.63 cm., and the average velocity is $2.63 \times 5 = 13.15$ cm. per sec. This may be taken as the velocity at $b$, $\frac{3}{10}$ sec. from the beginning.

Continuing this process for all the vibrations we find the

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| velocities at | $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ | |
| are | 4.50, | 13.15, | 20.40, | 27.70, | 35.15, | 42.65, | 49.45, | 56.60 | cm. per sec. |

These are given in column III.

We find, then, that at $a$ the velocity of the car is 4.50 cm. per sec., and at $b$, $\frac{1}{5}$ sec. later, the velocity is 13.15 cm. per sec. During this fifth of a sec. the increase in the velocity $= 13.15 - 4.50 = 8.65$ cm. per sec. At $c$ the velocity is 20.40 cm. per sec. and the increase in the previous fifth of a second $= 20.40 - 13.15 = 7.25$ cm. per sec. Proceeding in the same way, we obtain the increase in the velocity during the successive fifths of seconds to be

8.65, 7.25, 7.30, 7.45, 7.50, 6.80, 7.15 cm. per sec.

These are given in column IV.

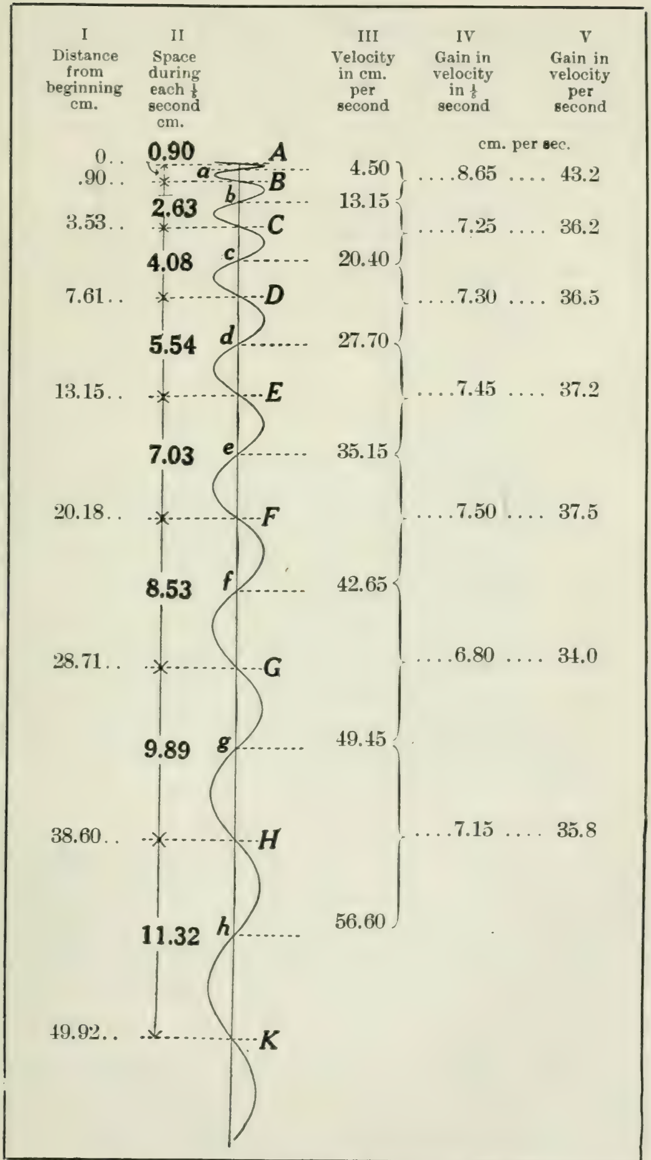



FIG. 22.—Trace of a body moving with uniform acceleration.

The measure of the acceleration is the *rate of change of the velocity*, or the *change of velocity per unit of time.* While the car was passing from $b$ to $c$ (Fig. 22) the change in the velocity was 7.25 cm. per sec. The time taken to gain this was $\frac{1}{5}$ sec., and hence the acceleration was

7.25 cm. per sec. per $\frac{1}{5}$ sec., which is

the same as 36.25 cm. per sec. per sec.

The acceleration as determined from the next $\frac{1}{5}$ sec. was

7.30 cm. per sec. per $\frac{1}{5}$ sec.

or, 36.50 cm. per sec. per sec.

So on for the rest of the values. They are given in columns IV and V.

**30. Was the Acceleration Uniform?** On examining these values we see that the last six are nearly equal, but the first one is somewhat higher. Now make a graph as shown in Fig. 23, in which horizontal distances (abscissas) represent time, and vertical distances (ordinates) represent velocity. The dots $a'$, $b'$, $c'$ . . . . . represent the velocities at $a$, $b$, $c$ . . . . (Fig. 22). Looking along them it is seen that the last six lie very nearly on a straight line. There are two reasons why they are not *exactly* on a straight line:—First, the acceleration may not have been perfectly uniform; second, there are unavoidable errors in all physical measurements.

The first velocity can easily be somewhat in error, since the point $A$ of the curve (Fig. 22) may not represent the precise instant when the car started to move. Indeed, producing the line in Fig. 23 backward past $a'$, it is found to cut the 'time-line' about $\frac{2}{25}$ sec. to the left of 0, and this indicates that the car began to move about $\frac{2}{25}$ sec. before the point $A$ on the curve.

Omitting the first value, which we see is in error, take the average of

7.25, 7.30, 7.45, 7.50, 6.80, 7.15. It is 7.24

We conclude, therefore, that the acceleration was very nearly uniform and that its value was approximately

$$7.24 \text{ cm. per sec. per } \tfrac{1}{5} \text{ sec.,}$$

$$\text{or,} \quad 36.2 \text{ cm. per sec. per sec.}$$

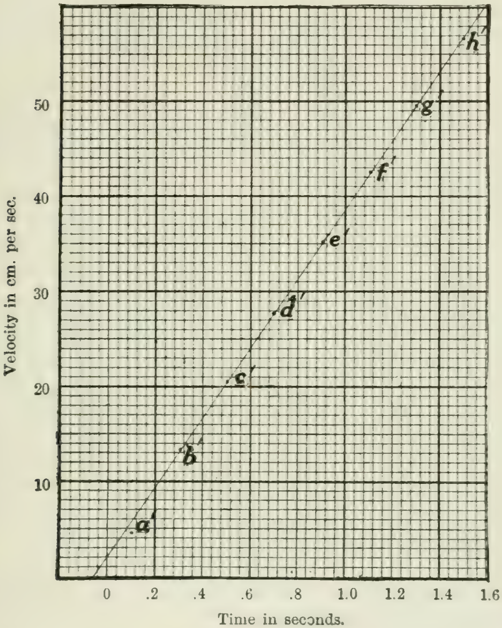



FIG. 23.—Graph showing uniform accelerations.

**31. Further Experiments.** On putting blocks under one end of the plane until its inclination to the horizon was about $4\frac{1}{2}°$ a tracing was obtained on which the following measurements were made:

$AB = 2.29$, $AC = 7.12$, $AD = 14.43$, $AE = 24.29$, $AF = 36.55$, $AG = 51.34$ cm.

From these deduce the approximate velocities at $a$, $b$, $c$ . . . . (Fig. 22) and then the mean acceleration in cm. per sec. per sec. Then from a graph as in Fig. 23 find how long before

the instant represented by $A$ on the curve the car started to move. (Acc. = 62.5 cm. per sec. per sec.; time, .08 sec.).

With an inclination about $6\frac{1}{4}°$ the following values were obtained:

$$AB = 2.92,\ AC = 9.45,\ AD = 19.58,\ AE = 33.22,\ AF = 50.55 \text{ cm.}$$

Make similar use of these values. (Acc. = 90.0; time .06 sec.).

### PROBLEMS

1. A railway train changes its velocity uniformly in 2 minutes from 20 kilometres an hour to 30 kilometres an hour. Find the acceleration in centimetres per second per second.

2. A stone sliding on the ice at the rate of 200 yards per minute is gradually brought to rest in 2 minutes. Find the acceleration in feet and seconds.

3. A point is moving with a uniform acceleration of 10 feet per second per second. (1) What is the total change in velocity in a minute? (2) What is the measure of the acceleration in feet per second per minute?

4. What velocity will a body acquire in half-an-hour if the acceleration is (1) 10 centimetres per second per minute, (2) 10 centimetres per second per second?

5. A point is travelling with an acceleration of 12 feet per second per hour. Find (1) what will be its change in velocity in a minute, (2) the measure of its acceleration in feet per second per second.

6. A train acquires a velocity of 30 feet per second in one hour. If its velocity is uniformly accelerated, find (1) the velocity which it will acquire in one minute, (2) the measure of its acceleration in feet per second per second.

7. A point is travelling with an acceleration of 12 feet per second per hour. How long will it take to acquire a velocity of 2 feet per second?

8. A train, moving with a uniform acceleration, acquires a velocity of 75 feet per second in a quarter of a minute. How long will it take to acquire a velocity of 100 yards per minute?

9. A point, moving with a uniform acceleration, acquires a velocity of 60 feet per second in 10 minutes. What is the measure of its acceleration in (1) feet per second per minute, (2) yards per second per minute, (3) feet per second per second, (4) yards per second per second?

10. A point travelling with a uniform acceleration, has its velocity increased 50 metres per second each minute. What is the measure of the acceleration in (1) metres per second per minute, (2) centimetres per second per minute, (3) metres per second per second, (4) centimetres per second per second ?

11. A train moving with a uniform acceleration, acquires an additional velocity of 60 feet per second each minute. Find (1) the measure of its acceleration in feet per second per second, (2) the measure of the velocity it acquires each minute in feet per minute, (3) the measure of the acceleration in feet per minute per minute, (4) the measure of the acceleration in feet per minute per second.

12. A point is moving with a uniform acceleration and acquires an additional velocity of 20 cm. per second each second. Find the measure of the acceleration in (1) centimetres per second per minute, (2) centimetres per minute per minute, (3) metres per minute per minute, (4) metres per minute per second, (5) metres per second per second.

13. What is the measure of an acceleration of 30 feet per second per second when the units of displacement and of time are respectively (1) the foot and the second, (2) the foot and the minute ?

**32. Space, Acceleration, Velocity, Time.** Let a body move with a uniform acceleration of $a$ cm. per sec. per sec., and suppose that its velocity at a given instant is $u$ cm. per sec.

| | | | | | |
|---|---|---|---|---|---|
| At the beginning, velocity | | | | $v = u$ | cm. per sec. |
| At the end of | 1 | second, | velocity | $v = u + a$ | " " |
| " " | 2 | seconds, | " | $v = u + 2a$ | " " |
| " " | 3 | " | " | $v = u + 3a$ | " " |
| ... ... | ... | ... | ... | ... ... | ... ... |
| and " " | $t$ | " | " | $v = u + ta$ | " " |

Here the gain in velocity in 1 second is $a$ cm. per second; the gain in $t$ seconds is $at$ cm. per second; and the velocity at the end of the $t$ seconds is the original velocity + the gain, *i.e.*,

$$v = u + at \text{ cm. per sec.} \qquad (1)$$

If the initial velocity is zero we have $u = 0$, and

$$v = at \text{ cm. per sec.}$$

Next let us find the space traversed.

*First*, let the initial velocity be zero. In $t$ seconds, with an acceleration $a$ cm. per sec. per sec., the final velocity $= at$ cm. per sec.

The average or mean velocity $= \frac{1}{2}$ (Initial + Final velocity).

$$= \tfrac{1}{2}(0 + at).$$
$$= \tfrac{1}{2}\, at \text{ cm. per sec.}$$

This is the velocity when one-half the time has elapsed.

Now the space passed over

$$= \text{average velocity} \times \text{time};$$

hence, if $s$ represent space,

$$s = \tfrac{1}{2}\, at \times t = \tfrac{1}{2}\, at^2 \text{ cm.}$$

*Next*, let the initial velocity be $u$ cm. per sec. Then we have:

$$\begin{aligned} \text{Initial velocity} &= u \text{ cm. per sec.} \\ \text{Final} \quad " \quad &= u + at \text{ cm. per sec.} \\ \text{Average} \quad " \quad &= \tfrac{1}{2}(u + u + at) \text{ cm. per sec.} \\ &= u + \tfrac{1}{2}\, at \qquad " \quad " \\ \text{Then space} \quad s &= \text{average velocity} \times \text{time} \\ &= (u + \tfrac{1}{2}\, at)\, t = ut + \tfrac{1}{2}\, at^2 \text{ cm.} \qquad (2) \end{aligned}$$

In this expression note that $ut$ expresses the space which would be traversed in time $t$ with a uniform velocity $u$, and $\frac{1}{2}at^2$ is the space passed over when the initial velocity $= 0$. The entire space is then the sum of these.

Again, from (1)

$$t = \frac{v - u}{a},$$

and on substituting this value of $t$ in (2) we obtain

$$v^2 = u^2 + 2as. \qquad (3)$$

**33. Further Study of Motion on an Inclined Plane.** The motion of a ball rolling down an inclined plane may be studied by means of the apparatus shown in Fig. 24. It consists of a board 5 or 6 feet long in which is a circular groove 4 inches wide and having a radius of 4 inches. The surface is

painted black and is made very smooth. Along the middle of the groove is scratched or painted a straight line; and near one end of the board is fastened a strip of brass, accurately at right angles to the length of the groove and extending just to the middle of it. The board must be accurately made to give satisfactory results.

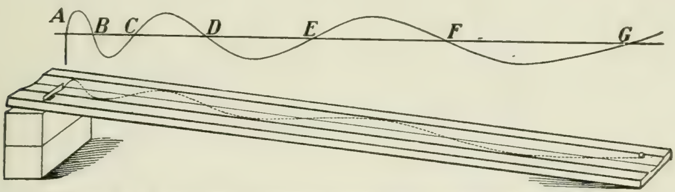



FIG. 24.—Apparatus to illustrate motion with uniform acceleration.

Lay the board flat on the floor, and place a sphere (a steel ball 1 in. to $1\frac{1}{2}$ in. in diameter), at one side of the groove and let it go. It will run back and forth across the hollow, performing oscillations in approximately equal times. By counting a large number of these and taking the average, we can obtain the time of a single one.

Next let one end of the board be raised and over the groove dust (through 4 or 5 thicknesses of muslin) lycopodium powder. Put the ball alongside the brass strip at one side of the groove and let it go. It oscillates across the groove and at the same time rolls down it, and the brass strip insures that it starts downwards without any initial velocity. By blowing the lycopodium powder away a distinct curve is shown like that in the upper part of Fig. 24.

It is evident that while the ball rolls down a distance $AB$ it rolls from the centre line out to the side of the groove and back again; while it rolls from $B$ to $C$, it rolls from the centre line to the other side of the groove and back again. These times are equal; let each be $\tau$ sec. (about $\frac{1}{3}$ sec.). In the same way $CD$, $DE$, $EF$ and $FG$ are each traversed in the same interval. This interval is one-half of a complete oscillation and sometimes it is better to take the spaces traversed during complete oscillations. Such spaces are $AC$, $CE$, $EG$. By laying a metre scale along the middle of the groove the distances $AB$, $AC$, $AD$, ...... may be measured.

The following are measurements obtained with 1 inch and $1\frac{1}{4}$ inch balls, rolling down a board 6 feet long. In the third, fifth

and seventh columns are shown the ratios of $AB$, $AC$, $AD$, $AE$, $AF$, and $AG$ to $AB$, which are referred to below.

| | 1 inch ball. End raised 20 cm. | | $1\frac{1}{4}$ inch ball. End raised 22 cm. | | $1\frac{1}{4}$ inch ball. End raised $22\frac{1}{2}$ cm. | |
|---|---|---|---|---|---|---|
| | cm. | Ratio. | cm. | Ratio. | cm. | Ratio. |
| $AB$........... | 4.55 | 1.0 | 4.40 | 1.0 | 4.45 | 1.0 |
| $AC$........... | 18.80 | 4.1 | 18.35 | 4.2 | 18.65 | 4.2 |
| $AD$........... | 40.40 | 8.9 | 39.50 | 9.0 | 40.25 | 9.0 |
| $AE$........... | 70.28 | 15.4 | 70.90 | 16.1 | 72.95 | 16.4 |
| $AF$........... | 111.90 | 24.6 | 108.45 | 24.6 | 111.00 | 24.9 |
| $AG$........... | 161.30 | 35.4 | 157.10 | 35.7 | 161.00 | 36.2 |

The curves recording the motion in this experiment are quite similar to those obtained with the trolley, and if we know the period of oscillation of the ball we can calculate the acceleration.

**Example**:—From the second column in the table we find the distances $AC = 18.80$, $CE = 51.48$, $EG = 91.02$ cm. Each is traversed in the time of a complete oscillation, which we shall take as $\frac{2}{3}$ sec.

Hence, the average velocity while describing $AC = 18.80 \times \frac{3}{2} = 28.20$ cm. per sec. This is approximately the velocity at point $B$, midway as to time between $A$ and $C$.

Similarly the

approximate velocity at $D$ is $51.48 \times \frac{3}{2} = 77.22$ cm. per sec.
and that at $F$ $= 136.53$ " " "

The increase in velocity in the $\frac{2}{3}$ sec. between $B$ and $D = 49.02$ cm. per sec.
" " " " " " " $D$ and $F = 59.31$ " " "

Average increase in $\frac{2}{3}$ sec. $= 54.17$ " " "

Acceleration or increase in 1 sec. $= \frac{3}{2} \times 54 = 81$ " " "

On comparing the increase in velocity between $B$ and $D$ with that between $D$ and $F$, it is seen that the acceleration varies considerably from uniformity,—more so than in the case of the trolley.

The acceleration in this case can easily be obtained by using the relation $s = \frac{1}{2}at^2$ obtained in the last section.

Here, $s = AG = 161.30$ cm.

$$t = 3 \times \tfrac{2}{3} = 2 \text{ sec.}$$

$$a = \frac{2s}{t^2} = \frac{2 \times 161.30}{4} = 81 \text{ cm. per sec. per sec.}$$

### 34. Space Traversed

Applying the formula $s = \frac{1}{2} at^2$, we have

$$\begin{aligned}
AB &= \tfrac{1}{2} a\tau^2, & &= 1 \times AB, \\
AC &= \tfrac{1}{2} a (2\tau)^2 &= 4 \times \tfrac{1}{2} a\tau^2 &= 4 \times AB, \\
AD &= \tfrac{1}{2} a (3\tau)^2 &= 9 \times \tfrac{1}{2} a\tau^2 &= 9 \times AB, \\
AE &= \tfrac{1}{2} a (4\tau)^2 &= 16 \times \tfrac{1}{2} a\tau^2 &= 16 \times AB, \text{ etc.,}
\end{aligned}$$

*i.e.*, the spaces $AB$, $AC$, $AD$, $AE$, etc., are proportional to 1, 4, 9, 16, etc., or the distance is proportional to the square of the time.

The actual measurements of the spaces are given in the above table, and also the ratios obtained on dividing each space by the first one. These ratios are very close to the theoretical values 1, 4, 9, 16, 25, étc., the discrepancies being due to imperfections in the board and small errors in measurement.

#### PROBLEMS

1. What is the initial velocity of a point which, moving with a uniform acceleration of 10 centimetres per second per second, acquires in 10 seconds a velocity of 200 centimetres per second?

2. A body, moving at a certain instant with a velocity of 30 miles per hour, is subject to a uniform acceleration in the opposite direction, and comes to rest in 11 seconds. What was the measure of its velocity, in feet per second, 5 seconds before it stopped?

3. Find the initial velocity of a point which moves with a uniform acceleration of 20 centimetres per second per second, and acquires a velocity of 15 centimetres per second in 10 seconds. Interpret the result.

4. The velocity of a point increases uniformly in 20 seconds from 100 centimetres per second to 200 centimetres per second. Find (1) the measure of the acceleration in centimetres per second per second, (2) the velocity 3 seconds after it was 150 centimetres per second, (3) when the body was at rest.

5. A point, which has an acceleration of 32 feet per second per second, is moving with a velocity of 10 feet per second. At the same place and at the same time another point, which has an acceleration of 16 feet per second per second, is moving in the same direction with a velocity of 170 feet per second. Find (1) when the two points will have equal velocities, (2) when the velocity of the second will be double that of the first.

6. A body, moving, with a velocity of 5 centimetres per second, has a constant acceleration of 10 centimetres per second per second, in the direction of its motion. Find (1) how far it will go in 10 seconds, (2) how long it will take to go 10 centimetres.

7. A body starts with a velocity of 15 centimetres per second, and has a constant acceleration of 10 centimetres per second per second in the opposite direction. When and where will it come to rest?

8. A body, starting from rest, moves with a uniform acceleration of 20 feet per second per second. Find (1) how far the body goes in 4 seconds, (2) how far it goes in 5 seconds, (3) how far it goes in the 5th second.

9. A body starts with a velocity of 6 feet per second and has a uniform acceleration of 3 feet per second per second in the direction of its motion. At the end of 4 seconds the acceleration ceases. How far does the body move in 10 seconds from the beginning of its motion?

10. With what uniform acceleration does a point, starting from rest, describe 640 feet in 8 seconds?

11. A point, starting from rest and moving with a uniform acceleration, has a displacement of 66 feet in the 6th second. What is the measure of the acceleration in feet per second per second, and what is its displacement in the 7th second?

12. A train, having a velocity of 20 feet per second, attains a velocity of 30 miles per hour in passing over 128 feet. If the train is moving with a uniform acceleration, what is its acceleration?

13 A trolley car, moving at the rate of 24 feet per second, is stopped with a uniformly decreasing motion in a space of 9 feet. What is the acceleration of the car?

14. A particle starts with a velocity of 23 feet per second, and its velocity is uniformly decreased at the rate of 8 feet per second per second. Find how long it will take to describe a distance of 30 feet, and how much longer to come to rest.

**35. Acceleration Due to Gravity.** The most familiar illustration of uniformly accelerated motion is that of a body falling freely, but the velocity is gained so rapidly that it is difficult to measure accurately the time occupied in falling through a measured space. It is necessary to determine a very short interval of time, and a small error in the measurement produces a large error in the result deduced.

A simple form of experiment is illustrated in Fig. 25. *P* is a straight wooden rod about 4 feet long with a hole near one end and it swings on a pin *p* in a block *B* on the wall. A metal ball *b* is attached to a silk thread which passes over a round disc *C* mounted eccentrically.

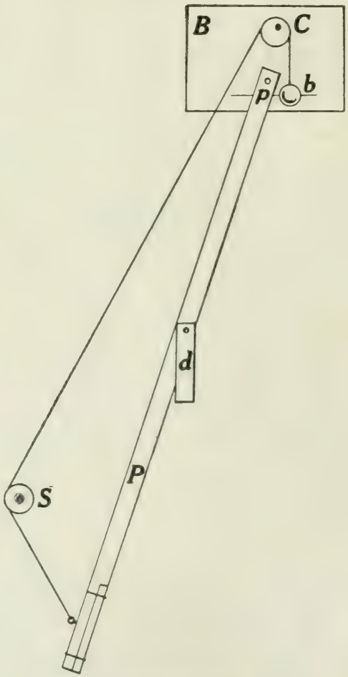



Fig. 25.—Determination of 'g.'

First, let the rod hang vertically, and turn *C* until the ball hangs just clear of the rod near the top. Then lower the ball and test if it hangs just free of the rod near the bottom. By turning a small weight *d* this latter adjustment can be made, since a motion in *d* slightly changes the position of the rod.

Cover about 20 cm. of the right-hand face of the rod with white paper, and by means of rubber bands fasten carbon paper over this with the ink-face inwards. Then pull the rod aside as shown in Fig. 25. By having the thread pass over a movable pin *S* the centre of

the ball may be made to be on the same level as a mark on the block $B$.

Then burn the thread near $C$, thus releasing $b$ and $P$ at the same time. At the moment when the pendulum is rapidly passing through its middle position it will strike the ball and a mark will be made on the white paper. By measuring from the centre of this mark to the mark on $B$ the distance the ball has fallen can be determined.

Next, pull the pendulum aside and count the number of swings in 30 sec., or the time required for any number of swings. Repeat both parts of the experiment and take the average of the values of the distance and the time of swing.*

**Example**:—From a number of counts, 31 complete oscillations of the pendulum = 57 sec., or $\frac{1}{4}$ oscillation = 0.46 sec.

Average of measured distances = 104.3 c.m.

Here the ball falls 104.3 cm. in 0.46 sec.

$$\text{Average velocity} = \frac{104.3}{0.46} = 226.7 \text{ cm. per sec.}$$

This is the velocity at half-time, or 0.23 sec. after the start.

In 0.23 sec. velocity gained = 226.7 cm. per sec.

" 1 " " " = 986 cm. per sec.

This is the measure of the acceleration.

This result might have been obtained somewhat more briefly by using the relation $s = \frac{1}{2}gt^2$, in which $g$ is the acceleration of gravity.

Here, $s = 104.3$ cm.

$t = 0.46$ sec.

$$\text{and} \quad g = \frac{2 \times 104.3}{(0.46)^2} = 986 \text{ cm. per sec. per sec.}$$

**36. Value of 'g.'** When it is desired to determine with accuracy the value of $g$ at any station a special form of pendulum is used. The period† of complete oscillation ($t$) of a

*Fuller instructions for performing this experiment are given in *Elements of Physics*, by E. H. Hall (N.Y., 1912).

† See Edser's "General Physics," pages 90, 94.

pendulum depends upon its length ($l$) and the value of $g$, the relation connecting these quantities being

$$t = 2\pi\sqrt{\frac{l}{g}}.$$

If we can measure $t$ and $l$, we can deduce the value of $g$.

In a number of countries measurements of $g$ have been made at many stations, since in this way the form of the earth can be determined. This work is called a gravimetric survey.

At the equator, $g = 978.1$; at the pole, 983.1; at Washington, 980.1; at Toronto, 980.6.

For middle latitudes the value may be taken as 981; using feet and seconds as units, $g = 32.2$.

### PROBLEMS

Unless otherwise stated, take as the measure of the acceleration of gravity, with centimetres and seconds, 980; with feet and seconds, 32.

1. A body moves 1, 3, 5, 7 feet during the 1st, 2nd, 3rd, 4th seconds, respectively. Find the average speed.

2. Express a speed of 36 kilometres per hour in cm. per second.

3. A body falls freely for 6 seconds. Find the velocity at the end of that time, and the space passed over.

4. The velocity of a body at a certain instant is 40 cm. per sec., and its acceleration is 5 cm. per sec. per sec. What will be its velocity half-a-minute later?

5. What initial speed upwards must be given to a body that it may rise for 4 seconds?

6. The Eiffel Tower is 300 metres high, and the tower of the City Hall, Toronto, is 305 ft. high. How long will a body take to fall from the top of each tower to the earth?

7. On the moon the acceleration of gravity is approximately one-sixth that on earth. If on the moon a body were thrown vertically upwards with a velocity of 90 feet per second, how high would it rise, and how long would it take to return to its point of projection?

8. A body moving with uniform acceleration has a velocity of 10 feet per second. A minute later its velocity is 40 feet per second. What is the acceleration?

9. A body is projected vertically upward with a velocity of 39.2 metres per second. Find

(1) how long it will continue to rise;

(2) how long it will take to rise 34.3 metres;

(3) how high it will rise.

10. A stone is dropped down a deep mine, and one second later another stone is dropped from the same point. How far apart will the two stones be after the first one has been falling 5 seconds?

11. A balloon ascends with a uniform acceleration of 4 feet per second per second. At the end of half-a-minute a body is released from it. How long will it take to reach the ground?

12. A train is moving at the rate of 60 miles an hour. On rounding a curve the engineer sees another train $\frac{1}{4}$ mile away on the track at rest. By putting on all brakes a retardation of 3 feet per second per second is given the train. Will it stop in time to avoid a collision?

13. A body drops vertically from rest. What velocity will it have (1) at the end of 5 seconds, (2) when it has fallen 1600 feet?

14. A body is thrown vertically downward with an initial velocity of 100 feet per second. Find what velocity the body will have (1) at the end of 10 seconds, (2) when it has fallen 900 feet.

15. A body is thrown vertically upward with an initial velocity of 4900 centimetres per second. Find its velocity (1) at the end of 3 seconds, (2) when it has risen 117.6 metres.

16. A body falls from rest for 4 seconds. Find the distance fallen (1) in the four seconds, (2) in the fourth second, (3) when it has a velocity of 100 feet per second.

17. A body is thrown vertically downward with an initial velocity of 1470 centimetres per second. Find the distance traversed in the fourth second.

18. A body is thrown vertically upward with an initial velocity of 100 feet per second. Find the height to which it will rise.

19. A body is projected vertically upward with an initial velocity of 160 feet per second. Find the distance traversed (1) in 5 seconds, (2) in the 5th second.

20. A body is thrown vertically upward with an initial velocity of 50 feet per second. What is its height when its velocity is 30 feet per second?

21. A stone is thrown vertically into the shaft of a mine with a velocity of 5 4 metres per second, and reaches the bottom in 4 seconds. What is the depth of the mine?

22. A particle is projected vertically upward with a velocity of 96 feet per second. In what time (1) will its velocity be 48 feet per second, (2) will its displacement be 144 feet?

23. A body drops vertically from rest. Find (1) when its velocity is 2450 centimetres per second, (2) when the body is 99.225 metres from the point from which it dropped.

24. A stone is projected vertically downward with a velocity of 100 feet per second. Find (1) when its velocity is 292 feet per second, (2) when it is 900 feet from the point of projection.

25. With what velocity must a body be thrown vertically upward (1) that it might rise for 3 seconds, (2) that it may have a velocity of 30 feet per second at the end of the 3rd second, (3) that it may rise 100 feet?

26. With what velocity must a body be thrown vertically downward (1) that it may have a velocity of 100 feet per second at the end of the 2nd second, (2) that it may describe 204 feet in 3 seconds?

27. A body, thrown vertically upward, passes a point 173 feet from the point of projection with a velocity of 50 feet per second. How much further will it go, and what was the velocity with which it was projected?

# CHAPTER V

## Momentum

**37. Mass and Velocity.** A lead bullet has small mass (about half-an-ounce) and if thrown against a wooden wall it will do little or no damage; but if fired from a modern rifle, with a speed of 2000 feet per second, it would pass through several feet of wood and can do great damage. A small mass when combined with a great velocity can produce a great result.

Sometimes a large vessel on entering a lock of a canal fails to stop, and though its speed may be quite small (no greater than a walk) it breaks through the strong gates of the lock. A great mass, even though moving with a small velocity, can produce a great effect.

When a person wishes to drive a nail he does not choose a light wooden stick for the purpose but takes a hammer with massive steel head, and if the nail is a large one he gives to the hammer a great velocity.

In ancient times the walls of fortifications were broken down by means of the battering-ram. This was a long heavy log suspended in a horizontal position by ropes and made to swing back and forth in a lengthwise direction. When vigorously operated by a large number of men even the strongest wall could not stand against the continued blows of the end of the log.

In each of the above illustrations it was the combination of mass with velocity which produced the result named, and of course the greatest effect is obtained when a great mass is moving with a great velocity.

Now the word *momentum* is used to express the combination of mass with velocity. Thus, by definition,

$$\text{Momentum} = \text{Mass} \times \text{Velocity}$$
$$\text{or } M = mv.$$

**38. Force and Time.** It is pleasant to spend some weeks during the summer beside the water. Often some friends will come to call upon you, and you go down to the landing to bid them good bye as they are leaving in a row-boat. Suppose the boat and its occupants to weigh 480 pounds. Taking hold of the end of the boat you push with a force of 25 pounds (that is, the force required to lift a 25 lb. mass) for 3 seconds and give the boat a velocity of 5 ft. per sec. Had you pushed for only 1 sec. the velocity given of course would have been $\frac{1}{3}$ of $5 = 1\frac{2}{3}$ ft. per sec. This is the gain of velocity in unit of time, or the acceleration.

On another day you push on a motor-boat, the mass of which is 2400 lb. (5 times as great as the row-boat). You apply the same force but the boat moves much more slowly. How long will it require to give it the same velocity? You find that it takes 15 sec., just 5 times as long.

It is evident that the momentum developed is proportional to the magnitude of the force applied and to the length of time it is applied.

We can say then that the effect produced is proportional to the force combined with the time, or

$$\text{Force} \times \text{time} = \text{Mass} \times \text{velocity},$$

$$\text{or} \qquad Ft = mv.$$

The quantity $Ft$, or the entire effect of the force, is sometimes called the *impulse.*

This can be written thus,

$$F = m \times \frac{v}{t}$$
$$= ma,$$

where $a$ is the acceleration produced.

$$\text{Also, } \frac{mv}{t} = ma = \text{rate of change of momentum.}$$

The magnitude of a force is measured by the rate at which it changes momentum.

**39. Newton's Second Law.** Arrange the trolley as in Fig. 26. Before attaching the light cord, however, raise the left-hand end of the board slightly so that the car will just not

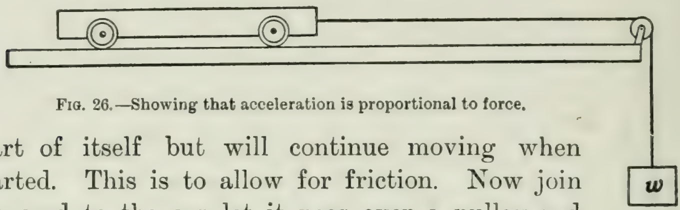



FIG. 26.—Showing that acceleration is proportional to force.

start of itself but will continue moving when started. This is to allow for friction. Now join the cord to the car, let it pass over a pulley and attach a small weight $w$ to it. This weight will keep a constant tension in the cord during the motion of the car and it should move with uniform acceleration. Use different weights for $w$ and obtain tracings with each. The experiments must be made with great care. The masses of the wheels and the pulley should be as small as possible, and it is difficult to get rid entirely of friction.

With a car of mass 1250 grams and $w = 20$ grams the tracing shown in Fig. 27$a$ was obtained; with $w = 40$ grams that in $b$ was obtained.

By measuring the curves as explained in Sec. 29.

In first case, acceleration = 16 cm. per sec.
" second " " = 31 " " "

The second acceleration is approximately twice as great as the first. The force producing the acceleration is the attraction of the earth on $w$, and we see that the *acceleration is proportional to the force.*

On using a car of mass 2500 grams the acceleration observed is just one-half as great as before, and the product $ma$ remains the same with the same force.

Now we have just seen that $ma$ is the rate of change of momentum; and we, therefore, conclude that:

*Change of momentum is proportional to the applied force and takes place in the direction in which the force acts.*

This is NEWTON'S SECOND LAW OF MOTION, though the words he uses are not quite the same. He speaks of "change of motion," where we say "change of momentum," but by 'motion' he means what we understand by 'momentum.'

**40. Measurement of Force.** We are now in a position to discuss more fully the units used in measuring a force. We start from the fact that the magnitude of a force is to be determined from the amount of momentum which it can produce (or destroy) in unit of time.

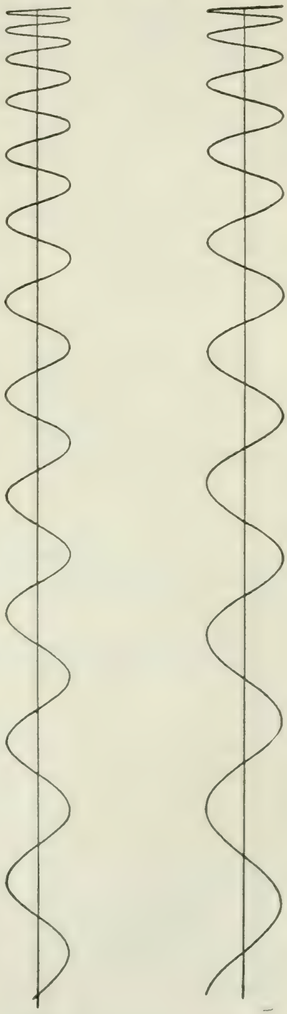
**41. Absolute Units of Force.** In the metric (or C. G. S.) system the units of length, mass and time are 1 cm., 1 gram, and 1 sec., respectively.

Unit velocity = 1 cm. per sec.
Unit of momentum = 1 gram mass × 1 cm. per sec.
Unit acceleration = 1 cm. per sec. per sec.

Unit force is that force which generates unit momentum in 1 sec., or which gives to unit mass unit acceleration.

It is called a *dyne*.

$F$ (in dynes) = $m$ (in grams) × $a$ (in cm. per sec. per sec.).



(a) (b)

FIG. 27.—Actual tracings with forces 20 grams-wt. and 40 grams-wt.

Remember that

1 dyne-force acting on 1 gm.-mass for 1 sec. gives it a vel. of 1 cm. per sec.

In the English (or F. P. S.) system the units of length, mass and time are 1 foot, 1 pound, and 1 sec., respectively.

Unit force is that force which acting on 1 lb. mass for 1 sec. will give it a velocity of 1 ft. per sec.; or that force which gives to 1 lb. mass an acceleration of 1 ft. per sec. per sec.

Such a force is called a *poundal* (pdl.)
As before, remember that

1 pdl.-force acting on 1 lb. mass for 1 sec. gives it a vel. of 1 ft. per sec.

$$F \text{ (in pdl.)} = m \text{ (in lb.)} \times a \text{ (in ft. per sec. per sec.)}.$$

These units are called 'absolute' because they do not depend on any particular place on the earth, or indeed, in the universe. Should one be transported to the moon he could use dynes and poundals as units of force just as we do on the earth.

### PROBLEMS

1. Two masses, $3m$ and $5m$, are acted on by forces which produce in their motions accelerations of 7 and 9 respectively. Compare the magnitude of the forces.

2. A force acts on a mass of $m$ grams. Compare the acceleration with that produced by the same force acting on a mass of (1) $am$ grams, (2) $\frac{m}{a}$ grams.

3. A force is capable of producing in a certain mass an acceleration of $f$ cm. per sec. per sec. and in another mass an acceleration of $af$ cm. per sec. per sec. Compare the masses.

4. What force expressed in poundals will give a mass of 25 pounds an acceleration of 25 feet per second per second ?

5. A force of 225 poundals gives a certain mass an acceleration of 15 feet per second per second. Find the measure of the mass.

6. Find the magnitude of the force expressed in dynes in each of the following cases :

(1) The force which will produce in a mass of 20 grams an acceleration of 10 cm. per sec. per sec.

(2) The force which will produce in a mass of 5 kg. an acceleration of 5 cm. per sec. per sec.

(3) The force which will produce in a mass of 30 grams an acceleration of 10 metres per sec. per sec.

(4) The force which will produce in a mass of 10 kg. an acceleration of 20 cm. per min. per min.

(5) The force which acting on a mass of 3 grams for 12 seconds will impart to it a velocity of 120 cm. per sec.

(6) The force which acting on a mass of $a$ grams for $t$ seconds will impart to it a velocity of $v$ cm. per sec.

7. Find the acceleration expressed in cm. per sec. per sec. in each of the following cases :

(1) A force of 10 dynes acts on a mass of 10 grams.

(2) A force of 15 dynes acts on a mass of 5 kg.

(3) A force of 9800 dynes acts on a mass of 5 grams.

8. Find the mass of the body acted upon by the force in each of the following cases :

(1) A force of 5 dynes produces in a body an acceleration of 10 cm. per sec. per sec.

(2) A force of 10 dynes acting for 5 seconds imparts to a body a velocity of 20 cm. per second.

(3) A force of 30 dynes produces in a body an acceleration of 5 metres per min. per min.

(4) A force of 1,960,000 dynes acting for 2 minutes imparts to a body a velocity of 60 cm. per sec.

9. A mass of 400 grams is acted on by a force of 2000 dynes. Find the acceleration. If it starts from rest, find, at the end of 5 sec., (1) the velocity generated, (2) the momentum.

10. A force of 10 dynes acts on a body for 1 min., and produces a velocity of 120 cm. per sec. Find the mass, and the acceleration.

11. Find the force which in 5 sec. will change the velocity of a mass of 20 grams from 30 cm. per sec. to 80 cm. per sec.

12. A force of 50 poundals acts on a mass of 10 lb. for 15 sec. Find the velocity produced, the acceleration and the momentum.

**42. Gravitation Units of Force.** However, we live on the earth and the force with which we are best acquainted is the force of gravity. It is quite natural, therefore, that there should have arisen a unit of force which depends upon gravity. In the metric system, as was stated in Sec. 23, the unit is the *gram-force,* and it is defined thus:

1 gram-force = the attraction of the earth on 1 gram-mass;

or, it is the *weight* of 1 gram-mass.

Thus a gram-force and a gram-mass are quantities of entirely different kinds. A gram-mass is a certain quantity of matter, which will remain the same wherever it may be taken; while a gram-force varies with one's position on the earth's surface and would change entirely if one should go off into space.

We can compare a dyne with a gram-force in the following way:

Allow a gram-mass to fall freely. At the end of 1 sec. its velocity $= g$ cm. per sec., and its momentum $= 1 \times g = g$ units. Thus,

1 gm.-force acting on 1 gm.-mass for 1 sec. generates $g$ units of momentum.
But 1 dyne " " 1 " " 1 " " 1 " " "

Hence, 1 gm.-force $= g$ dynes.

$g = 981$, and hence 1 dyne $= \frac{1}{981}$ of the weight of 1 gram of matter.

In the English system the gravitation unit of force is the earth's attraction on 1 pound-mass, or the *weight* of 1 pound-mass. Thus 1 pound-mass differs in nature from 1 pound-force, and it is convenient to write 1 pound as "1 lb." when referring to mass, and "1 pd." when referring to force or weight.

If 1 lb. mass is allowed to fall freely, in 1 sec. it gains a velocity of $g$ ft. per sec., and its momentum $= g$ units.

We can then say,

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 pd.-force | acting on | 1 lb. mass | for | 1 sec. | generates | $g$ units | of momentum. |
| But 1 pdl. | " " | 1 " | " | 1 " | " | 1 " | " " |

Hence, 1 pd.-force = $g$ poundals.

Here, $g$ = 32 (approx.), and 1 pdl. = $\frac{1}{32}$ of wt. of 1 lb. mass (approx.)
= wt. of $\frac{1}{2}$ oz. (approx.)

**43. Examples.** (i) Calculate the acceleration of the car under the circumstances given in Sec. 39 above.

Here, force producing the motion = weight of $w$.

Mass put in motion = mass of car + mass of $w$.

($a$) Take $w$ = 20 grams; total mass moved = 1270 grams.

By definition of gravitation unit of force,

| | | | | | |
|---|---|---|---|---|---|
| 1 gm.-force acting on | 1 gm.-mass | for 1 sec. | generates a vel. of | 981 | cm. per sec. |
| 1 " " " " | 1270 " " " | 1 " | " " " | $\frac{981}{1270}$ | " " " |
| 20 " " " " | 1270 " " " | 1 " | " " " | $\frac{981 \times 20}{1270}$ | " " " |
| | | | Acceleration = | 15.4 | " " " |

More briefly,

$F$ (in dynes) = mass (in grams) × $a$ (in cm. per sec. per sec.)

In the present case, $F = 20 \times 981$ dynes.

$m = 1270$ grams.

From which, $a = 15.4$ cm. per sec. per sec.

($b$) Take $w$ = 40 grams; whole mass moved = 1290 grams.

In this case, $F = 40 \times 981$ dynes.

$m = 1290$ grams.

and, $a$ = 30.4 cm. per sec. sec. per sec.

These do not differ greatly from the values obtained by experiment.

(ii) Two equal masses $M$, $M$ (Fig. 28) hang by a light cord over a pulley. A mass $w$ is laid on one: find the acceleration of $M$, $M$.

In this case,

Force producing the motion = $w$ grams force,
= $wg$ dynes;

Total mass moved = $w + 2M$ grams.

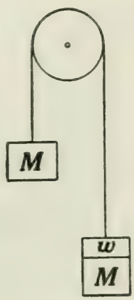


Fig. 28.

Reasoning from definition,

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 gram-force acting on | 1 gm.-mass | for 1 sec. | gives it a vel. of | $g$ cm. | per sec. | per sec. |
| $w$ " " " | 1 | " 1 | " " " | $wg$ | " " | " |
| $w$ " " " | $w+2M$ | " 1 | " " " | $\frac{wg}{w+2M}$ | " " | " |

More briefly, from formula,

$$F = ma,$$

$$\text{or,} \qquad wg = (w + 2M)\, a,$$

$$\text{or,} \qquad a = \frac{wg}{w+2M}.$$

### PROBLEMS

1. Express:

    (1) A force of 10 kg. in dynes.

    (2) A force of 10 dynes in grams force.

    (3) A force of 12 pounds in poundals.

    (4) A force of 320 poundals in pounds.

2. A mass of 25 pounds lies on a table. Find the force it exerts on the table in ($a$) pounds, ($b$) poundals.

3. A mass of 5 kg. is acted on by a force which imparts to it an acceleration of 100 cm. per sec. per sec. Find the force in ($a$) dynes, ($b$) grams.

4. A certain force acts on a mass of 150 grams for 10 seconds, and produces in it a velocity of 50 metres per second. Compare the force with the weight of a gram.

5. A certain force acts on a mass $m$ and generates in it an acceleration $a$. Find the mass which the force would support.

6. How long must a force of 5 units act upon a body in order to give it a momentum of 3000 units?

7. What force acting for one minute upon a body whose mass is 50 grams will give it a momentum of 2250 units?

8. A force of 980 dynes acts vertically upward upon a mass of 5 grams, at a place where $g = 981$ cm. per sec. per sec. Find the acceleration of the body.

9. A mass of 10 kg. is acted upon for one minute by a force which can support a mass of 125 grams. Find the momentum which it will acquire.

10. A falling weight of 160 grams is connected by a string to a mass of 1800 grams lying on a smooth flat table. Find the acceleration and the tension of the string.

11. A mass of 3 kg. is drawn along a smooth horizontal table by a mass of 4 kg. hanging vertically. Find the displacement in 3 seconds from rest.

12. A body of mass 9 grams is placed on a smooth table at a distance of 16 cm. from its edge, and is connected by a string passing over a pulley at the edge with a body of mass 1 gram. Find (1) the time that elapses before the body reaches the edge of the table, (2) its velocity on leaving the table.

13. A mass of 10 grams hanging freely draws a mass of 60 grams along a smooth table. Find (1) the displacement in 5 seconds, (2) the displacement in the 8th second, and (3) the velocity acquired between the 7th and the 12th seconds.

14. Two masses of 100 and 120 grams are attached to the extremities of a string passing over a smooth pulley. If the value of $g$ is 975 cm. per sec. per second, find the velocity after 8 seconds.

15. A mass of 52 grams is drawn along a table by a mass of 4 grams hanging vertically. If at the end of 4 seconds the string breaks, find the space described by each body in 4 seconds more.

16. Masses of 800 and 180 grams are connected by a string over a smooth pulley. Find the space described in (1) 5 seconds, (2) the 5th second.

17. To the ends of a light string passing over a small smooth pulley are attached masses of 977 grams and $x$ grams. Find $x$ so that the former mass may rise through 200 cm. in 10 seconds. ($g = 981$).

18. If bodies whose masses are $m_1$ and $m_2$ are connected by a string over a smooth pulley, find the ratio of $m_1$ to $m_2$ if the acceleration is $\frac{1}{4}$ $g$.

**44. Forces Acting Simultaneously on a Body.** Suppose a person to be at the top of a tower 64 ft. high. If he drops a stone it will fall vertically downwards and will reach the ground in 2 sec. If now it is thrown outwards in a horizontal direction, will it reach the ground as quickly?

In this case a force acting in a horizontal direction gives the body a velocity in that direction, and the question is, will that force in any way change the action of the force of

gravity? Can the horizontal velocity change in any way the vertical velocity?

The best way to answer this question is to test it by experiment. Many pieces of apparatus have been devised for this purpose, one of the simplest being shown in Fig. 29.

*A* and *B* are two upright supports through which a rod *R* can slide. *S* is a spring so arranged that when *R* is pulled back and let go it flies to the right. *D* is a metal sphere through which a hole is bored to allow it to slip over the end of *R*. *C* is another sphere, at the same height above the floor as *D*.

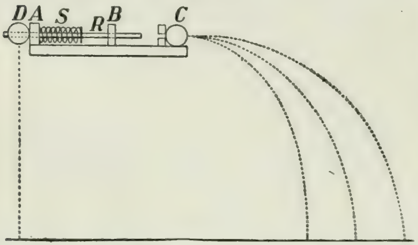


FIG. 29.—The ball *C*, following a curved path, reaches the floor at the same time as *D* which falls vertically.

The rod *R* is just so long that when it strikes *C*, the sphere *D* is set free. Thus *C* is projected horizontally outwards while *D* drops directly down.

By pulling *R* back to different distances, different velocities can be given to *C*, and thus different paths described, as shown in the figure.

It will be found that, no matter which of the curved paths *C* takes, it will reach the floor the same time as *D*.

#### PROBLEMS

1. From a window 16 ft. above the ground a ball is thrown in a horizontal direction with a velocity of 50 ft. per sec. Where will it strike the ground?

2. A cannon is discharged in a horizontal direction over a lake from the top of a cliff 19.6 m. above the water, and the ball strikes the water 2500 m. from the shore. Find the velocity of the ball outwards, supposing it to be uniform over the entire range.

3. An aeronaut when 2112 ft. above the earth's surface and rising vertically at the rate of 16 ft. per sec., throws an object in a horizontal direction with a velocity of 40 ft. per sec. How long will it take to reach the earth and where will it strike?

**45. Pressure Produced by a Fluid in Motion.** A wind is simply a portion of the atmosphere in motion and when it strikes a surface a pressure is produced upon it,—sometimes sufficient to do great damage, as shown by the destruction caused by a tornado. The pressure from a current of air can cause a windmill to rotate and thus pump water or grind grain, while that from a current of water can put in motion great turbine water-wheels, sometimes with the power of several thousand horses. A stream of water from a fire-hose can break a window, tear up shingles or do other things. Whole hills have been removed by directing streams of water against them, thus loosening the earth and then carrying it away.

Consider a cube of matter 1 cm. to the edge and having mass 1 gram. If a pressure of 1 dyne act at right angles to one face of this for 1 sec. it will be given a velocity of 1 cm. per sec., and it will possess 1 unit of momentum.

If this matter were water the dyne force would give it unit of momentum as before.

Next imagine 1 c.c. of water to be projected perpendicularly against 1 sq. cm. of a surface with such a speed that its momentum is destroyed in 1 sec. Then it will exert upon the surface a pressure of 1 dyne per sq. cm.

Of course it is impossible actually to perform this experiment on account of the attraction of the earth pulling the water downwards, but one can easily conceive of the action.

Suppose that a stream of water, moving with a speed of 3 metres per sec., falls at right angles upon a wall, and suppose further that the water simply falls downwards without rebounding from the wall.

In this case a cylinder of water 1 sq. cm. in section and 300 cm. long falls upon 1 sq. cm. in 1 sec. The volume of this cylinder = 300 c.c. and its mass = 300 grams. As its velocity = 300 cm. per sec., its

$$\text{Momentum} = 300 \times 300 = 90{,}000 \text{ units},$$

and this is destroyed in 1 sec.

The pressure on the sq. cm. must therefore

$$= 90{,}000 \text{ dynes} = 92 \text{ gms.-wt. (nearly)}.$$

**Examples**:—(1) A stream of water moving with a speed of 20 metres per sec. strikes a wall at right angles. Find the pressure.

Vol. of water falling on 1 sq. cm. in 1 sec. = 2000 c.c. = 2000 grams.

Its velocity = 2000 cm. per sec.

Hence, its momentum = 2000 × 2000 = 4,000,000 units, and this is destroyed in 1 sec.

$$\begin{aligned} \text{Now } Ft &= mv \\ \text{and } t = 1,\ mv &= 4{,}000{,}000. \\ \text{Hence, } F &= 4{,}000{,}000 \text{ dynes per sq. cm.} \\ &= 4077 \text{ gm.-wt. } \text{,, } \text{,, (approx.)} \end{aligned}$$

(2) A hose delivers 600 gallons per minute with a speed of 60 ft. per sec. against a wall which it strikes at right angles. Find the total pressure against the wall.

$$\begin{aligned} 600 \text{ gal. per min.} &= 10 \text{ gal. per sec.} \\ &= 100 \text{ lb. ,, ,,} \end{aligned}$$

$$\begin{aligned} \text{Mass of water striking wall in 1 sec.} &= 100 \text{ lb.} \\ \text{Its velocity} &= 60 \text{ ft. per sec.} \\ \text{Momentum destroyed in 1 sec.} &= 100 \times 60 \\ &= 6000 \text{ units.} \\ \text{Hence total pressure} &= 6000 \text{ poundals} \\ &= 187.5 \text{ pd.} \end{aligned}$$

**46. Pressure by an Air Current.** Similarly pressures are produced by currents of air, but as the density of air is much less than that of water the pressures are ordinarily much smaller.

**Examples**:—(1) Consider a wind of 48 km. (30 ml.) per hour and find its pressure on the wall of a building which it strikes at right angles.

The air in a cylinder 1 sq. cm. in section and of length

$$\frac{48 \times 1000 \times 100}{60 \times 60} = 1333\tfrac{1}{3} \text{ cm.}$$

will strike on 1 sq. cm. of the wall each sec.

$$\begin{aligned} \text{The volume of this} &= 1333\tfrac{1}{3} \text{ c.c.} \\ \text{and mass} &= 1333\tfrac{1}{3} \times .001293 \text{ grams.} \end{aligned}$$

The momentum $= 1333\frac{1}{3} \times .001293 \times 1333\frac{1}{3}$ units
$= 2298\frac{2}{3}$ units

Hence, the pressure $= 2298\frac{2}{3}$ dynes per sq. cm.
$= 2.34$ gm.-wt. " "

If this wind had been upon the stern of a steamship going 16 km. per hr. the pressure would have been equal to that of a wind of 32 km. per hr. upon a body at rest, and if the wind was directly against the ship's bow a pressure would have been exerted equal to that of a wind of 64 km. per hr. against a body at rest.

(2) A wind of 15 ml. per hr. is directly upon the stern of a ship sailing 6 ml. per hour. Find the pressure.

This is equivalent to a wind of 9 ml. per hr. upon a body at rest, and 9 ml. per hr. = 13.2 ft. per second.

Hence, the pressure is produced by a cylinder of air 1 sq. ft. in section and 13.2 ft. long falling upon 1 sq. foot each sec.

Mass of 13.2 cu. ft. $= 13.2 \times .08$ lb.
$= 1.056$ "

Consequently the pressure $= 1.056 \times 13.2 = 13.94$ pdl. per sq. ft.
$= .436$ pd. per sq. ft.

On an area of 120 sq. ft. (a sailing dinghy)
pressure = 52.32 pd.

**47. General Formula.** It will be useful for us to obtain the general formula for the pressure produced by a stream striking a surface at right angles.

Let the velocity of the stream = $v$ cm. per sec. Then a cylinder of fluid $v$ cm. long and 1 sq. cm. in cross-section will fall upon 1 sq. cm. of the surface in 1 sec. Let $\rho$ = density of the fluid.

Mass of fluid $= \rho v$ grams,
Velocity $= v$ cm. per sec.

Hence, momentum $= \rho v^2$ units, which is destroyed in 1 sec.

Consequently $Ft = \rho v^2$
and as $t = 1$, $F = \rho v^2$ dynes per sq. cm.

In the F.P.S. system, $F = \rho v^2$ pdl. per sq. ft.

It is to be observed that the pressure is not proportional to the velocity, but to its square.

### PROBLEMS

1. A jet of water, 1 sq. cm. in section and moving with a velocity of 20 m. per sec., strikes a board at right angles. Find the total force against the board.

2. A wind with a velocity of 40 ml. per hr. strikes at right angles a sign 30 ft. long and 6 ft. high. Find the pressure per sq. ft. and the total force against the sign.

3. A fire-hose delivers 400 gal. of water per minute at a speed of 20 ft. per sec. The water strikes upon an area of 4 sq. inches. Find the pressure per sq. inch.

# CHAPTER VI

## Action and Reaction

**48. Action and Reaction Equal.** On pressing together the thumb and a finger the force exerted by the thumb upon the finger is obviously equal to that exerted by the finger upon the thumb; or the *action* of the thumb upon the finger is equal to the *reaction* of the finger upon the thumb.

Again, consider a small vessel which is experiencing some difficulty in coming up to a dock. Someone in the vessel throws a rope to a person standing on the dock. He pulls steadily upon it and slowly the boat is brought in to the dock. In this case by exerting muscular effort a tension is produced in the rope, and it is evident that this pulls the boat in one direction and with an equal force pulls the man in the opposite direction. The action of the man upon the boat is equal and opposite to the reaction of the boat upon the man.

Someone may ask, if such is the case why does not the man move towards the boat just as the boat moves towards the man? The answer to this becomes clear if we consider the man and the boat separately.

The forces acting upon the boat are:

(i) the pull of the rope in one direction, and

(ii) the friction of the water in the opposite direction.

The former is greater than the latter and so the boat moves forward.

The forces acting upon the man are:

(i) the pull of the rope in one direction, and

(ii) the friction of the dock on which he stands in the opposite direction.

The pull of the rope is not sufficient to overcome the friction of the dock and so he remains where he is.

But if two boats are floating on still water and a line is thrown from one to a person in the other, when he pulls on it each boat will move towards the other. In this case the friction of the water is not sufficient to balance the pull of the rope in the opposite direction and both bodies move.

A magnet attracts a piece of iron; does the iron attract the magnet? Lay the magnet on one piece of wood and the iron on another and float them on the surface of water. There is no doubt about what happens, each moves towards the other.

We are thus led to conclude that,

*Reaction is always equal and opposite to action;*

or in other words,

*The actions of two bodies upon each other are always equal and in opposite directions.*

This is Newton's THIRD LAW OF MOTION.

**49. Conservation of Momentum.** Suspend two exactly similar ivory or steel balls, $A$ and $B$, side by side, as in Fig. 30. Draw $A$ aside to $C$ and let it go. Its velocity continually increases until it strikes $B$, when it suddenly comes to rest while $B$ starts off.

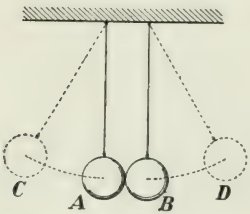


FIG. 30.—The action of $A$ on $B$ is equal to the reaction of $B$ on $A$.

Repeat the experiment and observe closely the distance through which $B$ swings. It will be found to move to $D$, approximately as far from $B$ as $C$ is from $A$. From this we conclude that $B$ starts off with approximately the velocity which $A$ has when it strikes $B$. The momentum possessed by $A$ is thus transferred to $B$.

The action of $A$ consists in exerting a force upon $B$ which gives to $B$ a certain velocity, that is, produces a certain momentum. The reaction of $B$ consists in exerting on $A$ an equal force in the opposite direction. As the balls are exactly similar this force gives to $A$ an equal backward velocity which brings it to rest.

In this particular case the momentum of $A$ is handed on, practically without loss, to $B$; so that in the impact of $A$ and $B$ there is no change in the amount of momentum possessed by the two bodies in the horizontal direction from left to right.

If the balls are of wood the transfer of momentum is not complete. $A$ loses some but not all of its momentum, and $B$ gains the amount that $A$ loses. If made of wax or putty they may stick together, but in every case what momentum $A$ loses $B$ gains.

Next, try an experiment with two trolleys (Fig. 31). First raise one end of the track until the cars will just run down

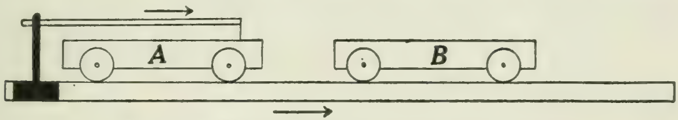



FIG. 31.—Experiment with two cars and a single tracing.

with uniform velocity when they are started. This is to allow for friction.

Place $B$ at rest and arrange a vibrating brush so that it will write a tracing upon $A$ just before $A$ strikes $B$ and for some time afterwards. Also arrange that when $A$ strikes $B$ they will be automatically coupled together and so must move off with the same velocity.

Find the masses of $A$ and $B$ by weighing them and from the tracing find the velocity of $A$ before impact and of the two combined after impact.

Let $m_1$ = mass of $A$,
$m_2$ = mass of $B$,
$u$ = velocity of $A$ before impact,
$v$ = velocity of $A$ and $B$ after impact.
Then momentum before impact $= m_1u$,
and " after " $= (m_1 + m_2)v$

Now the *action* of $A$ on $B$ is to give to $B$ a momentum in the horizontal direction from left to right, and the *reaction* of $B$ on $A$ reduces the momentum of $A$ by the same amount, and so the entire momentum in the direction named will be the same after and before impact, or $$m_1u = (m_1 + m_2)v.$$

**Example**:—In Fig. 32 is shown a tracing obtained with two cars, each having a mass of 1.12 kg. The long waves of the tracing give

FIG. 32.—Tracing giving velocity before and after impact.

the velocity before impact. The wave-length comes out 10.0 cm., which is the distance travelled in $\frac{1}{5}$ sec., or the velocity was 50.0 cm. per sec.

The short waves give a velocity after impact of $4.9 \times 5 = 24.5$ cm. per sec., and the mass moving at this velocity = 2.24 kg.

Hence, before impact momentum $= 1.12 \times 50.0 = 56.0$ units,
after " " $= 2.24 \times 24.5 = 54.9$ "
These are approximately equal.

Again, try this experiment but do not use the automatic coupling. A brush for each car will be required as in Fig. 33.

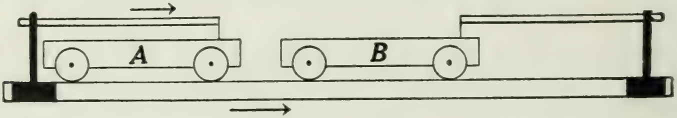



FIG. 33.—Experiment with two cars and two tracings.

Have $B$ at rest, start $A$ with a smart push, and at the same time start both brushes vibrating. There will be a tracing on $A$ showing its velocity before and after impact, and one on $B$ showing its velocity after impact.

Let $m_1$ = mass of $A$,

$u_1$ and $u_2$ = its velocities before and after impact,

$m_2$ = mass of $B$,

$v$ = its velocity after impact.

Then by the impact $A$ loses $m_1(u_1 - u_2)$ units of momentum and $B$ gains $m_2 v$ units.

According to the Third Law these quantities should be equal.

**Example** :—The following are the results of an experiment :

Two cars, each = 1.12 kg.

| | | | |
|---|---|---|---|
| Velocity of $A$ before impact | = | $10.7 \times 5$ = 53.5 | cm. per sec. |
| " " $A$ after " | = | $2.8 \times 5$ = 1.40 | " " " |
| " " $B$ " " | = | $8.1 \times 5$ = 40.5 | " " " |

Momentum lost by $A$ = 1.12 (53.5 − 14.0) = 44.2 units
" gained " $B$ = $1.12 \times 40.5$ = 45.4 "

These quantities are approximately equal.

The experiments with the trolleys can be varied in many ways, by loading with different masses and giving different velocities. If care is taken to make allowance for friction it will be found in every case that *the momentum lost by one body is equal to that gained by the other*; or, *the total amount of momentum is constant.*

This is known as the principle of the CONSERVATION OF MOMENTUM.

### PROBLEMS AND QUESTIONS

1. If the sphere $B$ (Fig. 30) has a mass twice as great as $A$, what will happen (1) when $A$ and $B$ are of ivory ? (2) when they are of sticky putty ?

2. A hollow iron sphere is filled with gunpowder and exploded. It bursts into two parts, one part being one quarter of the whole. Find the relative velocities of the fragments.

3. A rifle weighs 8 lb. and a bullet weighing 1 oz. leaves it with a velocity of 1500 ft. per sec. Find the velocity with which the rifle recoils.

4. Sometimes in putting a handle in an axe or a hammer it is accomplished by striking on the end of the handle. Explain how the law of inertia applies here.

5. On stepping from a row-boat to the shore the boat moves backward, but on stepping from a steamboat no backward motion is noticeable. Why is this?

**50. Centrifugal and Centripetal Force.** Tie a stone securely to the end of a string and whirl it about in a circle. You feel a distinct pull on the hand, and the faster the stone moves the stronger is the pull. We recognize that there is a tension in the string. Due to this tension there is a pull upon the hand and an equal pull on the stone. Considering the hand and the stone as two bodies acting upon each other through the string connecting them, we may say that the action of the hand upon the stone is equal to the reaction of the stone upon the hand and in the opposite direction.

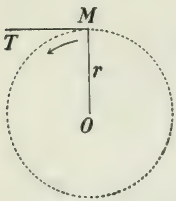


FIG. 34.—Motion in a circle.

Suppose that at some moment the stone is at $M$ (Fig. 34). If the string broke the stone would move off in the line $MT$, which is the tangent to the curve at $M$. The force acting upon $M$, in circular motion, is always directed (along the string) towards the centre $O$, that is, the direction of the force is at right angles to the tangent and draws the stone out of the tangent into the circular curve. This force thus appears to cause the body to *seek* the centre and it is known as the *centripetal force.* The force acting upon the hand is directed from the centre and is called the *centrifugal force.*

The pull on the hand leads most persons to think of a whirled body as endeavouring to fly off radially from the centre, but such is really not the case. The body, according to Newton's First Law, simply tries to continue in the straight line in which it at any moment may be considered as moving. It is clear also that the greater the mass of the whirled body,

the greater is its inertia and consequently the greater is the centripetal force required to make it move in its curved course.

The grandest examples of bodies moving under centripetal forces are to be found in the solar system. If the attraction of gravitation should cease, the planets and their satellites would move off in straight lines.

**51. Experiments with Rotating Bodies.** The apparatus

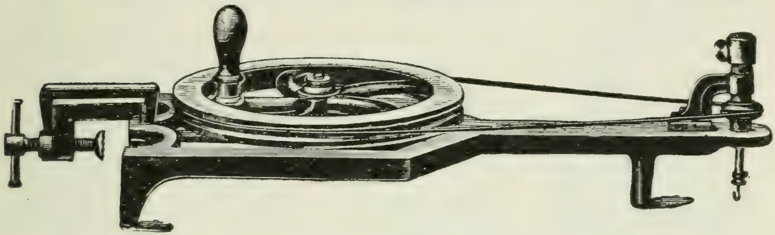
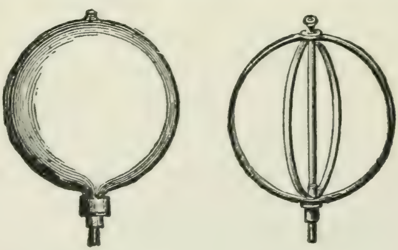


FIG. 35.—A whirling table.

shown in Fig. 35 is called a whirling table. By means of it various bodies can be rotated rapidly.

Place upon the spindle a glass globe (Fig. 36*a*), containing some coloured water and a little mercury and start it rotating. Both liquids creep up and form a band at the equator, with the heavier substance next the glass.



(*a*) (*b*)

FIG. 36.—Attachments for the whirling table.

Next remove the globe and substitute the rings shown in Fig. 36*b*. When at rest they are circular, but when rotated rapidly they become flattened along the axis of rotation. Now the earth is somewhat flattened at the poles. It is believed that at one time the earth was in a plastic condition and that the flattening is due to its rotation upon its axis. The equatorial diameter is 7926.6

miles and the polar diameter 7899.6 miles, or 27 miles less. This is about $\frac{1}{300}$th part of the diameter. This is so slight that if a person could observe the earth from a point far out in space the eye could not detect the flattening.

But the flattening is much greater in the case of some of the other planets. Jupiter's equatorial diameter is 88,200 miles, its polar, 83,000 miles, or $\frac{1}{17}$th part less. The flattened form of the planet is easily observed in the telescope. Jupiter rotates on its axis in 9 h. 55 m., from which we deduce that a point on its equator moves with a velocity 29,437 miles per hour! We should not be surprised at the flattening.

The planet next in order from the sun and second in size in the system is Saturn. Its equatorial diameter is 75,000 miles, and polar diameter 68,000 miles, or $\frac{1}{11}$th part less. Thus its flattening is greater than that of Jupiter, but it is not usually so easy to detect, chiefly on account of the wonderful rings which surround the ball of the planet. Every fifteen years, however, the rings are turned edgewise to us and then the

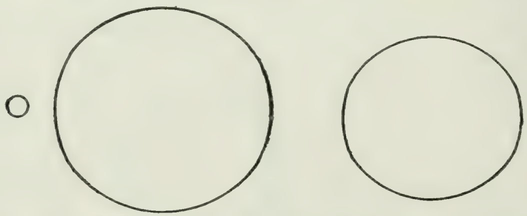


Fig. 37.—Shapes and relative sizes of the earth, Jupiter and Saturn.

flattening is very evident. In Fig. 37 the shapes and the relative sizes of the earth, Jupiter and Saturn are shown.

**EXERCISES**

Calculate the velocity of a point on the earth's equator, taking the diameter as 7926.6 miles and the period of rotation as 23 h. 56 m.

Do the same for Saturn, taking the diameter as 75,000 miles and the period as 10 h. 14 min.

**52. Some Practical Applications.** *The Centrifuge.*—The instrument shown in Fig. 38 is called a centrifuge. Near the ends of its two arms are suspended two small metal tubes closed at the lower end, and within these are placed glass tubes containing the substance to be investigated. The metal tubes can swing back and forth about horizontal pivots near the open end.

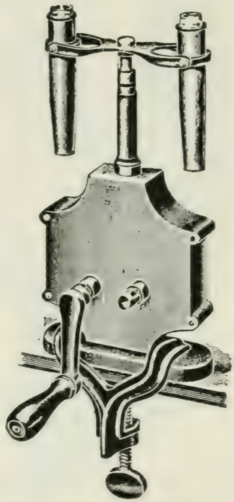


FIG. 38.—A centrifuge.

Inside the body of the instrument are multiplying gears and when the handle is turned the tubes can be made to revolve very rapidly, often more than 2,000 times per minute. This causes the tubes to point directly outwards from the axis of rotation and the heavier portion of the substance is driven to the bottom of the tubes. This apparatus is useful in testing blood and some other liquids.

*Babcock's Milk Tester.*—In Babcock's milk tester, used for determining the amount of butter-fat in milk, the fat is separated by a centrifugal machine similar to the above.

*The Cream Separator.*—Milk consists of a liquid with small globules of fat distributed through its mass. Not very many years ago the ordinary practice was to place the milk in pans and allow the globules, which are lighter than the liquid, slowly to rise and collect at the surface as cream. This was then skimmed off and afterwards churned into butter. It has been found, however, that the cream can be taken from the milk much more completely and in a small fraction of the time by means of the now familiar cream separator.

The essential portion of the machine is a steel bowl which is rotated very rapidly. That used in a well-known type of

separator is illustrated in Figs. 39, 40, 41, 42. The outside of the bowl, together with the gears for rotating it, is shown in Fig. 39. The lower part (see Fig. 40) is hollowed out underneath and is heavier around the rim, so that the bowl rests upright when placed on the end of a vertical axis, which is shown beneath. The conical shell of the bowl fits snugly into the lower part and rests on a rubber washer. By screwing down the nut at the top the bowl is made milk-tight. In Fig. 40, is shown the arrangement within the bowl. The central shaft is a projection upwards from the lower part of the bowl. It is hollow, thus forming a tube, and in the wall of the tube are slots opening into three enclosed gutters which lead into the bowl and end at some distance from the shaft. (See 2, 2, in Fig. 40). A series of conical 'discs' pressed from thin sheet-metal fit over the shaft. In these, holes are punched, being arranged to be just above the ends of the gutters.

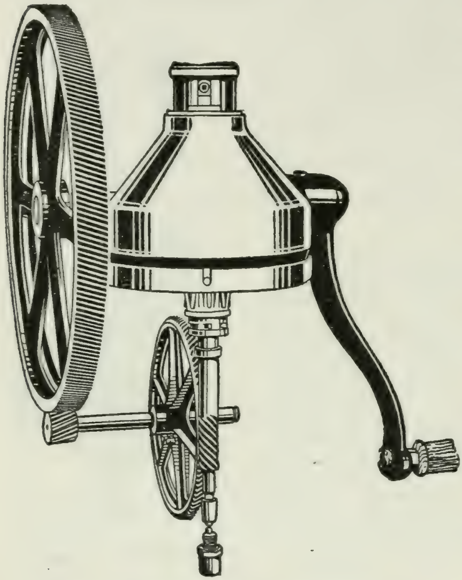


FIG. 39.—Bowl and gears of the separator.

The operation of the machine is somewhat as follows:—The bowl is put into very rapid rotation and the milk is admitted at 1. It passes down and comes out of the openings 2, 2, and is thus delivered between the discs some distance from

the axis of the bowl. The centrifugal motion causes the heavier liquid portion of the milk to go outwards along the under surface of the discs, and collect at the outer wall of the bowl. The cream finds its way inwards along the upper surface of the discs (Fig. 41). In this way the cream gathers at the centre and rises and, passing through 3, 3, comes out at the square outlet in the cap at the top. The skim-milk rises at the outer wall of the bowl and finds its way out of a rectangular opening beneath that for the cream. The cream and the skim-milk spurt out into vessels placed over the top of the bowl (see Fig. 42) and

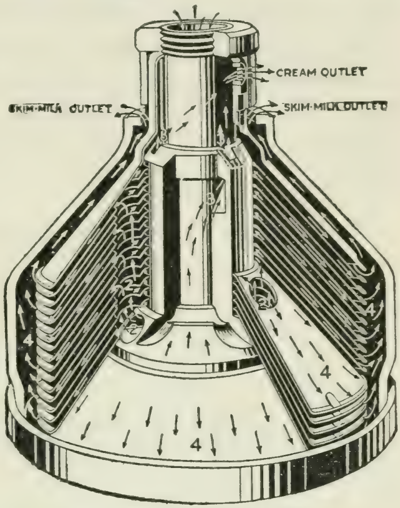



FIG. 40.—Inner view of separator bowl.

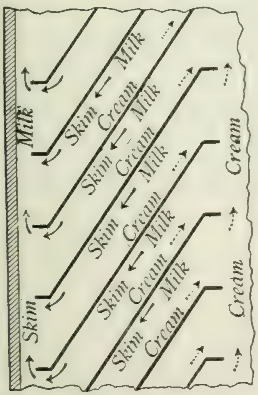



FIG. 41.—Showing passage of milk outwards and cream inwards.

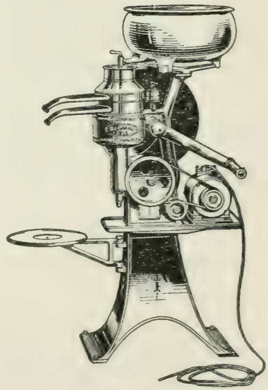


FIG. 42.—Separator, with electric motor.

pass off by way of spouts leading from these vessels. The entire operation requires only a few minutes.

The speed is very high, ranging (in the type of machine shown here) from 6000 revolutions per minute for the larger sizes to over 8000 for the smaller ones and on account of this high speed the machine must be well made, and must be kept well oiled and in good order.

### EXERCISES AND PROBLEMS

1. In a separator the crank revolved 60 times per minute, the large gear-wheel on the crank-shaft had 177 teeth, the small pinion into which it meshed had 19 teeth, the large worm wheel on its shaft had 103, and the worm screw on the axis supporting the bowl had 7 teeth. Calculate the revolutions per minute of the bowl.

2. In another machine the crank revolved 48 times per minute and the teeth on the gears were 247, 19, 93, 8, respectively. Find the revolutions per minute of the bowl.

3. A gun weighing 6 tons fires an 18-pound shell with a muzzle velocity of 1500 ft. per sec. Find the velocity of the recoil.

4. A shell of mass 12 lb. is discharged into a box of sand suspended by a rope and weighing 900 lb., and the combined mass begins to swing with a velocity of 25 ft. per sec. Calculate the velocity of the shell.

5. A railway train of mass 200 tons and moving at 6 ft. per sec. strikes a freight car of mass 40 tons standing still, and is automatically coupled to it. Find the speed with which the entire train begins to move.

6. A base-ball weighing 5 oz. and travelling forward at the rate of 40 ft. per sec. is struck and driven directly backward at the rate of 60 ft. per sec. What is the change in momentum? If the bat was in contact with the ball for $\frac{1}{25}$ sec. find the average value of the force exerted by the bat.

## CHAPTER VII

### Composition of Forces

**53. Addition of Forces in the Same Direction.** When an automobile gets stuck on the road all hands step out and try to release it. Two may pull on the fender in front while three may push at the rear, and thus move the car to the hard level track again. But in place of the five people we might get a good team of horses to do the job for us.

The forces exerted by the men are all in the same direction, all aiming to cause the car to move forward, and it is perfectly evident that the single force exerted by the team of horses is equal to all the forces exerted by the men added together. For instance, if the forces exerted by the men were 150, 125, 160, 100, 165 pounds, respectively, the force exerted by the horses must have been the sum, or 700 pounds.

We can say then that if a number of forces act in the same direction upon a rigid body, the total force urging the body in that direction is the sum of the individual forces.

**54. Forces Inclined at an Angle.** But if several forces act upon a body in directions inclined to each other it is not so easy to see what the result will be. As usual in a scientific problem, it is wise to study it experimentally.

Drive nails in a horizontal bar $AB$ (Fig. 43), which may conveniently be the frame above the blackboard, and hang two spring-balances $S$, $S'$ on two of them. Tie three strings together at $O$ and attach the other ends of two of

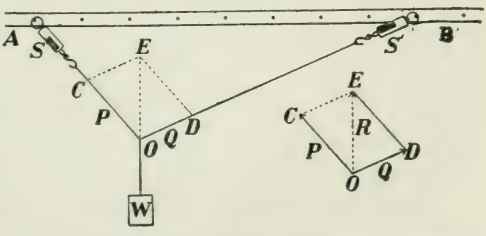


FIG. 43.—How to demonstrate the law of parallelogram of forces.

them to the hooks of the balances. On the third string hang a weight $W$ pounds. This string will take a vertical direction and the tension in it will be $W$ pounds. The tensions in the other strings will be given by the readings on the spring-balances. Let $S$ show $P$ pounds and $S'$ show $Q$ pounds. It is plain that the knot at $O$ is kept in equilibrium by the three forces, $P$ acting along $OC$, $Q$ along $OD$, and $W$ acting vertically downwards.

The force $W$ may be looked upon as balancing the other forces $P$ and $Q$, and hence if $R$ is the resultant of $P$ and $Q$ (that is, the single force which is equivalent to $P$ and $Q$ acting together), it must be equal in magnitude to $W$ but be acting in the opposite sense, that is, upwards.

Now draw on the blackboard immediately behind the strings (or in some other convenient place), lines parallel to the strings $OS$, $OS'$, and make $OC$, $OD$ as many units long as there are pounds shown on $S$, $S'$, respectively.

Then carefully complete the parallelogram $OCED$ and measure the diagonal $OE$. It will be found to be in the vertical and to be $W$ units long.

A slightly different form of the experiment is as follows:

Fasten three cords (fish-line) to a small ring, and hook spring-balances on the other ends of the cords (Fig. 44). By means of pins in the top of the table, over which the rings of the balances may be placed, or in any other convenient way, exert force on the balances so that the cords are under considerable tension. The balances should move free of the table top.

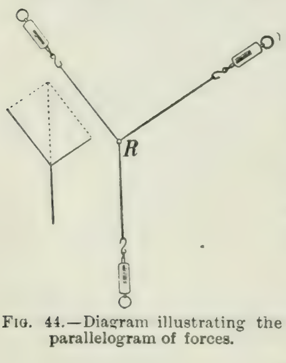


Fig. 44.—Diagram illustrating the parallelogram of forces.

Pin a sheet of paper under the strings and mark a dot precisely at $R$, the centre of the ring; also make dots exactly under each string and as far from $R$ as possible.

Read each balance. Then loosen them, and when they are lying on the table observe if the index returns to zero. If it does not, a correction to the reading on the balance must be made.

With great care draw lines from $R$ through the points under the cords, and on these lines take distances proportional to the tensions of the corresponding strings. Thus, if the tensions be 1000, 1500, 2000 grams, take lengths 10, 15, 20 cm. or 4, 6, 8 inches.

Using any two of these lines as adjacent sides, complete a parallelogram, taking care to have the opposite sides accurately parallel. Draw the diagonal between these sides and carefully measure its length. Compare it as to length and direction with the third line.

From these experiments we deduce the following proposition known as the PARALLELOGRAM OF FORCES which states that *if two forces are represented in magnitude and direction by two sides of a parallelogram, then their resultant will be represented, in magnitude and direction, by the diagonal between the two sides.*

#### PROBLEMS AND EXERCISES

1. Taking a line one centimetre in length to represent a gram-force, draw a line to represent a force of 12.3 grams acting (1) in a horizontal direction, (2) in a vertical direction, (3) in a direction making an angle of 45° with the horizontal.

2. Taking a line three-quarters of an inch long to represent a pound-force, draw a line which represents a force of $5\frac{3}{4}$ pounds acting in a direction making an angle of 60° with the vertical.

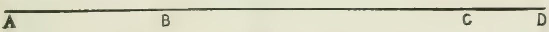



FIG. 45.

3. If $AB$ (Fig. 45) represents a force of 60 grams, what force will be represented by (1) $AC$, (2) $BC$, (3) $BD$, (4) $AD$, (5) $CD$?

4. If $BC$ (Fig. 45) represents a force of 24 pounds, what force will be represented by (1) $AB$, (2) $AC$, (3) $AD$, (4) $BD$, (5) $CD$?

5. If $CD$ (Fig. 45) represents a force of 3 kilograms, what force will be represented by (1) $AB$, (2) $AC$, (3) $AD$, (4) $BC$, (5) $BD$?

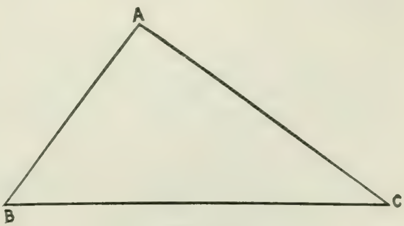



FIG. 46.

6. If 2 cm. in length represents a force of 3 grams, what are the magnitudes of the forces represented by $AB$, $BC$, $CA$, the sides of the triangle $ABC$? (Fig. 46).

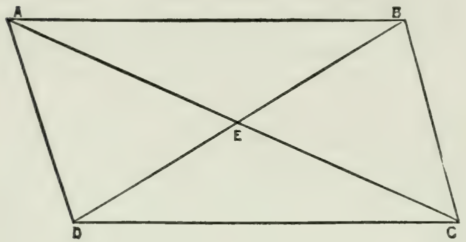


Fig. 47.

7. If $AD$ (Fig. 47) represents a force of 2 pounds, what are the magnitudes of the forces represented by $AB$, $AE$ and $ED$?

8. Find the greatest and the least resultants of two forces whose magnitudes are 15 grams and 20 grams.

9. Find the greatest and least resultants of two forces whose magnitudes are $P + Q$ and $P - Q$.

10. Find the resultant of forces of 15 pounds and 36 pounds, acting at right angles to each other.

11. Find the resultant of two forces of 12 kilograms and 35 kilograms acting at a point, the one acting north and the other east.

12. The resultant of two forces acting at right angles is 82 pounds. If one of the forces is 80 pounds, what is the other?

13. A force of 5 $P$ acts in a northerly direction, and the resultant of it and another force acting at the same point in an easterly direction is 13 $P$. What is the other force?

14. Determine the resultant of the following forces acting concurrently at the same point:—12 pounds N., 24 pounds E., 7 pounds S., and 36 pounds West.

15. A weight is supported by two strings. If the strings make an angle of 90° with each other, and the tension of the one is 9 pounds, while that of the other is 12, what is the weight?

16. A boat is moored in a stream by a rope fastened to each bank. If the ropes make an angle of 90° with each other, and the force of the stream on the boat is 500 pounds, find the tension of one of the ropes if that of the other is 300 pounds.

**55. Triangle of Forces.** On the blackboard, or on a sheet of paper, draw a line $OD$ (Fig. 48) parallel to $OS'$ (Fig. 43) and make it $Q$ units long. From $D$ draw $DC$, parallel to $OS$ and make it $P$ units long. Then measure $OC$. It will be found to be $W$ (or $R$) units long and it will also be found to be parallel to $OW$.

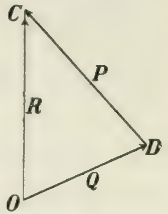


Fig. 48.—The triangle of forces.

Indeed it is evident that the triangle $COD$ is simply the half-parallelogram $EOD$ (Fig. 43), and we can make the following statement:

*If two forces acting on a body be represented in magnitude and direction by two sides $OD$, $DC$, of a triangle taken in order, their resultant will be represented by the third side $OC$.*

This can be put in a slightly different form, known as the Triangle of Forces, as follows:

*If three forces acting at a point can be represented in magnitude and direction by the sides of a triangle taken in order they will be in equilibrium.*

#### EXERCISES AND PROBLEMS

1. Can a particle be kept at rest by each of the following systems of forces acting at a point?

   (1) 4 pounds, 3 pounds, 7 pounds.

   (2) 1 gram, 3 grams, 5 grams.

   (3) 4 pounds, 3 pounds, 2 pounds.

   (4) $P + Q$, $P - Q$, $P$, when $P$ is greater than $Q$.

2. Draw lines to represent the directions of the following forces acting in one place at a point, when each system is in equilibrium:

   (1) 4 grams, 5 grams, 3 grams.

   (2) Three forces each equal to $P$.

   (3) $2P$, $P$, $\sqrt{3}P$.

   (4) 5 grams, 9 grams, 4 grams.

3. Forces $5P$, $12P$, $13P$ keep a particle at rest. Show that the directions of two of the forces are at right angles to each other.

4. Find the directions in which three equal forces must act at a point to produce equilibrium.

5. Forces $A + B$, $A - B$, and $\sqrt{2(A^2 + B^2)}$ keep a particle at rest. Show that the directions of two of the forces are at right angles to each other.

**56. Resolution of Forces.** Sometimes in felling a tree a rope is tied to the trunk, as high up as possible, and then pulled. This is done to help to make the tree to topple over and also to cause it to fall in a safe direction (Fig. 49). Now

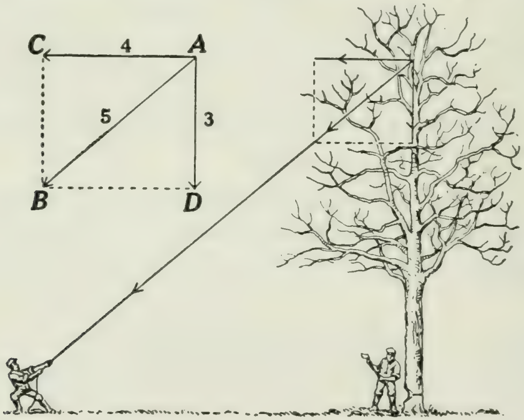



FIG. 49.—A pull along the rope pulls the tree over and also pulls it vertically downwards.

it is evident that the tension in the rope has the effect not only of pulling the tree over but also of pulling it vertically downwards. This latter force does not help in the removal of the tree at all.

It seems, then, that the same effect upon the tree could be produced if we substituted for the force along the rope two other forces, one in the horizontal, tending to topple the tree over, and the other in the downward vertical direction.

The magnitudes of these *component* forces can be determined from a consideration of the parallelogram of forces. Suppose the pull on the rope is 100 pounds. Draw a line $AB$ parallel to the rope and make it 5 inches long, and from $B$ draw horizontal and vertical lines meeting the vertical and horizontal lines through $A$ in the points $D$ and $C$. The lengths of $AC$ and $AD$ will represent the magnitudes of the forces in the horizontal and vertical directions.

For example, if the lengths of $AC$, $AD$ are 4 and 3 inches, respectively, the horizontal force is 80 pounds and the vertical force is 60 pounds.

$AC$ and $AD$ are said to be *components* of the force $AB$ in the horizontal and vertical directions, and the force is said to be *resolved* into these two components.

It is well to observe, however, that a force can be resolved into components in any two directions. We need only represent the original force by the diagonal of a parallelogram, and the two components will be represented by the two adjacent sides.

**57. Calculation of Resultant and Components.** (1) Suppose two forces acting on a body at a point in it to be represented

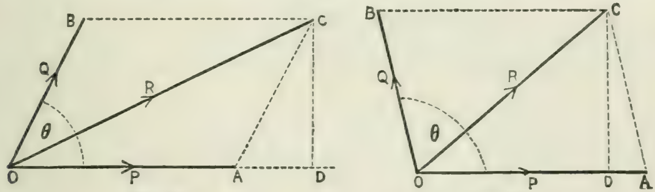



FIG. 50*a*. FIG. 50*b*.

Calculating the value of $R$ in terms of $P$ and $Q$.

in direction and magnitude by $OA$, $OB$ (Figs. 50*a*, 50*b*), and let the angle between them be $\theta$.

Then completing the parallelogram, the resultant is represented by $R$, and we wish to calculate its magnitude. From

$C$ drop a perpendicular upon $OA$; meeting it (produced if necessary) in $D$.

In Fig. 50$a$ we have

$$\frac{CD}{CA} = \sin CAD, \text{ or } CD = Q \sin \theta;$$

Also $$\frac{AD}{AC} = \cos CAD, \text{ or } AD = Q \cos \theta;$$

And $$OD = OA + AD = P + Q \cos \theta.$$

In Fig. 50$b$ we have

$$\frac{CD}{CA} = \sin CAD = \sin (180° - \theta) = \sin \theta;$$

Hence $CD = Q \sin \theta$;

Also $$\frac{AD}{AC} = \cos CAD = \cos (180 - \theta°) = -\cos \theta;$$

Hence $AD = -Q \cos \theta$;

And $$OD = OA - AD = P - [-Q \cos \theta],$$
$$= P + Q \cos \theta, \text{ as before.}$$

Now in the right-angled triangle $ODC$,

$$OC^2 = OD^2 + DC^2,$$

or $$R^2 = (P + Q \cos \theta)^2 + (Q \sin \theta)^2,$$
$$= P^2 + Q^2 + 2PQ \cos \theta, \text{ since } \sin^2 \theta + \cos^2 \theta = 1,$$

and $$R = \sqrt{(P^2 + Q^2 + 2PQ \cos \theta)}.$$

(2) A force $R$ is represented in magnitude and direction by

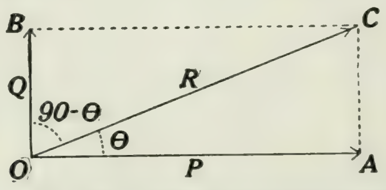



FIG. 51.—Finding the component of a force in any direction.

the line $OC$ (Fig. 51); we wish to find the components $P$, $Q$ of the force in the directions $OA$, $OB$.

Let angle $COA = \theta$, and $COB = 90° - \theta$.

Then $\frac{OA}{OC} = \cos\theta$, or $P = R\cos\theta$.

and $\frac{OB}{OC} = \cos(90 - \theta)$, or $Q = R\sin\theta$.

We thus see that the *component of a force resolved in any direction is equal to the product of the force into the cosine of the angle between the direction of the force and the new direction.*

## PROBLEMS

1. Find the resultant of the following forces:

(1) 36 pounds and 60 pounds at an angle of 60°.

(2) 10 pounds and 10 pounds at an angle of 45°.

(3) 10 pounds and 10 pounds at an angle of 150°.

(4) 30 pounds and 80 pounds at an angle of 120°.

(5) 2 pounds and 7 pounds at an angle of 30°.

(6) 2 pounds and 3 pounds at an angle of 135°.

(7) 3 pounds and 16 pounds at an angle of 15°.

(8) 4 pounds and 11 pounds at an angle of 75°.

(9) $P$ acting toward the west and $P\sqrt{2}$ toward the northeast.

2. Prove that the resultant of two forces, $P$ and $P + Q$, acting at an angle of 120°, is equal to the resultant of two forces, $Q$ and $P + Q$, acting at the same angle.

3. Find the resultant of two forces represented by the side of an equilateral triangle and the perpendicular on this side from the opposite angle.

4. Six posts are placed in the ground so as to form a regular hexagon, and an elastic cord is passed around them and stretched with a force of 50 pounds. Find the magnitude and the direction of the resultant pressure on each post.

5. Two forces of two pounds each, acting at an angle of 60°, have the same resultant as two equal forces acting at right angles. What is the magnitude of these forces?

6. The resultant of two forces, $P$ and $Q$, is $Q\sqrt{3}$, and its direction makes an angle of 30° with the direction of $P$. Show that $P$ is either equal to $Q$ or $2Q$.

7. Show that when two forces act at a point their resultant is always nearer the greater force, and the greater the angle between the forces the less is their resultant.

8. If a uniform heavy bar is supported in a horizontal position by a string slung over a peg and attached to both ends of the bar, prove that the tension of the string will be diminished if its length is increased.

9. A weight is suspended by means of two strings of equal length attached to points in the same horizontal line. Show that if the lengths of the strings are increased their tension is diminished.

10. The resultant of two forces which act at an angle of 60° is 13 grams. If one of the forces is 7 grams, find the other.

11. A particle is acted upon by two forces, one of which is inclined at an angle of 80° to the vertical, and the other at an angle of 40° to the vertical and on the other side of it. If one of the forces is 10 pounds, and the combined effect of the two is $2\sqrt{31}$ pounds, find the other force.

12. If one of two forces acting on a particle is 5 kilograms, and the resultant is also 5 kilograms, and at right angles to the known force, find the magnitude and the direction of the other force.

13. Find the resolved part of a force of 10 pounds in a direction making an angle with the direction of the force of (1) 30°, (2) 45°, (3) 75°.

14. Find the horizontal and the vertical resolved parts of a force of 20 pounds, making an angle of 30° with the horizontal.

15. Find the resolved part S.W. of a force of 12 pounds S.

16. A force of 100 pounds is resolved into two equal forces at right angles to each other. What is the magnitude of either force?

17. The resultant of two forces acting at right angles is 16 pounds, and makes an angle of 30° with one of the components. Find the magnitude of the components.

18. The horizontal resolved part of a force making an angle of 30° with the horizontal is 4 pounds. Find the vertical resolved part.

19. A horse, in towing a canal boat, pulls with a force of 200 pounds. If the tow-rope is horizontal and makes an angle of 5° with the direction of the canal, find the magnitude of the force that would have to be applied in the direction of the canal to draw the boat.

**58. Translation and Rotation.** It should be observed, however, that very generally when several forces act upon a body they tend not only to cause the body to move as a whole or to give it a *motion of translation*, but also to make it turn about an axis as well, that is, to give it a *motion of rotation.*

**59. The Airplane.** The sailing of a ship in a direction almost opposite to that from which the wind is coming has long been considered an interesting example of the resolution of forces; while investigations into the behaviour of airplanes involve the same principle and in addition many difficult problems.

Consider a plane surface $AB$ (Fig. 52) inclined to the horizontal at an angle $\theta$, moving from right to left with a velocity $v$. The

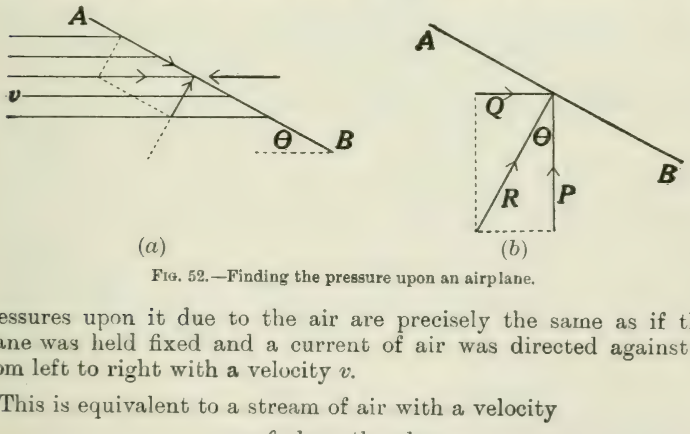



FIG. 52.—Finding the pressure upon an airplane.

pressures upon it due to the air are precisely the same as if the plane was held fixed and a current of air was directed against it from left to right with a velocity $v$.

This is equivalent to a stream of air with a velocity

$v \cos \theta$ along the plane,

and $v \sin \theta$ at right angles to the plane.

The former is assumed to slip without friction along the plane and so cannot produce any pressure on its surface. The latter exerts a pressure

$$R = \rho\,(v \sin \theta)^2 \begin{Bmatrix} \text{dynes per sq. cm.} \\ \text{pdl. per sq. ft.} \end{Bmatrix} \qquad \text{(Sec. 47)}$$
$$= .0025\,(v \sin \theta)^2 \text{ pd. per sq. ft.}$$

The pressure $R$ may be resolved into two components (Fig. 52*b*).

$$P = R \cos \theta \text{ vertically upward,}$$
$$Q = R \sin \theta \text{ horizontally to the right.}$$

Consequently as the plane is rushing forwards there is developed a lifting power $P$, and a resistance to the motion $Q$, which must be overcome by the engine.

$$\text{Hence, } P = \rho\,(v \sin \theta)^2 \cos$$
$$= .0025\,(v \sin \theta)^2 \cos \theta \text{ pd. per sq. ft.}$$
$$\text{and } Q = .0025\, v^2 \sin^3 \theta \text{ pd. per sq. ft. of plane.}$$

**Example**:—Calculate the lifting force and also the resistance to horizontal motion on a plane of area 260 sq. ft. travelling at 42 ml. per hr. and inclined at 12° to the horizontal.

$$\text{Total lifting force} = .0025\,(61.6)^2 \sin^2 12^\circ \cos 12^\circ \times 260$$
$$= 1043 \text{ pd.}$$
$$\text{Resistance} = .0025\,(61.6)^2 \sin^3 12^\circ \times 260$$
$$= 222 \text{ pd.}$$

In the actual construction of airplanes the wings are made arched, like a bird's wing, and this increases the lifting power.

**60. The Sailing Ship.** Let the ship be moving with a velocity $V$ ft. per sec. in the direction $CD$, and let the sail $AB$ make with

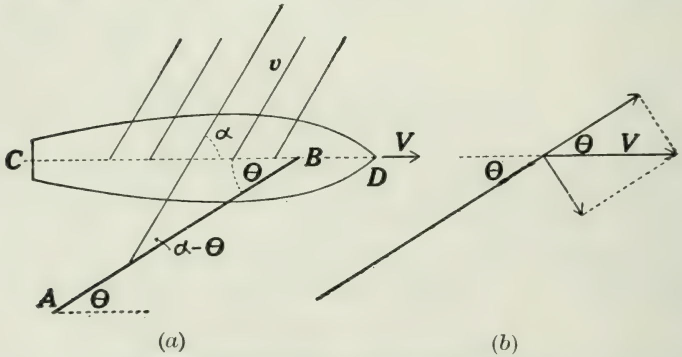



FIG. 53.—Finding the force propelling a sailing vessel.

$CD$ the angle $\theta$. Velocity of the wind $= v$ ft. per sec. in a direction making angle $\alpha$ with $CD$. (Fig. 53*a*).

It is assumed that the keel or centre-board prevents drifting sideways.

The direction of the wind makes with the sail the angle $(a - \theta)$.

Resolving the velocity of the wind we have (Fig. 53*a*)

$v \cos (a - \theta)$ along the surface of the sail, and
$v \sin (a - \theta)$ at right angles to " " .

Again, the sail's motion in the direction $CD = V$ ft. per sec.

This may be resolved into two components. (Fig. 53*b*)

$V \cos \theta$ along the plane of the sail, and
$V \sin \theta$ at rt. angles to " " .

Consequently the velocity of the wind with respect to the sail, in the direction at right angles to the sail

$$= v \sin (a - \theta) - V \sin \theta \text{ ft. per sec.,}$$

and the pressure on 1 sq. ft.

$$= \text{density} \times (\text{velocity})^2$$
$$= .08 \,[v \sin (a - \theta) - V \sin \theta]^2 \text{ poundals.}$$

Now the only part of this which is effective in driving the ship forward is that component in the direction $CD$, which

$$= .08 \,[v \sin (a - \theta) - V \sin \theta]^2 \sin \theta \text{ pdl. per sq. ft.}$$
$$= .0025 \,[v \sin (a - \theta) - V \sin \theta]^2 \sin \theta \text{ pd. " " "}$$

**Example** :—Find the total force on a sail 120 sq. ft. in area if the sail is set at 30° to the central line of ship, the wind is directly across the ship with a velocity of 5 ml. per hr., and the ship is moving at rate of 7 ml. per hr.

Here, $a = 90°$, $\theta = 30°$, $v = 7.33$ ft. per sec.,

$V = 10.27$ ft. per sec.

Pressure = .00368 pd. per sq. ft

Total force = .442 pd.

This example shows that the velocity of a ship may be greater than the velocity of the wind, which causes the motion. This has often been remarked in the case of an ice-boat, which meets with little resistance and can make great speed.

Read the fundamental principle

# CHAPTER VIII

## Moment of a Force

**61. Moment of a Force.** If you have to turn a nut which is rusted tight you can exert the greatest turning effort by using a wrench with a long handle. Again if you wish to turn a wheel which is hard to move you do not take hold of the hub, but of the rim (*i.e.*, as far as possible from the axis), and you exert a force at right angles to the spoke where you take hold. Similarly, in stormy weather, in order to keep the ship on her course the wheelsman grasps the wheel by the pins at the rim and exerts a force at right angles to the line joining the axis to the point where he takes hold (Fig. 54). If a machine is driven by a crank the longer the crank is the greater is the turning effort which can be exerted.

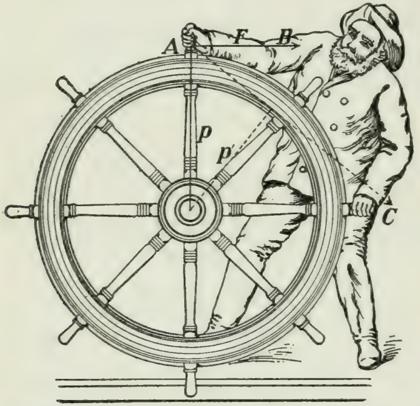



Fig. 54.—The *moment of a force* depends on the force applied and its distance from the axis of rotation.

From our experience we know that the turning effect upon the wheel is proportional to the force exerted and also to the distance from the axis of the point where the force is applied.

Let $F$ = the force applied,

$p$ = the perpendicular distance from the axis to the line $AB$ of the applied force.

By experience we know that the power to turn the wheel depends directly on $F$ and on $p$, and is therefore proportional to $Fp$. This product $Fp$ is called the *moment of the force* $F$ about the axis. It measures the tendency of the force to produce rotation.

If the direction of the force $F$ is not perpendicular to the line joining its point of application to the axis, the moment is clearly not so great, since part of the force is spent uselessly in pressing the wheel against its axis. In Fig. 54, if $AC$ is the new direction of the force, then $p'$, the new perpendicular, is shorter than $p$, and hence the product $Fp'$ is smaller.

**62. Experiment on Moments.** To the two ends $A$, $B$ of a rod which can turn freely about a pin driven in a board at $O$, attach two cords and allow them to pass over pulleys at the edge of the board, and on the ends hang weights $P$, $Q$ (Fig. 55). Allow the rod to come to rest. Measure the perpendicular distances from $O$ upon the two cords. Let them be $p$, $q$.

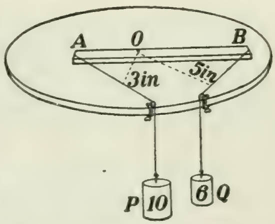


FIG. 55.—Apparatus for testing the law of moments.

Compare the products $Pp$ and $Qq$. One measures the tendency of the rod to turn in one direction, the other measures the tendency to turn in the other direction, and as these tendencies balance each other, $Pp$ should be found equal to $Qq$.

**Example**:—In an experiment

$$P = 10 \text{ oz.},\ Q = 6 \text{ oz.},$$
$$p = 3 \text{ in.},\ q = 5 \text{ in.},$$
$$Pp = 10 \times 3 = 30,$$
$$Qq = 6 \times 5 = 30.$$

**PROBLEMS**

1. $ABCD$ is a square, whose side is 2 ft. long. Find the moments about both $A$ and $D$ of the following forces:—(1) 3 pounds along $AB$, (2) 9 pounds along $CB$, (3) 2 pounds along $DA$, (4) 11 pounds along $AC$, (5) 1 pound along $DB$, (6) 20 pounds along $DC$.

2. A force of 12 acts along a median of an equilateral triangle whose side is 18. Find the measure of the moment of the force about each angle of the triangle.

3. A force of 6 acts along one side of an equilateral triangle whose side is 10. Find the measure of its moment about the opposite angle.

4. $ABCD$ is a rectangle, the side $AB$ being 12 cm. and the side $BC$ 5 cm. long. $O$ is the intersection of the diagonals. Find the algebraic sum of the moments about (1) $A$, (2) $O$, of the following forces:—14 dynes along $BA$, 19 dynes along $BC$, 3 dynes along $CD$, 4 dynes along $AD$, 10 dynes along $AC$, and 9 dynes along $DB$.

5. A force of 20 acts along a diagonal of a square whose side is $8\sqrt{2}$. Find the measure of its moment about each of the four angles.

6. At what point of a tree must one end of a rope whose length is 50 feet be fixed, so that a man pulling at the other end may exert the greatest force to pull it over?

7. $ABCD$ is a rhombus, the side $AB$ being 8 cm. long, and the angle $ABC$, 60°; $O$ is the intersection of the diagonals. Find the algebraic sum of the moments about (1) $A$, (2) $O$, of the following forces:—9 dynes along $AB$, 2 dynes along $CD$, 5 dynes along $DA$, 13 dynes along $AC$, 7 dynes along $BC$, 1 dyne along $BD$.

8. The connecting-rod of an engine is inclined to the crank-arm at an angle of 30°. Compare the moment of the force to turn the shaft when in this position with the moment when in the most favourable position.

**63. The Wheel and Axle.** The principle of moments is well illustrated in the apparatus shown in Figs. 56$a$, 56$b$. A weight $W$ is on one end of a cord which is wound about a cylinder (the axle) of radius $r$, while the weight $F$ is on the end of a cord which is wound about the circumference of a wheel of radius $R$.

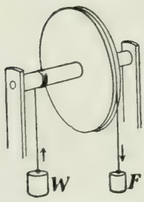



FIG. 56$a$.—The wheel and axle.

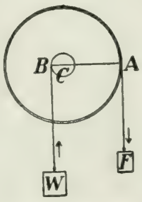



FIG. 56$b$.—Diagram to explain the wheel and axle.

If $F$ is just sufficient to balance $W$, its moment about the axis of the cylinder must be equal to the moment of $W$ about the axis, or

$$W \times r = F \times R,$$

$$\text{or } W/F = R/r.$$

Thus by taking a large wheel and a small axle the weight $W$ can be lifted by a much smaller force $F$.

**64. Resultant of Parallel Forces in the same Direction.** On a metre stick M (Fig. 57) hang a weight $W$ and support the two ends by spring-balances $B_1$, $B_2$, which are held up by a rod $AC$. Be careful to have the balances hanging vertically, and move $W$ to any point on the stick.

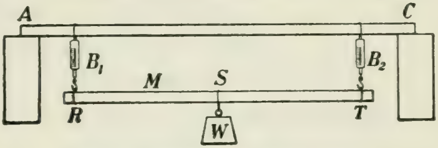


FIG. 57.—Resultant of parallel forces.

Take the readings of the balances $B_1$, $B_2$, let them be $P$ and $Q$, respectively. Also read the distances $RS$, $ST$.

Let the stick be so light that its weight compared to $P$, $Q$ and $W$ may be neglected.

We find (1) $P + Q = W$,

and (2) $P \times RS = Q \times ST$ or $P/Q = ST/RS$.

Notice that $P \times RS$ is the moment of $P$ about $S$, or is the measure of the tendency of $P$ to turn the stick about $S$; also $Q \times ST$ is the moment of $Q$ about $S$, or the measure of the tendency of $Q$ to turn the stick about $S$ in the opposite direction, and as the stick is at rest these must be equal.

Now $W$ balances $P$ and $Q$; hence the resultant of $P$ and $Q$ must be equal to $W$ and must act at $S$ in the upward direction, that is, parallel to $P$ and $Q$.

We conclude, then, that *the resultant of two parallel forces acting in the same direction is equal to the sum of the forces, and its point of application is situated so that its*

*distances from the lines of action of the forces are inversely as the magnitudes of the forces.*

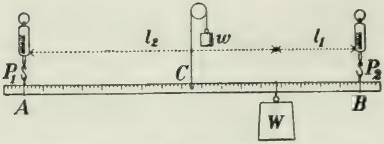


FIG. 58.—Finding the resultant of parallel forces.

If the weight of the stick is too great to be neglected we can perform the experiment as shown in (Fig. 58).

Support the rod at its centre of gravity $C$ (see Chap. XII) by a weight $w$ passing over a pulley. Attach spring-balances at $A$ and $B$ and hang a weight $W$ at any point of the stick.

Take the readings $P_1$, $P_2$ of the spring-balances and read the distances $l_2$, $l_1$ of the balances from the weight.

No matter where $W$ is placed, we shall find

$$P_1 + P_2 = W,$$

and $P_1 \times l_2 = P_2 \times l_1$ or $P_1/P_2 = l_1/l_2$.

Next, arrange that the metre stick shall not be horizontal, but in the position $AB$ (Fig. 59). It will be found to be in equilibrium still, with the balances showing the same readings.

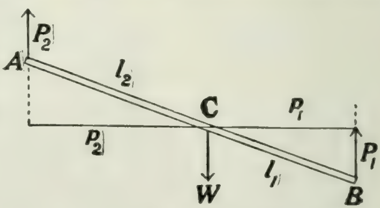


FIG. 59.—Forces on a rod not horizontal.

From $C$ drop perpendiculars on the line of action of $P_1$, $P_2$ and let their lengths be $p_1$, $p_2$.

Then moment of $P_1$ about $C = P_1 \times p_1$,

and " " $P_2$ " $C = P_2 \times p_2$,

and these are equal.

$$\text{Hence, } P_1/P_2 = p_2/p_1.$$

But from similar triangles $p_1/p_2 = l_1/l_2$,

and so $P_1/P_2 = l_2/l_1$, as before. (Notation changed).

Hence the resultant of $P_1$, $P_2$ has the same magnitude as before and its point of application is unchanged.


FIG. 60.—The two forces give a motion of translation.

**65. Couple.** Attach a string to each end of a rod lying on a table, and pull on these with equal force $P$ and in parallel directions (Fig. 60). The rod moves forward in the direction of the force.

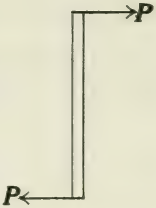



FIG. 61.—The two forces give a motion of rotation.

Next, pull with equal forces but in opposite senses (Fig. 61). Now the rod simply turns about a vertical axis without moving forward as a whole. Such a pair of forces is called a *couple.*

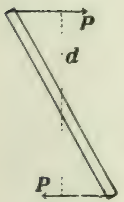



FIG. 62.—Two equal opposite parallel forces produce only rotation.

If $d$ is the perpendicular distance between the two equal forces (Fig. 62), the magnitude of the couple is $Pd$. This measures the rotating power.

Next pull one end with a force $P$, and the other with a greater force $Q$ (Fig. 63). This force $Q$ may be considered as made up of two components,

$$P \text{ and } Q - P.$$


FIG. 63.—Two opposite parallel but unequal forces produce both rotation and translation.

The two forces $P$, $P$ will form a couple and will produce rotation of the rod, while the force $Q - P$ will produce a motion of the rod as a whole, or a translation, in the direction of the force.

### PROBLEMS

1. Find the magnitude and point of application of the resultant of two parallel forces of 3 dynes and 2 dynes acting in the same direction at points 5 metres apart.

2. Find the magnitude and point of application of the resultant of two opposite parallel forces of 17 dynes and 25 dynes acting at points 8 metres apart.

3. The resultant of two parallel forces is 15 pounds, and acts at a distance of 4 feet from one of them whose magnitude is 7 pounds. Find the position and magnitude of the second force, when (1) the forces are in the same direction, (2) when opposite.

4. Two men of the same height, carry on their shoulders a pole 6 feet long, and a mass of 121 pounds is slung on it, 30 inches from one of the men. What portion of the weight does each man support?

5. Two men support a weight of 112 pounds on a weightless pole which rests on the shoulder of each. The weight is twice as far from the one as from the other. Find what weight each supports.

6. A man carries two buckets of water by means of a pole which he holds in his hand at a point three-fifths of its length from one end. If the total weight carried is 40 pounds, how much does each bucket weigh?

7. Two men, one stronger than the other, have to remove a block of stone weighing 270 pounds by means of a light plank whose length is 6 feet; the stronger man is able to carry 180 pounds. How must the plank be placed so as to allow him that share of the weight?

8. A plank weighing 10 pounds rests on a single prop at its middle point; if it is replaced by two others, one on each side of it, 3 feet and 5 feet from the middle point, find the pressure on each.

9. A straight weightless rod 2 feet in length rests in a horizontal position between two fixed pegs placed at a distance of 3 inches apart, one of the pegs being at one end of the rod. A weight of 5 pounds is suspended at the other end. Find the pressure on each of the pegs.

10. A light rigid rod 20 feet long is supported in a horizontal position on two posts 9 feet apart, one post is 4 feet from the end of the rod; from the middle point of the rod a weight of 63 pounds is suspended. Find the pressures on the posts.

## CHAPTER IX

### Equilibrium of a Rigid Body

**66. Translation and Rotation.** As has been remarked (Sec. 58) when forces act upon a body they may tend to give it a translation or a rotation or both at the same time. A body is translated when its centre of gravity (see Chap. XII) is displaced; it is rotated when any lines drawn in the body change their directions.

If a body is in equilibrium, of course there is no translation. We conclude, then, that if all the forces acting upon the body be resolved in any direction the sum of all the components in that direction must be zero.

Also, there is no rotation. Consequently the moments of all the forces about any axis must be just balanced, that is, the tendency to turn in any direction must be balanced by an equal tendency to turn in the opposite direction.

By making use of these two general principles many problems of equilibrium may be solved. The method to be followed can be best understood by considering a number of examples. In all cases the forces will be taken to be acting in one plane.

**67. First Example.** A straight rod $AB$ (Fig. 64), whose weight may be neglected, turns freely in a vertical plane about a pin through the end $A$. A 7-lb. mass is hung from the end $B$, and a string, attached at the middle point $C$, is pulled in such a way that it is always at right angles to the rod. What must be the tension of the string if the rod makes an angle of 30° with the vertical? Also, find the reaction of the hinge and its line of action.

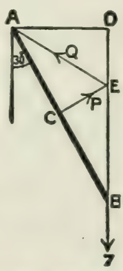



Fig. 64.—Rod pivoted at $A$.

Having drawn a good diagram, make a list of the forces acting on the rod. They are:

(*a*) A force of 7 pd. acting vertically downwards at $B$.

($b$) A force $P$ acting at right angles to the rod at its middle point $C$.

($c$) The reaction of the pin at $A$. Let the value of this force be $Q$. It will act in some line through $A$.

The lines of action of $P$ and the 7 pd. meet in the point $E$, and their resultant must be a force passing through $E$. Now the force $Q$ produces equilibrium and hence it must balance the resultant of $P$ and 7, but it cannot do so unless it passes through the same point $E$.

We, therefore, conclude that:

*If three forces acting in one plane upon a rigid body keep it in equilibrium, their lines of action must meet in a point.* (The forces are assumed to be not parallel).

This is an important proposition and can be proved in another way. Since the body is not being rotated the sum of the moments of the forces about any point must vanish.

Take the moments of $P$ and 7 about the point $E$. Both $= 0$. Now $Q$ is the only other force acting and so its moment must $= 0$. Hence, the line of action of $Q$ must also pass through $E$.

Having completed the diagram by drawing the line of action of $Q$, some geometrical relations can be deduced.

The triangles $ACE$ and $CBE$ are similar, and hence

$$\text{Angle } CAE = \text{angle } CBE = 30°.$$

Now $CAD = 60°$; hence $EAD = 30°$. Also $CEB = 60°$.

Next, resolve all forces in the horizontal and the vertical direction. All forces directed towards the right, will be considered positive; those towards the left, negative. Forces acting vertically upwards, will be considered positive; those downwards, negative.

Component of $P$ along the horizontal $= P \cos 30°$.

" " $Q$ " " " $= -Q \cos 30°$.

Their sum, $$P \cos 30° - Q \cos 30° = 0 \qquad (1)$$

from which $$P = Q.$$

Component of $P$ along the vertical $= P \cos 60°$,

" " $Q$ " " " $= Q \cos 60°$,

" " 7 " " " $= -7$.

Their sum, $$P \cos 60° + Q \cos 60° - 7 = 0, \qquad (2)$$

from which $$P + Q = 14,$$

and using (1) $$P = Q = 7 \text{ pd.}$$

This completes the solution of the problem.

The value of $P$ might have been obtained more directly by taking moments of the forces about $A$.

The force $P$ tends to turn the rod in a direction opposite to the motion of the hands of a clock, or anti-clockwise. This will be considered to be the positive direction. The 7 pd. tends to produce a clockwise rotation which will be considered negative.

$$\begin{aligned} \text{Moment of } P \text{ about } A &= P \times AC, \\ \text{"} \quad \text{"} \; 7 \quad \text{"} \quad A &= -7 \times AD, \\ \text{"} \quad \text{"} \; Q \quad \text{"} \quad A &= 0. \end{aligned}$$

The sum $P \times AC - 7 \times AD = 0,$ (3)

From the figure, $AC = \frac{1}{2} AB$, and $AD = AB \sin 30° = \frac{1}{2} AB$.

Substituting in (3), $P = 7$ pd.

**68. Second Example.** A uniform beam $AB$, 17 feet long, whose mass is 120 lb., rests with one end against a smooth vertical wall, and the other end on a smooth horizontal floor, this end being tied by a string 8 feet long to a peg at the bottom of the wall. Find (1) the tension of the string, (2) the reaction of the wall, (3) the reaction of the floor.

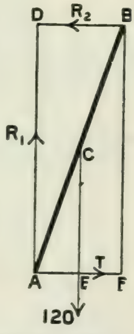


FIG. 65.—A beam resting upon a smooth wall and a smooth floor.

The forces acting on $AB$ (Fig. 65) are

(*a*) Its weight, 120 lb., acting vertically downward at its middle point $C$.

(*b*) The reaction of the floor, $R_1$, acting perpendicularly to the floor at $A$. The reaction of a smooth surface is at right angles to itself.

(*c*) The reaction of the wall, $R_2$, acting perpendicularly to the wall at $B$.

(*d*) The tension of the string, $T$, acting parallel to the floor at $A$.

In this case there are four forces acting and we cannot conclude that they meet at a point.

Equating to zero the algebraic sum of the horizontal forces,

$$T - R_2 = 0 \tag{1}$$

Equating to zero the algebraic sum of the vertical forces,

$$R_1 - 120 = 0 \tag{2}$$

or

$$R_1 = 120.$$

Equating to zero the algebraic sum of the moments of the forces about $A$,

$$R_2 \times AD - 120 \times AE = 0 \qquad (3)$$

or $$R_2 \times 15 - 120 \times 4 = 0$$

and $$R_2 = 32.$$

From (1) $$T = R_2 = 32.$$

**69. Third Example.** A uniform ladder, 40 feet long, whose mass is 160 lb., rests with one end on the top of a wall and is prevented from slipping by a peg driven into the ground at its lower end. If the inclination of the ladder to the horizon is 30°, find the pressure at the base and on the wall.

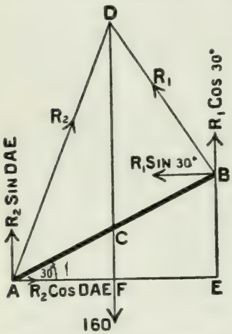



Fig. 66.—A ladder resting on a smooth wall and against a peg.

Let $AB$ (Fig. 66) be the ladder. The forces acting on it are:

($a$) Its weight, acting vertically downward at its middle point $C$.

($b$) The reaction of the wall, $R_1$, acting at right angles to the ladder. The ladder rests on the top of the wall at $B$, and its surface is supposed to be smooth.

($c$) The pressure at the peg, acting at $A$. The line of action of this force will be $AD$ because the three forces being in equilibrium their lines of action meet in a point.

Equating to zero the algebraic sum of the vertical components,

$$R_1 \cos 30° + R_2 \sin DAE - 160 = 0 \qquad (1)$$

Equating to zero the algebraic sum of the horizontal components,

$$R_2 \cos DAE - R_1 \sin 30° = 0 \qquad (2)$$

Equating to zero the algebraic sum of the moments of the forces about $A$,

$$R_1 \times 40 - 160 \times AF = 0 \qquad (3)$$

From (3)

$$40\, R_1 - 160 \times 20 \cos 30° = 0$$

or $$R_1 = 40\sqrt{3}.$$

Transposing, and dividing (1) by (2)

$$\frac{R_2 \sin DAE}{R_2 \cos DAE} = \frac{160 - R_1 \cos 30°}{R_1 \sin 30°}$$

or $$\tan DAE = \frac{160 - 60}{20\sqrt{3}} = \frac{5}{\sqrt{3}}.$$

Therefore, $\cos DAE = \dfrac{\sqrt{3}}{2\sqrt{7}}$

and, substituting in (2)

$$R_2 = 40\sqrt{7}.$$

**70. Fourth Example.** A uniform rod, 16 feet long, whose mass is 100 lb., is placed on two smooth planes whose inclinations to the horizon are 30° and 60° respectively. Find the pressure on each plane and the inclination of the rod to the horizon when in equilibrium.

Let $AB$ (Fig. 67) be the rod. The forces acting on it are:

($a$) Its weight, acting vertically downward at its middle point $C$.

($b$) The reaction of the plane at $A$, acting at right angles to the plane.

($c$) The reaction of the plane at $B$, acting at right angles to the plane.

Since the three forces are in equilibrium their lines of action meet in a point $D$.

The figure $ADBE$ is a rectangle.

Equating to zero the algebraic sum of the moments of the forces (1) about $A$, (2) about $B$, we have

$$R_2 \times AD - 100 \times AF = 0 \qquad (1)$$
$$100 \times BG - R_1 \times BD = 0 \qquad (2)$$

From (1)

$$R_2 \times AD - 100 \times AD \cos 30° = 0$$

or $$R_2 = 50\sqrt{3}.$$

From (2)

$$100 \times BD \cos 60° - R_1 \times BD = 0$$

or $$R_1 = 50.$$

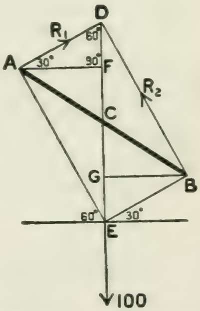


FIG. 67.—A rod resting on two smooth surfaces.

The inclination of the rod to the horizon =

$$CAF = CAD - DAF = CDA - DAF$$
$$= 60° - 30° = 30°.$$

**71. General Rules.** From the discussion of the above problems the following general rules may be given.

1. Construct a diagram of the system of forces which keep the body at rest, representing each force by a straight line

and its direction by an arrow. In drawing lines to represent the lines of action of the various forces the following facts should be observed:

($a$) The reactions of smooth surfaces are at right angles to these surfaces.

($b$) When three forces, not parallel, are in equilibrium their lines of action must meet in a point.

2. Equate to zero the algebraic sum of the components of the forces in two convenient directions at right angles. These relations will furnish two equations.

In choosing the directions for resolution, the solution is generally simplified by resolving along and at right angles to the directions of unknown forces. Forces not to be determined may thus be eliminated.

3. Equate to zero the algebraic sum of the moments of the forces about some convenient point. A third equation is thus furnished. If additional equations are required, they are obtained from the geometrical relations of the figure.

In choosing the point about which moments are to be taken, it is generally advisable to choose a point common to the directions of as many forces as possible. In this way also unknown forces not to be determined may be eliminated.

### PROBLEMS

1. A uniform beam is of length 12 metres and mass 50 kg., and from its ends are suspended bodies of masses 20 and 30 kg. respectively. At what point must the beam be supported that it may remain in equilibrium?

2. A lever with a fulcrum at one end is 3 feet in length. A mass of 24 pounds is suspended from the other end. If the mass of the lever is 2 pounds and acts at its middle point, at what distance from the fulcrum will an upward force of 50 pounds preserve equilibrium?

3. Masses of 7 lb., 1 lb., 3 lb., and 5 lb. are placed on a rod, supposed weightless, 1 foot apart. Find the point on which the rod will balance.

4. A bar 16 cm. long is balanced on a fulcrum at its middle. On the right arm are suspended 4 grams and 3 grams at distances of 5 cm. and 7 cm. respectively from the middle, and on the left arm 5 grams at a distance 5 cm. from the middle and $w$ at the end. Determine $w$.

5. A light rigid bar 30 feet long has suspended from its middle point a mass of 700 lb., and rests on two walls 24 feet apart, so that 1 foot of it projects over one of them. A mass of 192 lb. is suspended from a point 2 feet from the other end. What is the pressure borne by each of the walls?

6. Six parallel forces of 7 dynes, 6 dynes, 5 dynes, 4 dynes, 3 dynes and 2 dynes are applied to a rigid rod at points 1 metre apart. Find the magnitude and position of the resultant.

7. Five parallel forces 1, 6, 3, 4, 8 dynes act 1 metre apart on a straight horizontal rod. What force must be added to the 1 dyne, in order that if the rod is supported where the force of 3 dynes acts it may remain horizontal?

8. Four parallel forces 3, 2, 5, 7 dynes act at distances of 6 cm. apart along a straight rod and at right angles to it. Where must a force of 17 dynes act in order to maintain equilibrium?

9. A straight uniform heavy rod of length 6 feet has masses of 15 and 22 lb. attached to its ends, and rests in equilibrium when placed across a fulcrum distant $2\frac{1}{2}$ feet from the 22-lb. mass. Find the mass of the rod.

10. A straight rod 2 feet long rests in a horizontal position between two fixed pegs, placed at a distance of 3 inches apart, one of the pegs being at one end of the rod. If a mass of 5 lb. is suspended at the other end, find the pressure on each of the pegs.

11. A heavy uniform beam, whose mass is 40 kg., is suspended in a horizontal position by two vertical strings attached to the ends, each of which can sustain a tension of 35 kg. How far from the centre of the beam must a body, of mass 20 kg., be placed so that one of the strings may just break?

12. A heavy tapering rod, having a mass of 20 lb. attached to its smaller end, balances about a fulcrum placed at a distance of 10 feet from the end. If the mass of rod is 200 lb., find the point about which it will balance when the attached mass is removed.

13. A rod 6 inches long and 1 lb. mass is supported by two vertical strings at its ends. A mass of three pounds is attached to the rod at a distance of 1 inch from one end. At what distance from the other end must a mass of 4 lb. be attached in order that the tensions of the two strings may be equal?

14. A uniform rod whose mass is 60 lb. is movable about a hinge at one end. It is kept in equilibrium in a position making an angle of 30° with

the horizontal by a force making an angle of 30° with the rod at its other end. Find the reaction of the hinge and the directon of its line of action.

15. A uniform rod is suspended from a peg by two strings, one attached to each end. The strings are of such lengths that the angles between them and the rod are 30° and 60° respectively. Find the tensions of the strings, the mass of the rod being one kilogram.

16. A straight lever is inclined at an angle of 60° to the horizon, and a mass of 360 lb. hung freely at the distance of 2 inches from the fulcrum is supported by a force acting at an angle of 60° with the lever, at the distance of 2 feet on the other side of the fulcrum. Find the force.

17. A rod $AB$ movable about a hinge $A$ has a mass of 20 lb. attached at $B$. $B$ is tied by a string to a point $C$ vertically above $A$ and such that $CB$ is six times $AC$. Find the tension of the string $BC$.

18. A heavy uniform rod $AB$ whose mass is $W$ is hinged at $A$ to a fixed point, and rests in a position inclined at 60° to the horizon, being acted on by a horizontal force $F$ applied to the lower end $B$. Find the reaction of the hinge and the magnitude of $F$.

19. A light rod is hinged at one end and loaded at the other end with a weight of 6 pounds. The rod is supported in a horizontal position by a string which is attached to the loaded end, and which makes an angle of 30° with the rod. Find the tension of the string and the reaction of the hinge.

20. A uniform beam 32 feet long, whose mass is 200 lb., rests with one end on a smooth horizontal plane and the other end against a smooth vertical wall. If a string, 16 feet long, connects the lower end with the foot of the wall, find (1) the tension of the string, (2) the pressure against the wall, (3) the pressure on the plane.

21. A ladder, the weight of which is 90 pounds, acting at a point one-third of its length from the foot, is made to rest against a smooth vertical wall, and inclined to it at an angle of 30°, by a force applied horizontally at the foot. Find the force.

22. A uniform ladder, 40 feet long, whose mass is 180 lb., rests with one end against a smooth vertical wall and is prevented from slipping by a peg in the ground. Find the pressure against the wall and at the ground if the inclination of the ladder to the horizon is 60°.

23. A uniform beam, 12 feet long, whose mass is 50 lb., rests with one end $A$ at the bottom of a vertical wall, and a point $C$ in the beam 10 feet

from $A$ is connected by a horizontal string $CD$ with a point $D$ in the wall 8 feet above $A$. Find (1) the tension of the string, (2) the pressure against the wall.

24. A ladder, 14 feet long, whose mass is 50 lb., rests with one end against the foot of a vertical wall; and from a point 4 feet from the upper end a cord which is horizontal runs to a point 6 feet above the foot of the wall. Find the tension of the cord and the reaction at the lower end of the ladder.

25. A uniform heavy beam $AB$, whose mass is $W$, rests against a smooth horizontal plane $CA$ and a smooth vertical wall $CB$, the lower extremity $A$ being attached to a string which passes over a smooth pulley at $C$ and sustains a mass $P$. Find the pressure on the plane and the wall.

26. A carriage wheel, whose mass is $W$ and radius $r$, rests upon a level road. Show that the least force $F$ which will be on the point of drawing the wheel over an obstacle of height $h$ is

$$F = \frac{W\sqrt{(2rh - h^2)}}{r - h}$$

27. A weight of $10\sqrt{3}$ pounds hangs at the end of a string attached to a peg. If the weight is held aside by a horizontal force, so that the string makes an angle of 30° with the vertical, find the horizontal force and the tension of the string.

28. A weight is hung at the end of a string attached to a peg. If the weight is held aside by a horizontal force, so that the string makes an angle of 60° with the vertical, compare the tension of the string and the weight.

29. A weight of 10 pounds is supported by two strings, one of which makes an angle of 30° with the vertical. If the other string makes an angle of 45° with the vertical, what is the tension of each string?

30. A string fixed at its extremities to two points in the same horizontal line supports a smooth ring weighing 2 pounds. If the two parts of the string contain an angle of 60°, what is the tension of the string?

31. A weight of 12 pounds is supported by two strings, each of which is four feet long, the ends being tied to two points in a horizontal line 4 feet apart. What is the tension of each string?

32. A picture hangs symmetrically by means of a string passing over a nail and attached to two rings fixed to the picture. What is the tension of the string, if the picture weighs 6 pounds and the angle contained by the two parts of the string is 90°?

33. A ring, mass 9 lb., slides freely on a string of length $a\sqrt{2}$ whose ends are fastened to two points at a distance $a$ apart in a line making an angle of 45° with the horizon. Find the tension of string in the position of equilibrium.

34. A string is tied to two points. A ring, mass $W$, can slip freely along the string, and is pulled by a horizontal force $P$. If the parts of the string when in equilibrium are inclined at 90° and 45° respectively to the horizon, find the value of $P$.

35. A uniform bar, the weight of which is 100 pounds, is supported in a horizontal position by a string slung over a peg and attached to both ends of the bar. If the two parts of the string contain an angle of 120°, find the tension of the string.

36. A ball weighing 20 pounds slides along a perfectly smooth rod inclined at an angle of 30° with the vertical. What force applied in the direction of the rod will sustain the ball, and what is the pressure on the rod?

37. A body, the weight of which is 20 pounds, rests on a smooth plane, inclined to the horizon at an angle of 60°. Find (1) what force acting horizontally will keep the body at rest, (2) the reaction of the plane.

38. A body, the weight of which is 100 pounds, rests on a smooth plane inclined to the horizon at an angle of 30°. What force acting at an angle of 30° to the plane will keep the body at rest? What is the pressure on the plane?

39. Two weights of 2 pounds and $\sqrt{6}$ pounds respectively rest, one on each of two inclined planes which are of the same height and are placed back to back. The weights are connected by a string which passes over a smooth pulley at the common apex of the planes. If the first plane makes an angle of 60° with the horizon, find (1) the tension of the string, (2) the pressure on each plane, (3) the inclination of the second plane to the horizon.

# CHAPTER X

## FRICTION

**72. Friction is Resistance to Motion.** The word 'friction has already been used a number of times in the preceding chapters, as it is hardly possible to discuss the motion of a body without taking friction into account. In this chapter we shall study more closely some of its effects.

A heavy railway train may be running on a level track at the rate of a mile a minute, but if the steam is shut off the train will slow down and at last will come to rest. This is due to the friction in the bearings of the wheels and in the rolling of the wheels on the rails.

The machinery in a great factory may be 'humming,' but immediately after the 'power' is turned off at twelve o'clock the wheels slow up and come to rest in a few seconds.

FRICTION is that *resistance to the motion of a body when it slides or rolls over another.*

**73. Friction Depends on the Surfaces in Contact.** It is a common observation that the friction between two bodies depends upon the nature of the substances and the conditions of the surfaces which are in contact.

A sleigh may be drawn easily on a good road but when going over the planks or the rails at a railway crossing the horses have to exert much more of their strength, and if the ground is bare in some places the passengers may have to get out, to diminish the weight and so reduce the friction.

To slide a heavy plank over another requires more force if the surfaces are rough than if they are planed smooth.

**74. The Cause of Friction.** In the case of many surfaces the irregularities on them can be seen with the unaided eye, but even the smoothest surface when examined with a good microscope is seen to be covered with little projections with hollows between them. Hence when two surfaces are pressed together there is a kind of interlocking of these projections, which resists the motion of one surface over the other (Fig. 68).

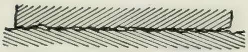


FIG. 68.—Section through two surfaces showing roughness as seen under a microscope.

**75. Experimental Study of Friction.** A simple apparatus like that shown in Fig. 69, enables us to investigate the laws of friction.

A flat block *M* rests on a board, which should be made as nearly horizontal as possible, and a cord attached to *M* passes over a pulley and bears a pan on the end of it. The block can be loaded to any desired amount and weights can be put on the pan.

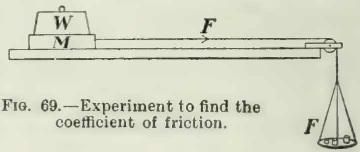


FIG. 69.—Experiment to find the coefficient of friction.

Let the horizontal board and the block *M* be both of dry pine. Clean the surfaces by rubbing with sand-paper and then wipe the dust off carefully. Also rub the block back and forth upon the board.

Weigh *M* and also the pan. Put a weight *W* on *M* and a smaller one on the pan, and let the weight of the pan and the mass on it be *F*. Then the block is pulled by a force *F* and (supposing there is no motion) sufficient friction between *M* and the board is called into action to balance *F*.

Continue adding weights to the pan, doing it carefully and avoiding jerks, until at last the block begins to move. Record the weight of the pan and its contents. Repeat the

experiment several times, and take the average of the several weights of the pan and its contents.

Let this average $= F$; also let $w =$ weight of the block $M$, and $W =$ the weight upon it.

Then find the value of $\frac{F}{W + w}$. This is called the *static coefficient of friction.*

By increasing the weight upon $M$ and obtaining the corresponding values of $F$ required to start the motion we secure a series of values of the coefficient of friction.

Next, try the same experiments, but instead of being very careful in placing the weights on the pan gently tap the board or give a slight jerk each time a new weight is put on. Continue to adjust the weights on the pan until the block moves forward with approximately uniform motion.

As before, obtain several values of $F$ with each value of $W$, and then calculate the value of $\frac{F}{W + w}$ for each average value of $F$. This quantity is now called the *kinetic coefficient of friction*, the word 'kinetic' meaning 'producing motion.'

The kinetic is considerably smaller than the static coefficient, which simply indicates that it is harder to *start* a body moving than to *keep it moving* when once motion has begun.

When speaking of the coefficient of friction the kinetic coefficient will always be understood unless the contrary is stated.

In the following table are given sample results for pine on pine, the grain of the block being parallel to that of the board.

COEFFICIENT OF FRICTION $\frac{F}{W+w}$, PINE ON PINE.

Block 15 cm. square, weight, 0.21 kg.

| $W + w$ in Kg. | Static | | Kinetic | |
|---|---|---|---|---|
| | $F$ in Kg. | Coeft. | $F$ in Kg. | Coeft. |
| 0.71 | 0.26 | 0.36 | 0.20 | 0.28 |
| 1.21 | .47 | .38 | .33 | .27 |
| 1.71 | .58 | .34 | .38 | .22 |
| 2.21 | .88 | .40 | .53 | .24 |
| 2.71 | .87 | .32 | .70 | .26 |
| 3.21 | .96 | .30 | .72 | .22 |
| 3.71 | 1.41 | .38 | .93 | .25 |
| 4.21 | 1.47 | .35 | 1.18 | .28 |
| 4.71 | 1.51 | .32 | 1.08 | .23 |
| 5.21 | 1.94 | .37 | 1.30 | .25 |
| Average........ | | 0.35 | | 0.25 |

From the results in this table we deduce

(1) *Friction varies directly with the pressure between the surfaces in contact;* or $F$ is proportional to $W + w$.

(2) *The static coefficient is considerably greater than the kinetic coefficient of friction;* or the friction at starting is greater than when uniform motion is maintained.

Next, take a block of different area, either larger or smaller. We find that the value required to cause motion in the case of a given value of $W$ is the same as before, and we conclude that

(3) *Friction is independent of the extent of the surfaces in contact.*

In this case the pressure per square inch is different but the total force of the block upon the board is the same as before when the area of the block was different.

There is another law which we cannot investigate with this apparatus but which is important. It is as follows:

(4) *Within wide limits the friction is independent of the rate of motion.*

This means that whether a machine is running rapidly or slowly the friction of the rubbing surfaces is approximately the same. However, this law is only approximately correct since, in general, friction with high speed is less than with slow speed of the moving surfaces. Thus when the engine driver sets the brakes on a train moving 60 miles per hour, the 'grip' on the wheels is not so powerful as when the speed is reduced to 20 miles or less per hour.

**76. Another Method of Determining the Coefficient of Friction.** Place the block with its load upon the board and continually raise one end of the board until the block slides down with uniform motion, the board being tapped to allow the block to start freely. Let the inclination be $i$ degrees (Fig. 70).

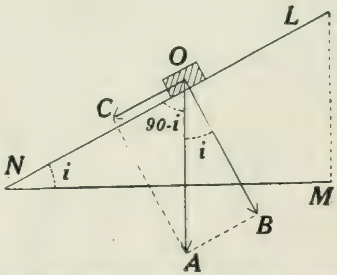



FIG. 70.—Finding the coefficient of friction with the inclined plane.

The weight of the block and its load acts vertically downwards. Let it be $W$ and be represented by $OA$.

This force may be resolved into two forces represented by $OB$ and $OC$, where $OB$ is perpendicular to the surface and $OC$ is parallel to it.

$$\text{Now, } OB = OA \cos i = W \cos i.$$

This component force is perpendicular to the surface, and is alone effective in producing friction between the surfaces. It

is the total force at right angles to the surface, and corresponds to $W + w$ in the experiments in the last section.

Again, $OC = OA \cos (90° - i) = OA \sin i = W \sin i$.

This component force is parallel to the surface and is that which causes the motion. It corresponds to $F$ in the last section.

Hence, if the body is moving uniformly,

$$\text{Coefficient of friction} = \frac{W \sin i}{W \cos i} = \tan i.$$

Now measure the inclination with a protractor* and the tangent of the angle = the coefficient of friction.

The angle $i$ is called the *angle of friction* or the *angle of repose.*

**Example**:—*Pine on Pine.*

$i = 14°$. From the mathematical tables $\tan 14° = 0.25$ = coefficient of friction.

**77. Magnitude of the Coefficients of Friction.** Since the surfaces of the bodies rubbing together are continually changing, the following values of the coefficients of friction must be considered as only roughly approximate:

Table of Coefficients of Friction

| | |
|---|---|
| Wood on wood, dry | 0.25 to 0.50 |
| " " " soapy | 0.20 |
| Metals on oak, dry | 0.50 to 0.60 |
| " " " soapy | 0.20 |
| Leather on oak | 0.27 to 0.38 |
| Metals on metals, dry | 0.15 to 0.20 |
| " " " wet | 0.30 |
| Iron on stone | 0.30 to 0.70 |
| Wood on stone | 0.40 |

The familiar stone-boat is made of wood or iron turned up in front, and is used for transporting stones, a barrel of water

* The tangent of the angle $i$ can be found by measuring the height $LM$ and the length $MN$, since $\tan i = LM/MN$.

or perhaps a plough from one part of the farm to another. When drawn over a dirt road the coefficient of friction is from 0.5 to 0.7. This is a large fraction of the weight, but yet the stone-boat is found useful for such jobs as those mentioned.

In launching a ship an abundance of soft soap is placed on the wooden ways down which the wooden cradle carrying the ship slides. From the above table the coefficient in this case is seen to be 0.20, which = tan $11\frac{1}{3}°$. Hence, if the ways make this angle with the horizontal the ship will slide down of itself when once the statical friction at starting has been overcome.

**78. Rolling Friction.** The resistance experienced by a body when rolling upon a surface is called *rolling friction* although in nature it is quite different from the resistance due to sliding.

Rolling friction is much smaller in magnitude than sliding friction. One may not be able to slide a heavy box over the floor and yet may move it without difficulty if rollers are put under it.

Consider the rolling of a wheel on a soft substance like india-rubber; it does not simply touch it (Fig. 71$a$) but sinks down, making a hollow with a mound on each side (Fig. 71$b$). As the wheel moves forward the mound behind practically disappears, but that in front continues there, and indeed is somewhat larger than when the wheel is at rest. The wheel is all the time trying to climb out of the hollow and "go over the top," but it never succeeds in doing so.

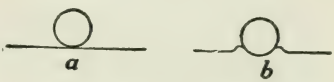


Fig. 71.—Illustrating rolling friction.

In the case of hard substances, like steel, the mound is small, but it exists, nevertheless; and indeed under a heavy load the wheel itself is slightly flattened.

The indentation in the surface can be reduced by increasing the diameter of the wheel and by widening its tire. As

self-binders and farm tractors have to pass over soft soil their driving wheels are large and broad.

In the ball-bearings and roller-bearings now so commonly used in automobiles and other high class machines the surfaces in contact are made very hard, thus reducing the rolling friction. In Fig. 72 is shown the bearings in the crank of a bicycle.

It is to be noted however, that while an ordinary carriage wheel rolls over the ground there is sliding friction in the hub—at the point *C* in Fig. 73.

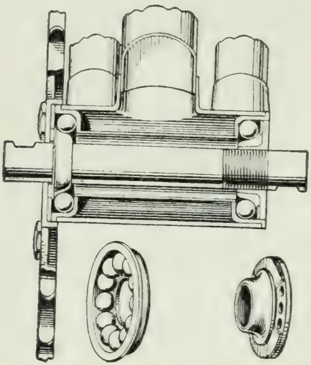


**Fig. 72.**—Section of the crank of a bicycle. The cup which holds the balls and the cone on which they run are shown separately below. Here the balls touch the cup in two points and the cone in one; it is a "three-point" bearing.

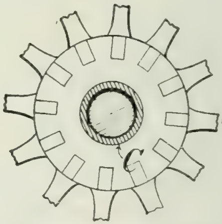



**Fig. 73.**—Section through a carriage hub, showing an ordinary bearing.

**79. Lubrication.** The amount of friction can be greatly reduced by lubricating the surfaces in contact. The oil or other substance used forms a thin film between the surfaces and in place of one solid rubbing upon another one layer of liquid moves over another and with much less friction.

**80. Utility of Friction.** But we must not think of friction simply as a waster of energy and an unmixed evil. As a matter of fact we make great use of it.

If it were not for friction we could not drive pulleys by means of belts, and nails and screws would be useless.

Thread and yarn would not hold together and textiles could not be woven. The experience of walking on smooth ice or on a polished floor suggests the difficulty of moving about if friction were altogether absent.

### QUESTIONS AND PROBLEMS

1. Explain the utility of friction in

    (*a*) Locomotive wheels on a railway track.

    (*b*) Leather belts for transmitting power.

    (*c*) Brakes to stop a moving car.

2. The current of a river is less rapid near its banks than in mid-stream. Can you explain this?

3. What horizontal force is required to drag a trunk weighing 150 pounds across a floor, if the coefficient of friction between trunk and floor is 0.3?

4. Give two reasons why is it more difficult to start a heavily-laden cart than keep it in motion after it has started?

5. A brick, 2 x 4 x 8 inches in size, is slid over ice. Will the distance it moves depend on what face it rests upon?

6. A mass of 10 pounds rests on a rough horizontal plane. If the coefficient of friction is .2, find the least horizontal force which will move the mass. Find also the reaction of the plane.

7. A force of 5 pounds is the greatest horizontal force that can be applied to a mass of 75 pounds resting on a rough horizontal plane without moving it. What is the coefficient of friction?

8. A mass of 10 pounds is resting on a rough horizontal plane, and is acted on by a force which makes an angle of 45° with the plane. If the coefficient of friction is .5, find the force.

9. A body resting on a rough horizontal plane is on the point of moving when acted on by a force equal to its own weight inclined to the plane at an angle of 30°. Find the coefficient of friction.

10. A body placed on a rough plane is just on the point of sliding down when the plane is inclined to the horizon at an angle of (1) 60°, (2) 45°, (3) 30°. What is the coefficient of friction in each case?

11. A body placed on a rough inclined plane is on the point of sliding when the plane rises 3 feet in 6 feet. What is the coefficient of friction?

12. A mass of 20 lb. rests on a rough plane inclined at an angle of 30° to the horizon. What force must be applied parallel to the plane that it may be on the point of moving up the plane, the coefficient of friction being .1?

13. A body, the mass of which is 30 lb., rests on a rough inclined plane, the height of the plane being $\frac{3}{5}$ of its length. What force must be applied to the body parallel to the plane that it may be on the point of moving up the plane, the coefficient of friction being .75?

14. The load on the driving wheels of a locomotive is 60 tons and the coefficient of friction between rails and wheels is $\frac{1}{6}$. What is the greatest force the locomotive can exert? If its entire mass is 75 tons, what is the greatest speed it can give to itself in 10 sec.?

15. A mass of 14 lb. when placed on a rough plane inclined to the horizon at an angle of 60° slides down unless a force of at least 7 pounds acts up the plane. What is the coefficient of friction?

16. A mass of 20 lb. is on the point of moving up a rough plane inclined to the horizon at an angle of 45° when a horizontal force is applied to it. Find the horizontal force, if the coefficient of friction is .1.

17. A body, the mass of which is 4lb., rests in limiting equilibrium when the inclination of the plane to the horizon is 30°. Find the force which acting parallel to the plane will support the body when the inclination of the plane to the horizon is 60°.

18. A body placed on a rough plane inclined to the horizon at an angle of 30° is just on the point of moving upward when acted upon by a horizontal force equal to its own weight. Find the coefficient of friction.

19. If the smallest force which will move a mass of 3 lb. along a horizontal plane is $\sqrt{3}$ pounds, find the greatest angle at which the plane may be inclined to the horizon before the mass begins to slide.

20. Show that in order to relieve a horse in drawing a sleigh the traces should be so placed as to make the angle of friction with the ground.

# CHAPTER XI

## Gravitation

**81. Bodies attracted by the earth.** "Whatever goes up must come down" is a truth learned in early childhood. Whether the body be set free when high up in the air or when near the earth's surface, it begins to fall at once and does not cease until it reaches some obstacle which blocks the descent. We recognize also that at whatever place we may be the body descends in the vertical direction, that is, along a line perpendicular to the earth's surface at that place. Now the earth is (very approximately) a sphere and a vertical line at the equator makes a right angle with the vertical at the pole. It is clear that we can describe the motion of a falling body as being along the radius of the sphere.*

We 'explain' this phenomenon of falling bodies by saying that it is due to the attraction of the earth which apparently tries to draw all bodies to its centre.

It was Galileo (1564-1642) who, as a result of experimental investigation, stated accurately the laws followed by bodies moving under the attraction of the earth. He showed that the acceleration of a falling body is uniform and is independent of the nature or the quantity of matter in it. Since his time it has been shown experimentally that the acceleration of gravitation, while constant at any place, varies with the position on the earth's surface. (See Secs. 35, 36).

**82. Newton's Law of Gravitation.** For many centuries it was commonly believed that the earth was at the centre of the universe and that the sun and the other heavenly bodies revolved about it. This view is known as the Ptolemaic

* On account of the earth being in rotation and also since it is not a perfect sphere this statement is not strictly accurate, but it is very approximately so. (See Sections 50, 51).

hypothesis. At last, however, it was overthrown, being superseded by the theory advocated by Copernicus* (1473-1543), and hence known as the Copernican theory. According to it, the sun is the central body of our system, and the planets, including the earth, revolve about it in circles, but with the sun not exactly at the centre of each. After this, Tycho Brahe (1564-1601) made many accurate observations of the planets, especially of Mars, though he did not have the telescope to help him. These observations were placed in the hands of his pupil, John Kepler (1571-1630), who spent many years studying them and at last enunciated his famous laws of planetary motion.†

Kepler did not assign any reason why the planets should obey his laws, but simply showed that they must move thus in order to satisfy Tycho's observations. It was felt by scientific men that there was some physical principle which would account for them and for fifty years the matter was a favourite subject for conjecture and discussion. At last Newton (1642-1727) proved that if we start with the simple hypothesis that *the sun attracts each planet with a force which is inversely proportional to the square of its distance*, Kepler's laws must necessarily follow.

On further consideration Newton was led to the view that *each* body attracts *every other* body in the same way that the sun attracts the planets. This is known as the PRINCIPLE OF UNIVERSAL GRAVITATION and may be stated as follows:

*The attraction between any two bodies is directly proportional to the product of their masses and inversely as the square of the distance between them.*

Let $m$, $m'$ be the masses of the two bodies, $r$ the distance between them.

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*Copernicus was born at Thorn, near the mouth of the Vistula River, in Poland. Much fighting occurred in this neighborhood during the Great War.

† For Kepler's laws and further information on this interesting portion of the history of science see any work on astronomy.

The force of attraction is proportional to $\frac{m\,m'}{r^2}$,

$$\text{or } F = k\,\frac{m\,m'}{r^2},$$

where $k$ is a numerical constant.

**83. Application to the Earth.** Consider a mass $m$ at $A$ on the earth's surface (Fig. 74), and let the mass of the earth be $M$.

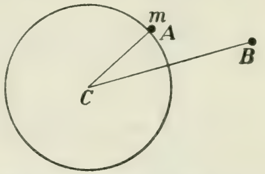

Fig. 74.—Attraction of the earth on a mass on its surface and also twice as far away from the centre.

Then, according to Newton's Law the force of attraction between the two bodies is proportional to $M \times m$. But what is the distance between them? They are actually in contact.

Now it can be shown by mathematical calculation that a homogeneous sphere* attracts as though all the matter in it were concentrated at its centre. This point is its *centre of mass.* Indeed every body has a centre of mass and the distance between two bodies is to be understood as the distance between their centres of mass. The centre of mass of a body coincides with its centre of gravity (see next chapter).

If $m$ is a pound-mass on the earth's surface, the attraction of the earth on it is 1 pound-force or 1 pound-weight.

If $m$ is a gram-mass, the attraction is 1 gram-force or 1 gram-weight.

If the mass of $m$ is 100 grams, the attraction is 100 grams-force.

If the mass of the earth could be doubled without altering its radius, the attraction would be doubled.

The force is proportional to the product of the masses.

Again, suppose the pound-mass to be at $B$, 2 radii or 8000 miles from $C$. The attraction is now not $\frac{1}{2}$, but $\frac{1}{2^2}$ or $\frac{1}{4}$ of 1 pound-weight or pound-force.

If it were 6000 miles or $\frac{3}{2}$ of the radius, the force $= \frac{1}{(\frac{3}{2})^2} = \frac{4}{9}$ of a pound-weight.

Read Section 23 again.

* A sphere is homogeneous when it is similar in all directions from the centre. It may differ at different distances from the centre but it is the same at the same distance.

**84. Attraction on the Moon.** Let us calculate the weight of a pound-mass on the surface of the moon.

The moon's diameter is 2163 miles and the earth's is 7918 miles, but for ease in calculation we shall take these numbers as 2000 and 8000 respectively (Fig. 75).

Assuming, then, the radius of the moon to be $\frac{1}{4}$ that of the earth, its volume is $\frac{1}{64}$ that of the earth, and if the two bodies were equally dense the moon's mass would also be $\frac{1}{64}$.

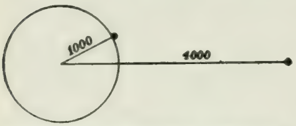

FIG. 75.—Attraction on the moon is one-sixth that on the earth.

In this case the attraction on a pound-mass at a distance of 4000 miles from its centre would be $\frac{1}{64}$ of a pound-force.

But the distance is 1000 miles, or $\frac{1}{4}$ of this, and the attraction on this account would be $4^2$ or 16 times as great.

Hence, attraction $= 16 \times \frac{1}{64} = \frac{1}{4}$ pound-force.

But the density of the moon is only $\frac{6}{10}$ that of the earth; and so the attraction

$$= \frac{6}{10} \times \frac{1}{4} = \frac{6}{40} = \frac{1}{6} \text{ approximately.*}$$

Hence, if we could visit the moon, retaining our muscular strength, we would lift 600 pounds with the same ease that we lift 100 on the earth. If you can throw a base-ball 100 yards here, you could throw it 600 there.

On the surface of the sun, so immense is that body, the weight of a pound-mass is 27 pounds-force.

**85. The Cavendish Experiment.** The mass of the earth is so great that its attraction upon a mass at its surface is easily detected and measured, but between ordinary bodies the attraction is extremely small and to measure it is a task of great difficulty.

The first successful attempt to determine experimentally the attraction between two known masses was made in 1798 by Henry Cavendish,† an eccentric but very able English physicist and chemist.

* A more accurate calculation is

$$\left(\frac{2163}{7918}\right)^3 \times \left(\frac{7918}{2163}\right)^2 \times \frac{6}{10} \approx \frac{6489}{39590} = \frac{1}{6.101}.$$

† Cavendish was the first to recognize hydrogen as a distinct substance.

The apparatus he used is illustrated in Figs. 76 and 77. Two lead balls *b*, *b*, 2 inches in diameter were hung from the

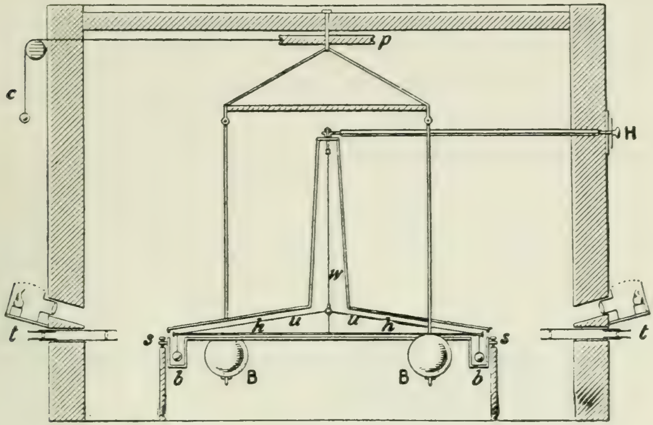


FIG. 76.—Elevation of Cavendish's apparatus.

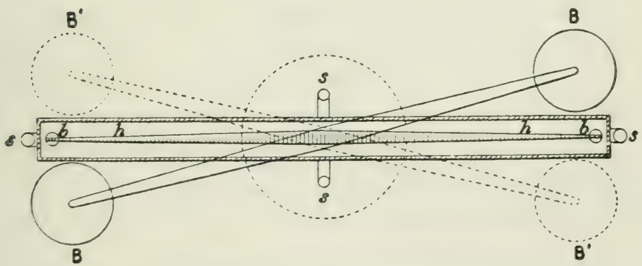


FIG. 77.—Plan of Cavendish's apparatus.

ends of a light wooden rod *h*, *h*, 6 feet long. This rod was stiffened by wires *u*, *u*, and was suspended from the top of an inner case by a wire *W*, $39\frac{1}{4}$ inches long. The upper end of the wire was attached to a torsion-head which rested on the top of the case and which could be rotated by a rod extending to the outside of an outer case. By turning *H* any twist could be taken out of the wire, and the pendulum, that

is the rod with the attached spheres, could be made to swing free from the case.

If we know the dimensions and mass of each part of the pendulum, and if we determine by observation the time it takes a pendulum to perform a to-and-fro oscillation, we can calculate the force which is required to turn it through any given angle, 1° for example.

Two large lead spheres $B$, $B$; 12 inches in diameter, were supported by a frame and suspended from the top of the outer case. By means of a cord $c$ passing about the circumference of a disc $p$ it was possible to swing these spheres from the position $B$, $B$ to the position $B'$, $B'$ shown in dotted outline (Fig. 77). In the first position the spheres $b$, $b$ will be drawn in one direction; in the second position they will be drawn in the opposite direction.

The lamps $L$, $L$ illuminated scales on the ends of the rod and by the telescopes $t$, $t$ the deflection of the rod when the heavy spheres were in the two positions could be read. From the deflection thus measured the turning force could be calculated and this was, of course, the measure of the attraction of the large spheres for the smaller ones.

In performing the experiment great difficulty was experienced through air currents, and the whole apparatus was placed in a large chamber which was kept constantly closed.

**86. Repetitions of Cavendish's Experiment.** The experiment has been repeated several time with many variations in the apparatus. In the investigation made by C. V. Boys (1890-93), the pendulum arm was only $\frac{1}{2}$ inch long and the spheres hung from its ends were of gold and $\frac{1}{4}$ inch in diameter. The attracting spheres were of lead, in one series of experiments being $4\frac{1}{4}$ inches, and in another, $2\frac{1}{4}$ inches in diameter. Notwithstanding the small dimensions, definite attraction between the spheres was detected and accurately measured.

**87. Results of Experiments.** From his experiments Boys calculated that two small spheres, each containing 1 gram of matter,* when placed with their centres 1 cm. apart, attract each other with a force of 0.000,000,064,8 dyne = $\frac{1}{15000000}$ dyne (approximately).

Having determined the attraction between two bodies of known mass, it is possible to calculate what must be the mass of the earth in order that it may exert the attraction upon a body at its surface which we have observed. Then, knowing its mass and its volume, we can calculate its average density. Cavendish found it to be 5.45 grams per cc.; Boys found 5.53 grams per cc.

This is about twice the density of substances in the crust of the earth; consequently (as we might expect), its density is greater as we descend below the surface.

#### QUESTIONS AND PROBLEMS

1. If the earth's mass were doubled without any change in its dimensions, how would the weight of a pound-mass vary?

Could one use ordinary balances and the same weights as we use now?

2. Find the weight of a body of mass 100 kilograms at 6000, 8000, 10,000 miles from the earth's centre.

3. The diameter of the planet Mars is 4230 miles and its density is $\frac{7}{10}$ that of the earth. Find the weight of a pound-mass on the surface of Mars.

4. The attraction of the earth on a mass at one of its poles is $\frac{1}{568}$ greater than at the equator. Why is this?

5. A spring-balance would have to be used to compare the weight of a body on the sun or the moon with that on the earth. Explain why.

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* A lead sphere 5.5 mm. in diameter (the size of a large pea), contains 1 gram of matter.

## CHAPTER XII

### Centre of Gravity

**88. A Unique Central Point in Every Body.** When one side of a carriage is somewhat lower than the other we experience an uncomfortable sensation, as we know that there is a definite position beyond which we must not go or the carriage will upset.

Next, consider a rectangular block of wood, or a brick, resting on a flat surface. Gradually raise one side until the middle point of the block is just over the line along which it touches the surface. This is a critical position, and if the body is turned any more it will topple over on another face. Try with the new face. When the central point gets beyond the line of support, over the block falls to a new position of rest.

Again, push a book or a piece of board slowly over the edge of a table. It rests safely on the table until it reaches a certain definite position, when it is seen to totter, and if pushed any farther it falls. When in the tottering position draw a line on the underside along the edge of the table. Now turn the object about and push it over the edge again, drawing another line when it is in its critical position. Repeat this several times and then look at the lines drawn. They all meet very approximately in a point, and thus it is seen that as soon as that particular point gets beyond the line of support the body falls over into a new position.

Our everyday experience leads us to believe that there is a unique central point in a body, and if that point goes beyond a certain position the body moves over into a new position of rest.

**89. Experiments with a Thin Flat Body.** Support an irregular-shaped sheet of metal, or a piece of cardboard, at $A$

(Fig. 78*a*), by hanging it on a pin or in some other convenient way. Have a cord attached to the pin, with a small weight on the end of it. Chalk the cord and snap it on the plate, thus making a straight white line across it. Next support the body at $B$ (Fig. 78*b*), and obtain another chalk line. Let it cut the other line at $G$. Support the plate at other places and get other lines on it. All the lines cut at a single point—the point $G$—which must be a unique point in the plate.

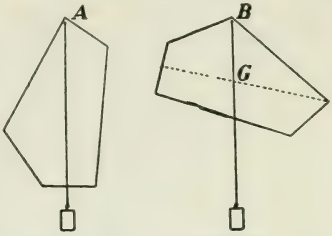



FIG. 78*a*. FIG 78*b*.
How to find the centre of gravity of a flat body.

Now try to balance the plate on the end of a finger. You find the plate balances if it is supported at $G$. But it is simply the weight of the plate that the finger has to overcome, and we conclude, then, that *the entire weight of the body may be considered as concentrated at $G$.* This point is called the CENTRE OF GRAVITY of the body. The abbreviation C.G. will be used for Centre of Gravity.

**90. Composition of Forces due to Gravity.** A body consists of a very great number of particles, and according to the principle of Universal Gravitation the earth attracts every particle with a force which we call its weight. The lines of action of these forces are directed to the centre of the earth, but since that point is 4000 miles away, the directions of the forces may be taken to be parallel.

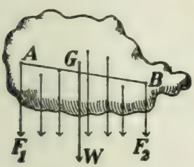



FIG. 79.—The weight of a body acts at its centre of gravity.

These forces will have a single resultant acting at a definite point fixed in the body, as can be seen in the following way :

The two forces $F_1$, $F_2$, acting on particles at $A, B$ (Fig. 79) will have a resultant acting at a point in $AB$, its distance from $A$ and $B$ being inversely as the magnitudes of the

forces acting there. Further, if the body moves into another position the magnitude and point of action of this resultant will be unchanged. (Sec. 64).

Next, combine this resultant with a third force, and obtain the resultant of the three forces.

Then combine this resultant with a fourth force; and continuing in this way we at last reach a single resultant of all the forces and acting at a definite point in the body.

The sum of all these separate forces is the weight of the body and the point of application of the resultant force is the centre of gravity of the body.

**91. To Find the Centre of Gravity of a Body of any Form.** Suspend the body by a cord attached to any point $A$ (Fig. 80) in it. Then there are two forces acting on the body, namely, the weight acting downwards at $G$ and the tension of the string acting upwards at $A$. These are equal in magnitude and form a couple. They cause the body to rotate until $G$ is directly beneath $A$, in which case the line of action of the weight coincides with the direction of the string, and the tension of the string will just balance the weight of the body. The body will then be in equilibrium.

FIG. 80.—How to find the centre of gravity of a body of any form.

Thus, if the body is suspended at $A$ and allowed to come to rest the direction of the supporting string will pass through the centre of gravity.

Next attach a cord at $B$ and hang up the body as before. The direction of the cord will again pass through the C.G. That point, therefore, will be where the two lines intersect.

This experimental method may be employed to determine the C.G. of any kind of body at all, and indeed in many cases it is the only available method. But when the body is of simple form it is often easy to determine the position of the C.G. from geometrical considerations.

**92. Centre of Gravity of Simple Geometrical Figures.** (1) *A uniform straight bar.* For a uniform straight bar $AB$ (Fig. 81) it seems evident that the C.G. is at its mid-point.

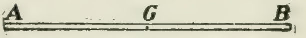



FIG. 81.—Centre of gravity of a uniform rod.

We may, however, consider it as made up of equal particles uniformly distributed from one end to the other. The C.G. of equal particles at $A$ and $B$ will be half-way between, at $G$. In the same way the C.G. of the particles next to $A$ and $B$ will be at $G$; and continuing in this way we find the C.G. of all to be at $G$.

(2) *A uniform parallelogram.* This may be considered as made up of uniform thin strips, such as $LM$, parallel to the side $AB$. The C.G. of $LM$ is at its mid-point $g$, and the mid-points of all such rods will be on a line $EF$, midway between the sides $AD$, $BC$. The C.G. of the parallelogram is evidently in this line.

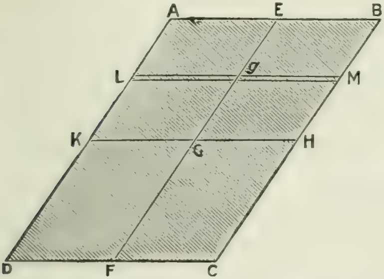



FIG. 82.—Finding the centre of gravity of a parallelogram.

In the same way we may consider the parallelogram as made up of uniform rods parallel to $AD$, and the C.G. of each will be on the line $KH$, midway between $AB$, $DC$; and the C.G. of the parallelogram will be somewhere in $KH$.

Hence, the C.G. of the parallelogram is where $EF$ and $KH$ intersect, that is, at $G$. This is the geometrical centre of the parallelogram, and is where the diagonals meet. (Fig. 83).

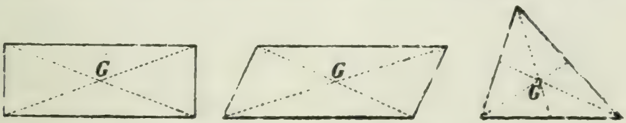



FIG. 83.—Centre of gravity of a parallelogram and a triangle.

(3) *A triangular plate.* The plate may be considered to be made up of a series of thin rods like $LM$, parallel to the side $BC$, and the C.G. of each rod is at its mid-point $g$. The median line $AE$ (that is, the line joining $A$ to the mid-point of the opposite side $BC$) bisects all such rods, and hence the C.G. of the triangle is on this line.

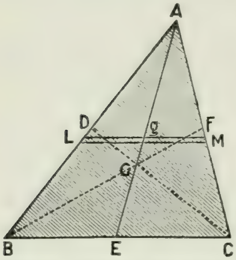


FIG. 84.—Finding the centre of gravity of a triangle.

In the same way it can be shown that the C.G. is on the median line $BF$ and also on the median line $CD$. Consequently it must be at $G$ where these three lines intersect.

It can be shown that $EG = \frac{1}{3}\ EA$.

It may be of interest to know the positions of the C.G. in some familiar geometrical forms. The C.G. of a pyramid or a cone is on the line joining the vertex to the C.G. of the base and one-fourth of the way up. The C.G. of a solid hemisphere is $\frac{3}{8}$ths of its radius from the flat face, that is, from its geometrical centre. The C.G. of a hemispherical shell is half-way between centre and circumference.

**PROBLEMS**

1. An isosceles triangle has its equal sides of length 5 cm. and its base of length 6 cm. Find the distance of the centre of gravity from each of the angular points.

2. If the angular points of one triangle lie at the middle points of the sides of another, show that the centres of gravity of the two are coincident.

3. The equal sides of an isosceles triangle are 10 feet, and the base is 16 feet in length. Find the distance of its centre of gravity from each of the sides.

4. The sides of a triangle are 3, 4, and 5 feet in length. Find the distance of the centre of gravity from each side.

5. The sides of a triangular lamina are 6, 8, and 10 feet in length. Find the distance of the centre of gravity from each of its angular points.

6. The sides $AB$, $AC$ of a triangle $ABC$, right-angled at $A$, are respectively 18 and 12 inches long. Find the distance of the centre of gravity from $C$.

7. An equilateral triangle is described upon one side of a square whose side is 16 inches. Find the distance of the centre of gravity of the figure

so formed from the vertex of the triangle, the vertex being without the square.

8. The length of one side of a rectangle is double that of an adjacent side, and on one of the longer sides an equilateral triangle is described externally. Find the centre of gravity of the whole.

9. A piece of cardboard is in the shape of a square $ABCD$ with an isosceles right-angled triangle described on the side $BC$. If the side of the square is 12 inches, find the distance of the centre of gravity of the cardboard from the line $AD$.

10. An isosceles right-angled triangle is described externally on the side of a square as hypotenuse. Find the centre of gravity of the whole figure.

11. A square is described on the base of an isosceles triangle. What is the ratio of the altitude of the triangle to its base when the centre of gravity of the whole figure is at the middle point of the base?

12. $ABCD$ is a square whose middle point is $E$ and whose side $= a$. If the triangle $ECD$ is removed, find the centre of gravity of the remainder.

13. $E$ and $F$ are the middle points of the sides $AB$, $AC$ of an equilateral triangle $ABC$. If the portion $AEF$ is removed, find the centre of gravity of the remainder.

14. $ABCD$ is a square, $O$ its centre, $E$ and $F$ the middle points of $AB$, $AD$. If $AEF$ is cut away, find $G$, the centre of gravity of the remainder.

15. From a square piece of paper $ABCD$ a portion is cut away in the form of an isosceles triangle whose base is $AB$ and altitude equal to one-third $AB$. Find the centre of gravity of the remaining portion.

16. $ABCD$ is a rectangle, $E$ the middle point of $CD$; the triangle $ADE$ is cut away. Find the centre of gravity of the remainder.

**93. Centre of Gravity of Weights on a Rod.** Let $AB$ be a light rod, of negligible weight and 40 cm. long, with 1 kg.


FIG. 85.—To find centre of gravity of three weights in a straight line.

at $A$, 4 kg. 30 cm. from $A$, and 5 kg. at $B$ (Fig. 85). We have to find the C.G. of the system.

First, find the C.G. of 1 kg. and 4 kg. It is at the point $D$, where $AD/DC = 4/1$, or $1 \times AD = 4 \times CD$.

$$\text{Hence, } AD = 24,\ DC = 6 \text{ cm.}$$

For 1 kg. and 4 kg. we can substitute 5 kg. at $D$, and we have now to find the C.G. of 5 kg. at $D$ and 5 kg. at $B$, and $DB = 16$ cm. The C.G. is at $E$, half-way between $D$ and $B$, or 8 cm. from $B$ and 32 cm. from $A$.

The process used here may be followed in the case of any number of weights, but it is generally easier to apply the principle of moments, as follows:

Let the distance of the C.G. from $A = x$ cm. Then for the three weights, 1, 4, 5 kg. at $A$, $C$, $B$, respectively, we can substitute their sum 10 kg. at the C.G. If now the rod be pivoted on a horizontal axis at $A$ at right angles to the rod, so that it can turn in the vertical plane, then the tendency of the rod to turn about this axis will be measured by taking the moments of the forces 4 kg. and 5 kg. about this axis.

$$\begin{aligned} \text{Moment of 4 kg. wt. about } A &= 4 \times 30 = 120 \\ \text{" \quad " 5 \quad " \quad " \quad " } A &= 5 \times 40 = 200 \\ \text{Entire tendency to turn} = \text{sum of moments} &= 320 \end{aligned}$$

Moment of entire weight at $E = 10 \times x = 10\,x$.

$$\text{Hence, } 10\,x = 320,$$

$$\text{and} \qquad x = 32 \text{ cm.}$$

This method may be used for any number of weights. As an example, find the C.G. of 3, 4, 5, 6, 7 kg., where 3 is at one end of a rod and the others are at distances 5, 9, 10, 25 cm., respectively. (Answer, $x = 12$ cm.).

### PROBLEMS

1. Masses of 2 lb., 4 lb., 6 lb., 8 lb., are placed so that their centres of gravity are in a straight line, and six inches apart. Find the distance of their common centre of gravity from that of the largest mass.

2. Two masses of 6 lb. and 12 lb. are suspended at the ends of a uniform horizontal rod, whose mass is 18 lb. and length 2 ft. Find the centre of gravity.

3. A rod, 1 foot in length and mass 1 ounce, has an ounce of lead fastened to it at one end, and another ounce fastened to it at a distance from the other end equal to one-third of its length. Find the centre of gravity of the system.

4. Four masses of 3 lb., 2 lb., 4 lb., and 7 lb., respectively, are at equal intervals of 8 inches on a lever without weight, 2 feet in length. Find where the fulcrum must be, in order that they balance.

5. A uniform bar, 3 feet in length and of mass 6 ounces, has three rings, each of mass 3 ounces, at distances 3, 15 and 21 inches from one end. About what point of the bar will the system balance?

6. A ladder, 50 feet long and mass 100 lb., is carried by two men; one lifts it at one end, and the other at a point 2 feet from the other end. The first carries two-thirds of the weight which the second does. Where is the centre of gravity of the ladder?

7. A pole, 10 feet long and mass 20 lb., has a mass of 12 lb. fastened to one end. The centre of gravity of the whole is 4 feet from that end. Where is the centre of gravity of the pole?

8. Four masses, 1 lb., 4 lb., 5 lb., and 3 lb., respectively, are placed 2 feet apart on a rod 6 feet long, whose mass is 3 lb. and centre of gravity 2 feet from the end at which the 1 lb. is placed. Find the centre of gravity of the whole.

9. A cylindrical vessel whose mass is 4 lb. and depth 6 inches will just hold 2 lb. of water. If the centre of gravity of the vessel when empty is 3.39 in. from the top, determine the position of the centre of gravity of the vessel and its contents when full of water.

10. A cylindrical vessel, without lid, one foot in diameter and one foot in height, is made of thin sheet metal of uniform thickness. If it is half filled with water, where will be the common centre of gravity of the vessel and the water, assuming the mass of the vessel to be one-fifth the mass of the contained water?

**94. Centre of Gravity of Weights in a Plane.** The method of moments may be applied to masses distributed over a plane.

Let $ABCD$ be a uniform square board with sides 26 inches long and of mass 8 lb., and let masses 4, 6, 5, 3 lb. be placed at the corners $A, B, C, D$ (Fig. 86). We wish to find the C.G. of the system.

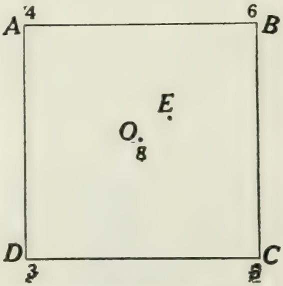



Fig. 86.—Weights at the corners of a board.

Let the C.G. be at $E$, and be $x$ inches from $AD$ and $y$ inches from $DC$. The total mass = 26 lb.

1st, Take moments about $AD$.

The masses 4 and 3 are on the line $AD$ and give no moments about $AD$.

$$\begin{aligned} \text{Moment of } 6 &= 6 \times 26 = 156 \\ \text{,, ,, } 5 &= 5 \times 26 = 130 \\ \text{,, ,, board} &= 8 \times 13 = 104 \\ \hline \text{Moment of whole} &= 390 \end{aligned}$$

$$\begin{aligned} \text{But moment of whole } &= 26 \times x = 26\,x \\ \text{Therefore } 26\,x &= 390, \\ \text{and } x &= 15 \text{ inches.} \end{aligned}$$

2nd, Take moments about $DC$.

$$\begin{aligned} \text{Moment of } 4 &= 4 \times 26 = 104 \\ \text{,, ,, } 6 &= 6 \times 26 = 156 \\ \text{,, ,, board} &= 8 \times 13 = 104 \\ \hline \text{Moment of whole} &= 364 \end{aligned}$$

$$\begin{aligned} \text{But moment of whole } &= 26 \times y = 26\,y. \\ \text{Therefore } 26\,y &= 364, \\ \text{and } y &= 14 \text{ inches.} \end{aligned}$$

Hence, the C.G. is at $E$, 15 in. from $AD$ and 14 in. from $CD$.

In this example moments were taken about $AD$ and $DC$, but any other lines might be chosen. As an exercise, solve the problem by taking moments about $BC$, $CD$.

**PROBLEMS**

1. Masses of 1, 1, 1 and 2 lb., are placed at the angular points of a square. Find their centre of gravity.

2. Masses of 2 lb., 1 lb., 2 lb., 3 lb., are placed at $A$, $B$, $C$, $D$ respectively, the angular points of a square. Find the distance of the centre of gravity from the centre $O$.

3. Masses of 1, 4, 2, 3 lb., are placed at the corners $A$, $B$, $C$, $D$ of a rectangle; a mass of 10 lb. is also placed at the intersection of the diagonals. If $AB = 7$ inches, and $BC = 4$ inches, find the distance of the centre of gravity of the whole from $A$.

4. At the angular points of a square, taken in order, there act parallel forces in the ratio 1 : 3 : 5 : 7. Find the distance from the centre of the square of the point at which their resultant acts.

5. Masses 5, 7, 10 are placed at the three angles of a square whose side = 4 ft. Find the distance of their centre of gravity from 5.

6. Three masses 3, 4, 5 lb. are placed at the angles of an equilateral triangle whose sides are 12 inches. Find the distance of the centre of gravity of the whole from the least mass.

7. $ABC$ is a triangle right-angled at $A$, $AB$ being 12 and $AC$ 15 inches in length. Masses in the ratio 2 : 3 : 4 are placed at $A$, $C$, and $B$ respectively. Find the distances of their centre of gravity from $B$ and $C$.

8. Prove that the centre of gravity of an equilateral triangular lamina coincides with that of three equal masses placed at its angular points.

**95. Condition for Equilibrium.** In the case of a body resting on a surface there are two forces acting on the body,—

(i) The weight of the body acting vertically downwards through its centre of gravity;

(ii) The reaction of the surface, which is the resultant of the various forces upwards exerted by the surface upon the body.

If the body is in equilibrium it is evident that these two forces must be equal in magnitude and must act in the same line but in opposite directions.

Consider the stool $C$ in Fig. 87. The reactions of the surface are at the points where the feet rest on the surface, and the resultant of these reactions must be a single force within the area formed by a cord drawn closely about the legs. This area is the *supporting base.* It is clear, then, that for a body to rest in equilibrium the vertical through the C.G. must fall within the supporting base.

This is seen to be the case in $A$ (Fig. 87), but it is not so in $B$, and the cylinder will topple over. In $D$ the wagon is in the critical position.

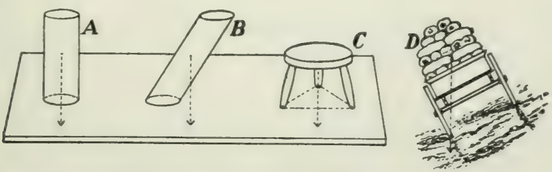



Fig. 87.—$A$ and $C$ are in stable equilibrium; $B$ is not, it will topple over; $D$ is in the critical position.

The famous Leaning Tower of Pisa is an interesting case of stability of equilibrium. It is circular in plan, 51 feet in diameter and 172 feet high, and has eight stages, including the belfry. Its construction was begun in 1174. It was founded on wooden piles driven in boggy ground, and when it had been carried up 35 feet it began to settle to one side. The tower overhangs the base upwards of 13 feet, but the centre of gravity is so low down that a vertical through it falls within the base and hence the equilibrium is stable.

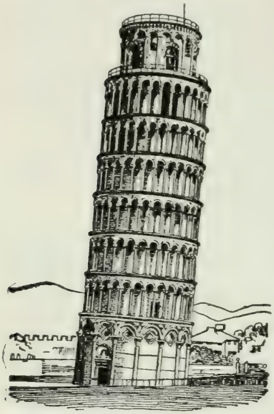


FIG. 88.—The Leaning Tower of Pisa. It overhangs its base more than 13 feet, but it is stable. (Drawn from a photograph.)

**96. The three States of Equilibrium.** The centre of gravity of a body will always descend to as low a position as possible, or the potential energy of a body tends to become a minimum. (See next chapter).

Consider a body in equilibrium, and suppose that by a slight motion this equilibrium is disturbed. Then, if the body tends to return to its former position, its equilibrium is said to be *stable.* In this case the slight motion raises the centre of gravity, and on letting it go the body tends to return to its original position.

If, however, a slight disturbance lowers the centre of gravity the body will not return to its original position, but will take up a new position in which the centre of gravity is lower than before. In this case the equilibrium is said to be *unstable.*

Sometimes a body rests equally well in any position in which it may be placed, in which case the equilibrium is said to be *neutral.*

An egg standing on end is in unstable equilibrium; if resting on its side, the equilibrium is stable as regards motion in an oval section and neutral as regards motion in a circular

section. A uniform sphere rests anywhere it is placed on a level surface; its equilibrium is neutral. (Fig. 89).

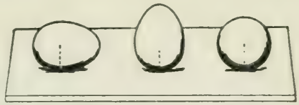


FIG. 89.—Stable, unstable and neutral equilibrium.

A round pencil lying on its side is in neutral equilibrium; balanced on its end, it is unstable. A cube, or a brick, lying on a face, is stable.

The amount of stability possessed by a body resting on a horizontal plane varies in different cases. It increases with the distance through which the centre of gravity has to be raised in order to make the body tip over. Thus, a brick lying on its largest face is more stable than when lying on its smallest.

### QUESTIONS AND PROBLEMS

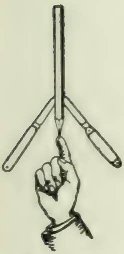


FIG. 90.—Why is the pencil in equilibrium?

1. Why is a pyramid a very stable structure?

2. Why is ballast used in a vessel? Where should it be put?

3. Why should a passenger in a canoe sit on the bottom?

4. A pencil will not stand on its point, but if two penknives are fastened to it (Fig. 90) it will balance on one's finger. Explain why this is so.

5. A uniform iron bar weighs 4 pounds per foot of its length. A weight of 5 pounds is hung from one end, and the rod balances about a point which is 2 feet from that end. Find the length of the bar.

6. Illustrate the three states of equilibrium by a cone lying on a horizontal table.

7. If a heavy uniform lamina, in the shape of an equilateral triangle, is suspended from any of its angles, show that the opposite side is always horizontal.

8. If a right-angled triangle is suspended from either of the points of trisection of the hypotenuse, show that it will rest with one side horizontal.

9. The wheels of a hay cart are 10 feet apart and the centre of gravity of the cart and load is 12 feet above the ground and midway between the wheels. How much could either wheel be raised without the cart falling over?

10. How many coins of the same size, having the thickness $\frac{1}{20}$ the diameter, can stand in a cylindrical pile on an inclined plane of which the height is $\frac{1}{6}$ the base, if there is no slipping?

11. A number of cent pieces are cemented together so that each just laps over the one below it by the ninth part of its diameter. How many may be thus piled without falling?

12. A brick is laid with a quarter of its length projecting over the ridge of a wall; a brick and a quarter are laid on the first with a quarter of its length over the edge of the first brick; a brick and a half laid on this and so on. Prove that four such courses can be laid, but that if the fifth course is added the mass will topple over.

13. A square table, whose mass is 10 kg., stands on four legs placed respectively at the middle points of its sides. Find the greatest mass which can be put at one of the corners without upsetting the table.

14. A circular table, of mass 50 lb., rests on three legs attached to three points in the circumference at equal distances apart. When the table rests on a horizontal plane what is the least mass which when placed on it will be on the point of upsetting it?

# CHAPTER XIII

## WORK, ENERGY, POWER

**97. Meaning of 'Work' in Mechanics.** When water is drawn from a well by means of a bucket on the end of a rope; or when bricks are hoisted during the erection of a building; or when land is ploughed; or when a blacksmith files a piece of iron; or when a carpenter planes a board; it is recognized that *work is done.*

Let us analyse these operations. In the case of drawing water from the well, the water is attracted towards the earth, and the person pulling on the rope must exert upon the bucket a force just sufficient to overcome this attraction, that is, its weight. The bucket is then displaced through a certain distance in the direction in which the force acts. When a force acts upon a body and causes it to move in the direction of the force we say that the *force does work,* though it would be more accurate to say that the agent exerting the force does work. The force in this case which resists the motion and which is overcome is gravity and it is customary to say that the work is done *against gravity.* We might also describe the production of work by stating that when motion takes place against resistance work is done.

In the raising of the bricks the circumstances are precisely similar to those just described. A force acts upon the bricks and displaces them in the direction of the force and the *force does work.*

In the other three cases a force has to be exerted sufficient to overcome the resistance opposed to it, a resistance similar to friction. The force is applied to the object (plough, file, plane) which moves in the direction of the force.

We thus see that the work done depends upon two factors,

(i) the force acting on the body ;

(ii) the distance through which the body moves in the direction of the force.

As a force acts *at a point* in a body it is rather more accurate to speak of the motion of the point of application of the force than of the motion of the body as a whole.

It is evident that if a force twice as great is exerted, twice the work will be done.

Also, if the displacement of the body is doubled the work will be doubled.

Hence, if $F$ = force,
and $s$ = space moved through,
the work done $= Fs$.

This is a very important formula,

Work done = force exerted × displacement.

It must be clearly understood that unless motion takes place no work is performed. The iron pillars supporting a building may be exerting great force but they are not doing any work. Atlas may hold up the world on his shoulders but he does not perform any work in doing so.

**98. Units of Work.** By choosing various units of force and of length we obtain different units of work.

If unit of force = 1 pound-force or pd.-wt.,
and " " length = 1 foot,
then " " work = foot-pound (ft.-pd.).

Thus if 5,000 gallons of water (weight of each* = 10 pd.) be raised 80 ft., the

$$\text{work done} = 50{,}000 \times 80 = 4{,}000{,}000 \text{ ft.-pd.}$$

In the practice of engineering many calculations have to be made regarding excavation of earth, pumping of water and

* The U.S. gallon of water weighs 8.34 pd.

other matters in which gravity is the force to be overcome, and this gravitation unit, the ft.-pd., is in general use by British and American engineers.

The corresponding metric engineering unit is the kilogram-metre, and as 1 kg. = 2.205 pd., and 1 m. = 3.28 ft.,

$$1 \text{ kg.-m.} = 2.205 \times 3.28 = 7.23 \text{ ft.-pd.}$$

In more purely scientific work the absolute units of force are used.

In the British (F.P.S.) system the absolute
unit of force = 1 poundal,
" " length = 1 foot,
and hence, " " work = 1 foot-poundal (ft.-pdl.).

Now, 1 pd.-force = $g$ pdl. ($g = 32$),
and consequently 1 ft.-pd. = $g$ ft.-pdl.

In the C.G.S. system the absolute
unit of force = 1 dyne,
" length = 1 cm.,
and hence, " " work = 1 dyne-cm.

To this unit has been given the special name *erg*.

Now, 1 gm.-force = $g$ dynes ($g = 981$),
and hence, 1 gm.-cm. of work = $g$ ergs,
and 1 kg.-m. of work = 98,100,000 ergs.

An erg is a very small quantity and another unit, introduced through its convenience in electrical calculations, is often used, namely the *joule*.

$$1 \text{ joule} = 10{,}000{,}000 \text{ or } 10^7 \text{ ergs},$$
$$= 0.737 \text{ (nearly } \tfrac{3}{4}) \text{ ft.-pd.}$$

**99. How To Calculate Work.** A bag of flour, 98 pounds, has to be carried from the foot to the top of a cliff, which has a vertical face and is 100 feet high.

There are three paths from the base to the summit of the cliff. The first is by way of a vertical ladder fastened to the

face of the cliff. The second is a zig-zag path 300 feet long, and the third is also a zig-zag route, 700 feet long.

Here a person if he were strong enough might strap on his back the mass to be carried and climb vertically up the ladder, or he might take either of the other two routes. The distances passed through are 100 feet, 300 feet, 700 feet, respectively, but the result is the same in the end, the mass is raised through 100 feet.

The force required to lift the mass is 98 pounds-force, and it acts in the vertical direction. The distance *in this direction* through which the body is moved is 100 feet, and therefore the

$$\text{Work} = 98 \times 100 = 9800 \text{ foot-pounds.}$$

Along the zig-zag paths the effort required to carry the mass is not so great but the length of path is greater and the total work is the same in the end.

Again, let a loaded sleigh be drawn on a level road a distance 5 ft. by a force $F$ pds. acting in a direction making an angle $\theta$ with the horizontal (Fig. 91).

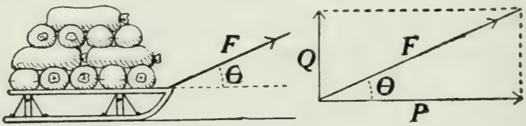



FIG. 91.—Calculation of work done in drawing a sleigh.

Here the displacement of the body is 5 ft., but it is not in the direction of the force.

The force $F$ may be resolved into components $P$ and $Q$ (Fig. 91), where $P = F \cos \theta$, $Q = F \sin \theta$.

The force $Q$ is perpendicular to the direction of motion, and hence does no work.

The force $P$ is in the direction of motion, and the work done by it $= Ps = F \cos \theta . s$.

**Examples**:—(1) Let $F = 25$ pd., $\theta = 20°$, $s = 1$ mile $= 5280$ ft.

Work done $= 25 \times \cos 20° \times 5280 = 12{,}403$ ft.-pd.

(2) A canal horse tows a boat by means of a rope which is inclined 30° to the direction of motion. The tension of the rope is 100 pd. Find the work done in going 2 miles.

Work $= 100 \times \cos 30° \times 5280 \times 2 = 91{,}435$ ft.-pd.

(3) A force is applied to a mass of 10 kg. and gives it an acceleration of 60 cm. per sec. per sec. It moves through a distance of 5 metres. Find the work done by the force.

We must first determine the magnitude of the force, and it will be best to use C.G.S. units throughout.

| | | |
|---|---|---|
| Mass | $m = 10{,}000$ grams. | |
| Acceleration | $a = 60$ cm. per sec. per. sec. | |
| Hence force | $F = ma = 600{,}000$ dynes | (Sec. 41) |
| and work done | $= 600{,}000 \times 500$ ergs, | |
| | $= 300{,}000{,}000$ ergs $= 30$ joules. | |

## PROBLEMS

1. A force 10 pounds acts through a space of 10 feet. Find the work done in (*a*) foot-pounds, (*b*) foot-poundals.

2. A force of 20 pounds acts through a space of 32 feet. Find the work done in (*a*) foot-pounds, (*b*) foot-poundals.

3. Find the work done in exerting a force of 1000 dynes through a space of 1 metre.

4. A block of stone rests on a horizontal pavement. A spring-balance, inserted in a rope attached to it, shows that to drag the stone requires a force of 90 pounds. If it is dragged through 20 feet, what is the work done?

5. The weight of a pile-driver, of 2500 pounds mass, was raised through 20 feet. How much work was required?

6. A coil-spring, naturally 30 centimetres long, is compressed until it is 10 centimetres long, the average force exerted being 20,000 dynes. Find the work done. Find its value in kilogram-metres. ($g = 980$).

7. Two men are cutting logs with a cross-cut saw. To move the saw requires a force of 50 pounds, and 50 strokes are made per minute, the

length of each being 2 feet. Find the amount of work done by each man in one hour.

8. To push his cart a banana man must exert a force of 50 pounds. How much work does he do in travelling 2 miles ?

9. Find the work done in raising 1000 litres of water from a well 10 metres deep.

10. Supposing that a man, whose weight is 100 kg., in walking raises his whole mass a distance of 10 cm. at every step, and that the length of the step is 50 cm., find how much work he does in walking 500 metres.

11. A ladder 10 metres long rests against a vertical wall, and is inclined at an angle of 60° to it. How much work is done in ascending it by a man weighing 80 kg. ?

12. How much work is done in lifting 8 kg. to a height of 12 metres above the surface of the moon, where $g$ is 150 cm. per sec. per sec. ?

13. A circular well 1.4 metres in diameter is 10 metres deep. Find the work expended in raising the material, supposing that a cubic metre of it weighs 2500 kg.

14. The cylinder of a steam engine has a diameter of 14 cm. and the piston moves through a distance of 20 cm. Find the work done per stroke if the pressure of the steam in the cylinder be constant, and equal to 5 kg. per square centimetre.

**100. Definition of Energy.** In preparing the foundation for a bridge, a wharf or other structure, frequently piles are driven into the ground, and the method of doing this is well known. After sharpening one end of a log it is stood upright and then a heavy iron mass is raised to a considerable height and allowed to fall upon the upper end of the log. This is repeated time after time, and with each blow the log sinks farther into the earth, until at last it is down far enough.

Now to thrust the log into the earth requires a great force and therefore in driving it home considerable work

is performed. The ability to do this work is possessed by the mass of iron moving with the velocity it has acquired in falling. *A mass in motion is able to do work.*

A hammer moving with some speed is able to drive in a nail against considerable resistance; and a rifle bullet of mass but half an ounce, moving with very great velocity, can penetrate almost anything in its path, and thus perform much work.

*Ability to do work* is called ENERGY.

Again, the velocity possessed by the pile-driver weight arises from its having fallen from a height. By doing work on this body, against the force of gravity, it is given an advantageous position and we can look on it as possessing energy by virtue of its position. The source of the energy, however, does not reside in the body but rather in its separation from the earth. The body and the earth form a *system* and by changing the shape of the system energy is given to it.

By the performance of work we give the body energy of position, and as it falls, its energy of position is changed into energy of motion, which is used up in doing work.

We see then that there are two kinds of energy:

(i) Energy of position, or *potential* energy (P.E.).

(ii) Energy of motion, or *kinetic* energy (K.E.).

As another example of a body possessing energy of position, a spring wound up may be mentioned. It is able to drive a clock, or a phonograph, or do other kinds of work.

**101. How to Measure Energy.** Since energy is the ability to do work it can be measured in the same units as are used in measuring work.

Suppose a mass $m$ grams to be lifted through a height $h$ cm. (Fig. 92).

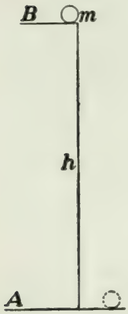


FIG. 92.—The potential energy at height $B$ is equal to the kinetic energy on reaching $A$.

Force exerted $= m$ gm.-force $= mg$ dynes.

Displacement of body $= h$ cm.

Hence work done $= mgh$ ergs.

Now allow the mass to fall. Upon reaching the former level $A$ it will have acquired a velocity $v$ such that

$$v^2 = 2gh, \text{ or } gh = \tfrac{1}{2} v^2 \qquad \text{(Sec. 32)}$$

The P.E. possessed by the body at $B$ is $mgh$ ergs (the work expended in putting it there), and if we assume for the present that this energy of position is completely changed into energy of motion on reaching $A$,

$$\text{the K.E. at } A = mgh \text{ ergs.}$$

$$\text{But } gh = \tfrac{1}{2} v^2,$$

$$\text{and therefore the K.E.} = \tfrac{1}{2} mv^2 \text{ ergs.}$$

In the F.P.S. system,

Let mass $= m$ lb., and velocity $= v$ ft. per sec.

$$\text{Then K.E.} = \tfrac{1}{2} mv^2 \text{ ft.-pdl.} = \tfrac{1}{2} \frac{mv^2}{g} \text{ ft.-pd.}$$

since 1 pd.-force $= g$ pdl. ($g = 32$) (Sec. 42)

**102. More General Solution.** In the last section the expression for the K.E. was obtained on the assumption that it was developed through the force of gravity, but as the result is very important it is desirable to show that we reach the same formula for any force.

Let a force $F$ dynes act for $t$ seconds on a mass $m$ grams initially at rest. (Fig. 93).

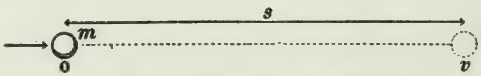


FIG. 93.—The calculation of kinetic energy.

Let the velocity produced be $v$ cm. per sec., and the space travered be $s$ cm.

The force $s$ dynes acts on the mass which moves through a space $s$ cm., and hence the force does $Fs$ ergs of work. Consequently at the end of the time the body possesses K.E. $= Fs$ ergs.

Now the initial velocity $= 0$ cm. per sec.

and the final " $= v$ " " "

Hence, average " $= \frac{1}{2}v$ " " "

and space traversed $s = \frac{1}{2}vt$ cm.

Again, the force $F$ dynes acts for $t$ sec.

Hence, momentum $= Ft = mv$, (Sec. 38)

$$\text{and } F = \frac{mv}{t},$$

Therefore, K.E. $= Fs = \frac{mv}{t} \times \frac{1}{2}vt = \frac{1}{2}\, mv^2$ ergs.

**103. Examples.** (1) Find the K.E. of 1 kg. after falling 1 metre.

In this case force acting on mass $= 1000 \times 981$ dynes.

Distance fallen through $= 100$ cm.

K.E. acquired $=$ work done $= 981 \times 1000 \times 100$.

$= 98{,}100{,}000$ ergs.

(2) A mass of 6 kg. is moving with a velocity of 60 cm. per sec. What is its K.E.? If brought to rest by a constant force in a distance 100 cm., what is the force?

Here, mass $m = 6000$ gm., velocity $v = 60$ cm. per sec.

K.E. $= \frac{1}{2}\, mv^2 = 10{,}800{,}000$ ergs.

Also, if $F =$ opposing force in dynes,

$Fs = \frac{1}{2}\, mv^2$, and $F = 108{,}000$ dynes.

(3) In Fig. 28 let $M = 14$ lb., $w = 1$ lb. Find the velocity, and the acceleration, when the masses have moved through 2 ft.

Mass $M$ rises and gains P.E. $= 14 \times 2$ ft. pd.

$= 14 \times 2 \times 32$ ft.-pdl.

Mass $M$ has also gained K.E. $= \frac{1}{2} \times 14 \times v^2$ "

Mass $M + w$ has lost P.E. $= 15 \times 2 \times 32$ "

" " " gained K.E. $= \frac{1}{2} \times 15 \times v^2$ "

Considering the two masses as one "system," there has been no change in the total energy, or the loss $=$ the gain.

Hence, $15 \times 2 \times 32 = 14 \times 2 \times 32 + \frac{1}{2} \times 14 \times v^2 + \frac{1}{2} \times 15 \times v^2$,
and $v^2 = 128/29$, or $v = 0.210$ ft. per sec.
Also $v^2 = 2as$, and $a = 0.110$ ft. per sec. per sec.

(4) A mass $m$ grams hangs at the end of a light cord $l$ cm. long. It is drawn aside through the angle $\theta$ from position $OB$ to $OA$ (Fig. 94), and allowed to swing. Find its velocity at its lowest point.

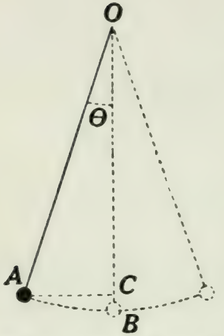


FIG. 94.—Finding velocity at lowest point $B$.

In position $A$ the energy of the mass is entirely potential.

P.E. $= m \times BC$ gm.-cm. $= mg \times BC$ ergs.

When in position $B$ its P.E. has been completely changed into K.E. and $= \frac{1}{2} mv^2$.

Hence, $\frac{1}{2} mv^2 = mg \times BC$ or $v^2 = 2g \times BC$.

Hence, the velocity at $B$ is the same as if the mass had simply fallen freely through a distance $CB$.

**104. Transformation and Transference of Energy.** Energy has been defined as ability to do work, and anything from which we can get work is a source of energy. We must therefore consider falling water, coal, an electric current, the sun, as among our sources of energy. It appears in many forms. The various effects due to heat, light, sound and electricity are simply manifestations of it.

In dealing with the motion of bodies we were led to believe that there are two distinct forms of energy, namely, energy of position and energy of motion. Now it is difficult to determine accurately the nature of some of the forms of energy met with, but the farther the investigation proceeds the more firmly becomes the conviction that all energy can be considered to be either potential or kinetic. When sound is produced the particles of air or other substance are in vibration. Heat and light are believed to be due to vibrations of some material particles, and similarly with electricity.

The utmost that a machine, whether a living body or an inanimate thing, can do is to transform energy from one form

into another or transfer it from one body to another. It can never create it. The energy of coal when burned in the furnace is changed into the energy of heat, and this is changed into the energy of steam. The steam drives the engine, which can pump water, saw wood or make a dynamo generate an electric current. The energy of the current may be conveyed to another place and there produce heat or light or chemical action or drive a motor.

It has been established, or at least made extremely probable, by numerous careful experiments extending over many years, that there is no change in the total amount of the energy in our universe. This is now looked upon as one of the grand laws of nature and is known as the law of the CONSERVATION OF ENERGY.

We start with a definite amount of energy in the coal, and if it were possible to keep a strict account of the different forms into which it is changed and the amount in each form and could add them all together, we would have at last precisely the same total that we had at the beginning.

It should be observed, however, that energy may be in existence without being available for use. Thus there is much heat energy in the ocean but we cannot use it. When a railway train is brought to rest, the energy of motion of the train is changed by the friction into heat energy, which is radiated or conducted away and is lost to us. In any 'system' the tendency is towards dissipation and degradation of its energy to a condition where it cannot be used.

The law of the CONSERVATION OF MATTER has been accepted for many years and is the basis of analytical chemistry. Matter can be changed into many forms but the sum total remains the same. It cannot be created or destroyed.

Force, on the other hand, is of an entirely different nature. On pulling a string, tension is developed in it, which disappears

when we let go. Matter and energy are bought and sold but force cannot be. (This subject is discussed further in Chapter XXVI).

### PROBLEMS

1. A mass of 64 pounds is moving with a velocity of 10 feet per second. Find its kinetic energy in (1) foot-pounds, (2) foot-poundals.

2. A mass of 10 grams is thrown vertically upward with a velocity of 980 cm. per second. Find its kinetic energy (1) at the instant of projection, (2) at the end of one-half second, (3) at the end of 1 second, (4) at the end of two seconds.

3. Find the kinetic energy of a cannon-ball whose mass is 10 lb. discharged with a velocity of 50 yards per second.

4. A stone of mass 6 kg. falls from rest. What will be its kinetic energy at the end of 5 seconds?

5. A 100-gram bullet strikes an iron target with a velocity of 400 metres per second and falls dead. How much kinetic energy has the bullet lost?

6. A stone whose mass is 100 lb. is carried to the top of a wall 40 feet high. What potential energy does the stone possess? If the stone is dropped, what kinetic energy will it have when it strikes the ground?

7. A hammer whose mass is one pound strikes a nail with a velocity of 20 feet per second. Find the kinetic energy possessed by the hammer when it is about to touch the nail. If it drives the nail a distance of $1\frac{1}{2}$ inches, find the average force in pounds exerted by the hammer upon the nail.

8. A cricket ball, whose mass is 100 grams, is given by a blow a velocity of 20 metres per second. What is the measure of the work done?

9. Calculate the kinetic energy possessed by a stone whose mass is 1 kg. after it has fallen from rest through a space of 1 metre.

10. Find the energy required to project a golf ball whose mass is 10 grams a distance of 100 metres vertically upwards.

11. A stone whose mass is 100 pounds falls freely from a point 400 feet above the ground. Find in foot-pounds (1) its kinetic energy, (2) its potential energy, at the end of the fourth second.

12. A mass of 20 pounds hanging at the end of a light cord 16 feet in length is drawn aside through an angle of 90° and then let go. Find (1) its kinetic energy in foot-poundals, (2) its velocity, when it reaches its lowest point.

**105. Power.** In stating the amount of work done the question of *time* does not enter at all. A man could dig a big cellar quite as well as a steam shovel can, if he were given time enough, but when he got through, the need for the building to be erected over it might be past. In ordinary life we must consider time, or the rate at which work is performed.

The POWER or ACTIVITY of an agent is its *rate of doing work.*

**106. The Horse-power; the Watt.** The chief use of the steam engine at first was to pump water from the mines. Horses had been utilized for this as well as for many other purposes, and it was natural that James Watt,* after making the engine really efficient, should rate it in terms of the power of a horse. In order to do this he made experiments with strong dray-horses and finally he decided to call a horse-power the ability to perform 33,000 ft.-pd. of work in 1 minute, or 550 ft.-pd. in 1 second. As a matter of fact, this is much greater than any ordinary horse can continuously perform, but it would seem that Watt was anxious that purchasers of his engines should be satisfied with their capabilities, and that they should be able to do more than their name would demand.

In the C.G.S. system the unit of power is the ability to do 1 erg per second, but this is an extremely small quantity and it is more convenient to choose a unit 10,000,000 times as great. This unit is called a *watt.*

1 watt = 10,000,000 ergs per second.

" = 1 joule per second.

1000 watts = 1 kilowatt (k.w.).

The horse-power can be expressed in watts as follows:

1 ft. = 30.48 cm.,

1 pd. = 453.59 gms.-wt. = $453.59 \times 981$ dynes.

---

* Watt died in 1819, one hundred years ago. The story of his life is very interesting.

Hence, 550 ft.-pd. $= 550 \times 30.48 \times 453.59 \times 981$ ergs.

$= 746 \times 10^7$ ergs,

$= 746$ joules.

Hence, 1 h.p. = 550 ft.-pd. per sec. = 746 watts = $\frac{3}{4}$ k.w. (approx.),

$$\text{and } 1 \text{ k.w.} = \frac{1000}{746} \text{ h.p.} = 1\tfrac{1}{3} \text{ h.p. (approx.)}$$

**PROBLEMS**

1. At what rate (measured in horse-powers) is work being done when 1100 pounds of water are lifted every second from a well 100 feet deep?

2. Find the horse-power of an engine that will pump every hour 660 tons of water from a mine 600 feet deep.

3. Water flows into a mine 990 feet deep at the rate of 100 cubic feet per minute. What is the horse-power of the engine that will keep the mine dry? (A cubic foot of water weighs 62.5 pounds).

4. It is estimated that 700,000 tons of water pass over Niagara Falls every minute, and fall 160 feet to the lower level. If it were permissible to take one-tenth of the water for commercial purposes, what horse-power could be developed therefrom?

5. A motor is capable of hoisting 1320 tons of coal from the bottom of a mine 1200 feet deep per hour. Find its horse-power.

6. A force of 10 dynes acting on a mass moves it through 60 cm. in 10 seconds. What is the power?

7. A force of 30 dynes acting on a mass moves it through 2 metres in a minute. What is the power?

8. A mass of 20 grams is lifted vertically a distance of 1 metre in 196 seconds. What is the rate of working?

9. On applying a dynamometer to a street car it is found that six million dynes are required to keep it in motion, while it passes over 1 kilometre in 10 minutes. Determine the rate of working in watts.

10. A force of ten million dynes is required to draw a car along a track at the rate of 36 kilometres per hour. What is the rate of working in watts?

11. A man pumps 600 kilograms of water from a well 10 metres deep in 49 minutes. At what rate, measured in watts, is he working?

12. Calculate the horse-power of a steam engine which will raise 1,200 kilograms of water per minute from a well 149.2 metres deep.

13. A man whose mass is 60 kilograms walks up a hill 298.4 metres high in 14 minutes. What is the average power which he exerts compared with a horse-power?

14. A boy can carry 300 litres of water to the top of a hill 80 metres high in 1 hour. State in watts his rate of working.

15. 596,800 litres of water flow per minute over a dam 6 metres high. What is the power of the fall?

16. A hoist used in the erection of a building raises in 2 hours 30,000 bricks, each weighing 5 pounds, and 2000 feet of lumber, weighing 3 pounds per foot, through a height of 50 feet. Calculate the work done.

Calculate also the horse-power developed by the engine running the hoist, supposing 20 per cent. of the energy developed to be lost in friction.

17. Find the horse-power necessary to haul a train, whose mass is 18 tons, up an incline of 1 foot in 100 feet at the rate of 10 miles per hour, neglecting friction.

18. An engine is drawing a train whose mass is 360,000 kilograms up a smooth inclined plane of 1 in 30, at the rate of 22,380 metres per hour. What is the horse-power of the steam engine?

19. A man cycles up a hill, whose slope is 1 in 14, at the rate of 6000 metres per hour. The mass of the man and the machine is 60 kilograms. At what rate is he working?

20. A train consists of 30 cars, and each car with its load weighs 14,920 kg., the resistance to motion on a level track is at the rate of 15 kg. per 1000 kg. of load. Find in horse-powers at what rate an engine is working that hauls this train at the rate of 30 km. per hour.

21. What is the horse-power of an engine which keeps a train whose mass is 60,000 kg. moving on a horizontal track at a uniform rate of 44,760 metres per hour, the resistance due to friction, etc., being $\frac{1}{50}$ of the weight of the train?

22. An engine, whose horse-power is 1000, pumps water from a depth of 1000 feet. Find the number of tons raised per hour.

23. An engine of 98 horse-power, working 10 hours a day, supplies 3000 houses with water, which it raises to a mean level of 149.2 metres. Find the average supply to each house.

24. The piston of a steam engine is 10 inches in diameter and the stroke is 16 inches long. If the average pressure of steam on this piston throughout the full length of the stroke is 70 pd. per square inch, and if the engine makes three double strokes (backward and forward movements) per second, determine its working capacity in horse-powers.

# CHAPTER XIV

## MACHINES

**107. Object of a Machine.** It is frequently necessary to raise the axle of an automobile in order to renew a tire, and everyone knows how easy this is when you have a suitable machine.

Or perhaps a barrel of oil or of flour is to be loaded on a wagon. It is too heavy to lift, but it can easily be put in place by rolling it up a plank.

Again, an electric current may be at your disposal. By suitable contrivances you can make it sew your clothes or separate the cream, or print the newspaper, or do a thousand other tasks.

In each case we use a suitable machine, and the function of the machine is to transfer energy from one place to another, or transform it from one kind to another.

The six simple machines, usually known as the *mechanical powers*, are the lever, the pulley, the wheel and axle, the inclined plane, the wedge and the screw. All other machines, no matter how complicated, are only combinations of these.

Since energy cannot be created or destroyed, but is simply changed from one form to another, it is evident that, neglecting friction, the amount of work put into or done upon a machine is equal to the amount which it will perform. Furthermore, since in every machine which man can make some friction is unavoidably present, it is clear that more work must be done in driving the machine than will be its output. Many attempts have been made to invent a machine which will continue to deliver as much work as is spent upon it, and indeed sometimes more work has been expected from a

machine than has been spent upon it. Such attempts have always failed, and if the law of Conservation of Energy is true such efforts cannot possibly succeed. If there is only five gallons of gasoline in your tank, that is all you can use,—unless you put more in. It is the same with stores of energy.

**108. The Lever; First Class.** The lever is a rigid rod movable about a fixed axis called the fulcrum. Levers are of three classes.

In Fig. 95 is shown a lever of the first class. By applying a force $F$ at $A$ a force $W$ is obtained at $B$, the lever turning about the fulcrum $O$.

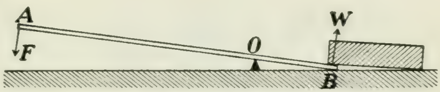



FIG. 95.—A lever of the first class.

The relation between the forces $F$ and $W$ follows from the principle of moments, and it can be determined experimentally as follows:

Lay a metre rod on a prism with the 50 cm. mark exactly over the edge of the prism (Fig. 96). If it does not balance add bits of lead to the lighter end until it does. Put blocks under the ends to reduce the vibrations.

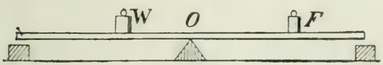



FIG. 96.—Investigating the law of the lever of the first class.

Place a weight $F$ on some graduation, noting its distance from $O$. This distance $OF$ is one arm of the lever, and the product $F \times OF$ is the moment of $F$ about $O$.

Move the weight $W$ until it just balances $F$ and note the length of the arm $OW$. The moment of $W = W \times OW$. Make 5 or 6 readings, changing weights and distances each time. Then compare the values of

$$F \times OF \text{ and } W \times OW.$$

They will be found to be equal.

Applying this result to either figure we see that

$$\text{Force obtained} \times \text{its arm} = \text{Force applied} \times \text{its arm},$$

$$\text{or } \frac{\text{Force obtained}}{\text{Force applied}} = \text{inverse ratio of length of arms.}$$

This is called the *Law of the Lever*, and the ratio $W/F$ is called the *Mechanical Advantage*.

Suppose, for instance, $AO = 36$ inches, $BO = 4$ inches.

Then $W/F = AO/BO = 36/4 = 9$, the mechanical advantage.

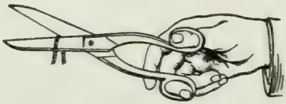


Fig. 97.—Shears, lever of the first class.

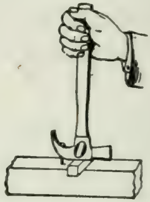



Fig. 98.—Claw-hammer, used as a lever of the first class.

There are many examples of levers of the first class. Among them are, the common balance, a pump handle, a pair of scissors (Fig. 97), a claw-hammer (Fig. 98).

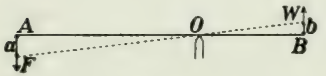



Fig. 99.

The law of the lever can be deduced by applying the principle of energy.

Suppose the end $A$ (Fig. 99) to move through a distance $a$ and the end $B$ through a distance $b$. It is evident that

$$a/b = AO/BO.$$

Now the work done by the force $F$, acting through a distance $a$ is $F \times a$, while the work done by $W$ acting through a distance $b$ is $W \times b$.

Neglecting all considerations of friction or of the weight of the lever, the work done by the applied force $F$ must be equal to the work accomplished by the force $W$.

Hence, $Fa = Wb$,

and the mechanical advantage $W/F = a/b = AO/BO$, which is the law of the lever.

**109. The Lever; Second Class.** In levers of the second class the weight to be lifted, or the resistance to be overcome,

is placed between the point where the force, or the effort, is applied and the fulcrum.

A lever of this class is shown in Fig. 100. The force $F$, or the effort, is applied at $A$, and the force obtained, or the resistance overcome, is at $B$, between $A$ and the fulcrum $O$.

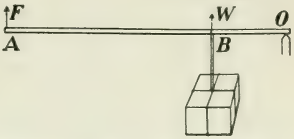



Fig. 100.—A lever of the second class.

The law in this case can be determined experimentally as follows:

Weigh the rod; let it be $w$ grams. Next find the position of the centre of gravity $C$ of the rod by balancing it on the prism. Support it at this point by a weight $w$ attached to a string as shown in Fig. 101. Let it be at $C$.

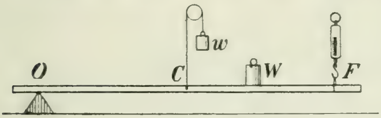



Fig. 101.—Demonstrating the law for a lever of the second class.

Now rest the rod on a prism at a point 2 cms. from one end and attach a spring-balance 2 cm. from the other end. Place a weight $W$ on the rod, noting its distance from the fulcrum $F$, and observe the reading $P$ of the spring-balance. Make 5 or 6 readings, varying the value of $W$ and the point where it is placed.

Compare the products $F \times OF$ and $W \times OW$: they will be found equal; and we have, as in the first class,

Mechanical Advantage $W/F = AO/BO$ (Fig. 102),

a ratio which is greater than 1.


Fig. 102.—Theory of the lever of the second class.

If we apply the principle of energy we have:

$$\text{Work done by } F = Fa; \text{ by } W = Wb,$$

and these must be equal, or $Fa = Wb$.

Hence, $W/F = a/b = AO/BO$, the law of the lever.

Examples of levers of the second class: nut crackers (Fig. 103), trimming board (Fig. 104), safety valve (Fig. 105), wheel-barrow, oar of a row-boat.

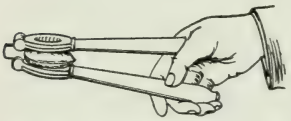


FIG. 103.—Nut-crackers, lever of the second class.

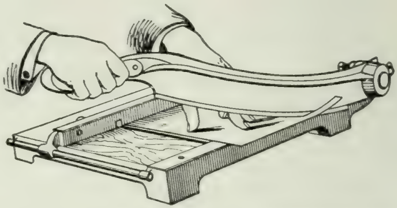


FIG. 104.—Trimming board for cutting paper or cardboard; a lever of the second class.

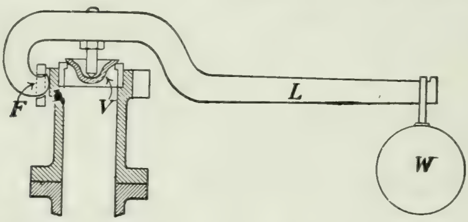



FIG. 105.—A safety-valve of a steam boiler. (Lever of the second class). $L$ is the lever arm, $V$ the valve on which the pressure is exerted, $W$ the weight which is lifted, $F$ the fulcrum.

**110. The Lever; Third Class.** In this case the force $F$ is applied between the fulcrum and the weight to be lifted. (Fig. 106).

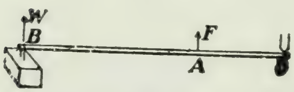



FIG. 106.—A lever of the third class.

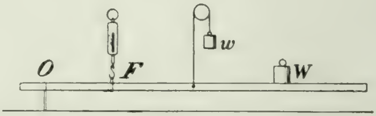



FIG. 107.—Demonstrating the law of the lever of the third class.

To investigate this arrangement experimentally the apparatus as shown in Fig. 107 may be used. The rod is supported at its centre of gravity as in the previous case, and one end of the rod is pushed through a wire loop fastened to the table.

As before, compare the products

$$F \times OF \text{ and } W \times OW$$

for various values and positions of $W$.

These will be found to be equal, and

$$W/F = AO/BO \text{ (Fig. 106), the law of the lever.}$$

Notice that the weight lifted is always less than the force applied, or the mechanical advantage is less than 1.

Examples of levers of this class: sugar-tongs (Fig. 108), the human forearm (Fig. 109); treadle of a lathe or a sewing machine.

Fig. 108.—Sugar-tongs, lever of the third class.

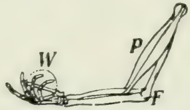



Fig. 109.—Human forearm, lever of the third class. One end of the biceps muscle is attached at the shoulder, the other is attached to the radial bone near the elbow, and exerts a force to raise the weight in the hand.

**PROBLEMS**

1. Explain the action of the steelyards (Fig. 110). To which class of levers does it belong? If the distance from $B$ to $O$ is $1\frac{1}{2}$ inches, and the sliding weight $P$ when at a distance 6 inches from $O$ balances a mass of 5 lb. on the hook, what must be the weight of $P$?

If the mass on the hook is too great to be balanced by $P$, what additional attachment would be required in order to weigh it?

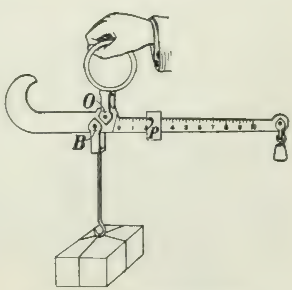



Fig. 110.—The steelyards.

2. A hand-barrow (Fig. 111), with the mass loaded on it weighs 210 pounds. The centre of gravity of the barrow and load is 4 feet from the front

handles and 3 feet from the back ones. Find the amount each man carries.

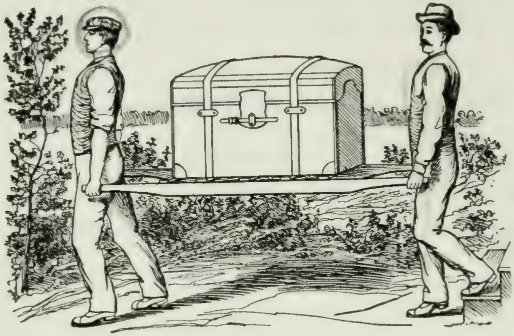


FIG. 111.—The hand-barrow.

3. To draw a nail from a piece of wood requires a pull of 200 pounds. A claw-hammer is used, the nail being $1\frac{1}{2}$ inches from the fulcrum $O$ (Fig. 98) and the hand being 8 inches from $O$. Find what force the hand must exert to draw the nail.

4. A cubical block of granite, whose edge is 3 feet in length and which weighs 4500 lb., is raised by thrusting one end of a crow-bar 40 inches long under it to the distance of 4 inches, and then lifting on the other end. What force must be exerted?

**111. The Pulley.** The pulley is used sometimes to change the direction in which a force acts, sometimes to gain mechanical advantage, and sometimes for both purposes.

The pulleys used in experiments should be of very light construction and with well-made bearings, in which there is little friction.

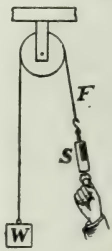



FIG. 112.—A fixed pulley simply changes the direction of force.

A single fixed pulley, such as is shown in Fig. 112, can change the direction of a force but cannot give a mechanical advantage greater than 1. $F$, the force applied, is equal to the weight lifted, $W$.

By this arrangement a lift is changed into a pull in any convenient direction. It is often used in raising materials during the construction of a building.

By inserting a spring-balance, $S$, in the rope, between the hand and the pulley, one can show that the force $F$ is equal to the weight $W$.

In order to apply the principle of energy, suppose the hand to move through a distance $a$, then the weight rises through the same distance.

$$\text{Hence, } F \times a = W \times a,$$
$$\text{or } F = W,$$

as tested by the spring-balance.

If the friction is not negligible, pull on the balance until $W$ rises slowly and uniformly. Then the difference between the weight $W$ and the reading on the balance will give the magnitude of the friction.

**112. A Single Movable Pulley.** Here the weight $W$ (Fig. 113) is supported by the two portions $B$ and $C$, of the

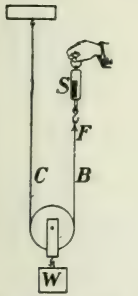



Fig. 113.—With a movable pulley the force exerted is only half as great as the weight lifted.

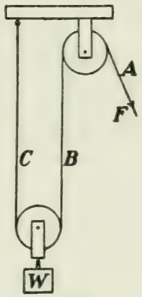



Fig. 114.—With a fixed and a movable pulley the force is changed in direction and reduced one-half.

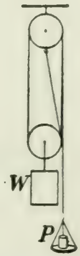



Fig. 115.—One fixed and one movable pulley.

rope, and hence each portion supports half of it.

Thus the force $F$, indicated by the balance $S$, is equal to $\frac{1}{2}$ $W$, and the mechanical advantage is 2.

This result can also be deduced from the principle of energy.

Let $a$ be the distance through which $W$ rises. Then each portion, $B$ and $C$, of the rope will be shortened a distance $a$, and so $F$ will move through a distance $2a$.

Then, since

$$F \times 2a = W \times a,$$
$$W/F = 2, \text{ the mechanical advantage.}$$

For convenience a fixed pulley is also generally used as in Figs. 114 or 115.

Here when the weight rises 1 inch, $B$ and $C$ each shorten 1 inch and hence $A$ lengthens 2 inches. That is, $F$ (or $P$) moves through twice as far as $W$, and $W/F = 2$, as before.

**113. Other Systems of Pulleys.** Various combinations of pulleys may be used. Two are shown in Figs. 116, 117, the latter one being very common.

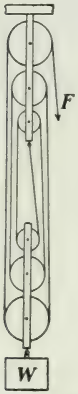



Fig. 116.—Combination of 6 pulleys; 6 times the force lifted.

Fig. 117.—A familiar combination for multiplying the force 6 times.

Here there are six portions of the rope supporting $W$, and hence the tension in each portion is $\frac{1}{6}$ $W$.

Hence, $F = \frac{1}{6}$ $W$,

or a force equal to $\frac{1}{6}$ $W$ will hold up $W$. This entirely neglects friction, which in such a system is often considerable, and it therefore follows that, to prevent $W$ from descending, less than $\frac{1}{6}$ $W$ will be required. On the other hand, to actually lift $W$ the force $F$ must be greater than $\frac{1}{6}$ $W$. In every case friction acts to prevent motion.

Let us apply the principle of energy to this case. If $W$ rises 1 foot, each portion of the rope supporting it must shorten 1 foot and the force $F$ will move 6 feet.

Then, work done on $W = W \times 1$ foot-pounds.

" " by $F = F \times 6$ "

These are equal, and hence

$$W = 6\,F$$

or $W/F = 6$, the mechanical advantage.

**PROBLEMS**

1. A clock may be driven in two ways. First, the weight may be attached to the end of the cord; or secondly, it may be attached to a pulley, movable as in Fig. 113, one end of the cord being fastened to the framework, and the other being wound about the barrel of the driving wheel. Compare the weights required, and also the length of time the clock will run in the two cases.

2. Find the mechanical advantage of the system shown in Fig. 118. This arrangement is called the Spanish Barton.


FIG. 118.—The Spanish Barton.

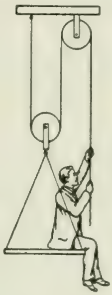


FIG. 119.—An easy method to raise one's self.

FIG. 120.—Find the pressure of the feet on the floor.

3. What fraction of his weight must the man shown in Fig. 119 exert in order to raise himself?

4. A man weighing 140 pounds pulls up a weight of 80 pounds by means of a fixed pulley, under which he stands (Fig. 120). Find his pressure on the floor.

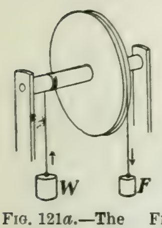



FIG. 121*a*.—The wheel and axle.

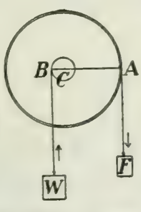



FIG. 121*b*.—Diagram to explain the wheel and axle.

**114. The Wheel and Axle.** This machine is shown in Figs. 121*a*, 121*b*. It is evident that in one complete rotation the weight $F$ will descend a distance equal to the circumference of the wheel, while the weight $W$ will rise a distance equal to the circumference of the axle.

Hence $F \times$ circumference of wheel $= W \times$ circumference of axle. Let the radii be $R$ and $r$, respectively; the circumferences will be

$2\pi R$ and $2\pi r$, and therefore

$$F \times 2\pi R = W \times 2\pi r,$$

$$\text{or } FR = Wr,$$

and the mechanical advantage $W/F = R/r$.

This result can also be seen from Fig. 121*b*. The wheel and axle turn about the centre $C$. Now $W$ acts at $B$, a distance $r$ from $C$, and $F$ acts at $A$, a distance $R$ from $C$.

Then, from the principle of the lever

$$F \times R = W \times r, \text{ as before.} \quad \text{(See Sec. 63)}$$

**115. Examples of Wheel and Axle.** The windlass (Fig. 122) is a common example, but in place of a wheel, handles are used. Forces are applied at the handles and the bucket is lifted by the rope, which is wound about the axle.

FIG. 122.—Windlass used in drawing water from a well.

If $F$ = applied force, and $W$ = weight lifted, $\dfrac{W}{F} = \dfrac{\text{length of crank}}{\text{radius of axle}}$.

The capstan, used on board ships for raising the anchor is another example (Fig. 123).

The sailors apply the force by pushing against bars thrust into holes near the top of the capstan. Usually the rope is too long to be all coiled up on the barrel, so it is passed about it several times and the end $A$ is held by a man who keeps that portion taut. The friction is sufficient to prevent the rope from slipping. Sometimes the end $B$ is fastened to a post or a ring on the dock, and by turning the capstan this portion is shortened and the ship is drawn in to the dock.


FIG. 123.—Raising the ship's anchor by a capstan.

**116. Differential Wheel and Axle.** This machine is shown in Fig. 124. It will be seen that the rope winds off one axle and on the other. Hence, in one rotation of the crank the rope is lengthened (or shortened) by an amount equal to the difference in the circumferences of the two axles; but since the rope passes round a movable pulley the weight to be lifted, attached to this pulley, will rise only one-half the difference in the circumferences.

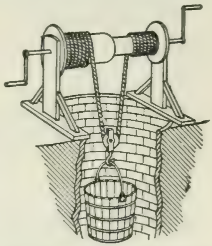


FIG. 124.—Differential wheel and axle.

Thus by making the two drums which form the axles nearly equal in size we can make the difference in their circumferences as small as we please, and the mechanical advantage will be as great as we desire.

**117. Differential Pulley.** This is somewhat similar to the last described machine (Figs. 125, 126).

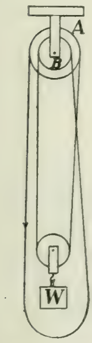



FIG. 125.—Explanation of the action of the differential pulley.

Two pulleys of different radii (Fig. 125), are fastened together and turn with the same angular velocity. Grooves are cut in the pulleys so as to receive an endless chain and prevent it from slipping.

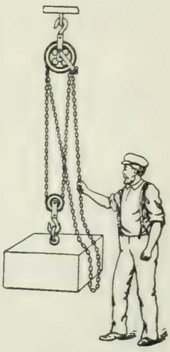


FIG. 126.—The actual appearance of the differential pulley.

Suppose the chain is pulled at $F$ until the two pulleys have made a complete rotation. Then $F$ will have moved through a distance equal to the circumference of $A$, and it will have done work

$$= F \times \text{circumference of } A.$$

Also, the chain between the upper and the lower pulley will be shortened by the circumference of $A$, but lengthened by the

circumference of $B$, and the net shortening is the difference between these two circumferences.

But the weight $W$ will rise only half of this difference. Hence, work done by $W$

$$= W \times \tfrac{1}{2} \text{ difference of circumferences of } A \text{ and } B,$$

and therefore $\dfrac{W}{F} = \dfrac{\text{circumference of } A}{\frac{1}{2} \text{ difference of circumferences of } A \text{ and } B}.$

**PROBLEMS**

1. A man weighing 160 pounds is drawn up out of a well by means of a windlass, the axle of which is 8 inches in diameter, and the crank 24 inches long. Find the force required to be applied to the handle.

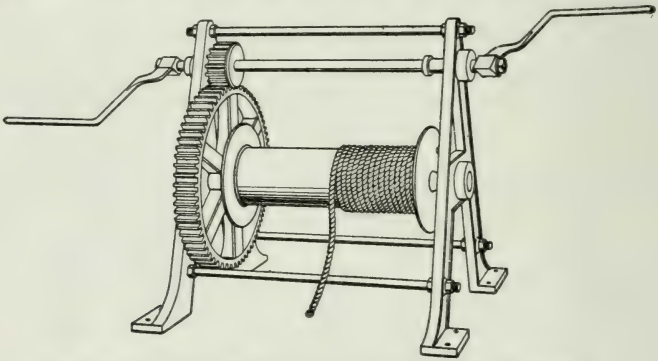


Fig. 127.—Windlass, with gearing, such as is used with a pile-driver.

2. Calculate the mechanical advantage of the windlass shown in Fig. 127. The length of the crank is 16 inches, the small wheel has 12 teeth and the large one 120, and the diameter of the drum about which the rope is wound is 6 inches.

If a force of 60 pounds be applied to each crank how great a weight can be raised? (Neglect friction).

**118. The Inclined Plane.** If we wish to load a heavy box or barrel on a wagon it is often convenient to slide or roll it up a plank having one end on the ground and the other on the wagon. The relation between the effort and the resistance overcome can be investigated by means of the apparatus shown in Fig. 128.

The carriage and the pulley should move with very little friction.

Place a weight on $W$ and then add to $P$ until $W$ just moves up the plane. Let $P_1$ be the weight in this case. Then lighten $P$ until $W$ just moves down the plane; let $P_2$ be the weight now. Take $\frac{1}{2}(P_1 + P_2) = P$ as the weight to balance $W$ if there were no friction.

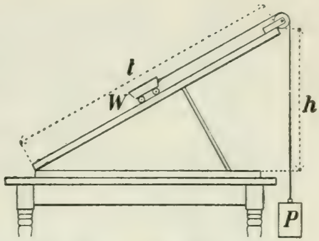


Fig. 128.—To show that $Pl = Wh$.

Now carefully measure $l$ the length of the plane and $h$ its height, and compare the products $P \times l$ and $W \times h$. They will be found to be equal, or very approximately so.

$$\text{Hence,} \quad W/P = l/h,$$

or the mechanical advantage is the ratio of the length to the height of the plane.

This result can be obtained easily by applying the principle of work. Try it.

Taking friction into account, the mechanical advantage is not so great, and to reduce the friction as much as possible the body may be rolled up the plane.

**119. The Wedge.** The wedge is designed to overcome great resistance through a small space. Its most familiar use is in splitting wood. Knives, axes and chisels are also examples of the wedge.

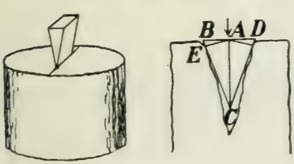


Fig. 129.—The wedge, an application of the inclined plane.

The resistance $W$ (Fig. 129) to be overcome acts at right angles to the slant sides $BC$, $DC$, of the wedge, and when the wedge has been driven in as shown in the figure, the work done in pushing back one side of the split block will be $W \times AE$, and hence the work for both sides is $W \times 2\,AE$.

But the applied force $F$ acts through a space $AC$, and so does work $F \times AC$.

$$\text{Hence,} \quad W \times 2\,AE = F \times AC,$$
$$\text{and } W/F = AC/2\,AE.$$

This is the mechanical advantage, and it is evidently greater the thinner the wedge is.

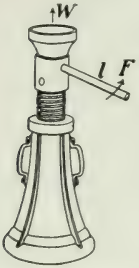



FIG. 130.—The jackscrew.

This result is of little practical value, as we have not taken friction into account, nor the fact that the force $F$ is applied as a blow, not as a steady pressure. Both of these factors are of great importance.

**120. The Screw.** The screw consists of a grooved cylinder which turns within a hollow cylinder or nut which it just fits. The distance from one thread to the next is called the *pitch*.

The law of the screw is easily obtained. Let $l$ be the length of the handle by which the screw is turned (Fig. 130) and $F$ the force exerted on it. In one rotation of the screw the end of the handle describes the circumference of a circle with radius $l$, that is, it moves through a distance $2\pi l$, and the work done is therefore

$$F \times 2\pi l.$$

Let $W$ be the force exerted upwards as the screw rises, and $d$ be the pitch. In one rotation the work done is

$$W \times d.$$

$$\text{Hence,}\quad W \times d = F \times 2\pi l,$$

$$\text{or } W/F = 2\pi l/d,$$

or the mechanical advantage is equal to the ratio of the circumference of the circle traced out by the end of the handle to the pitch of the screw.

In actual practice the advantage is much less than this on account of friction.

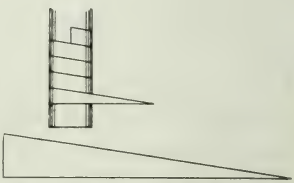


FIG. 131.—Diagram to show that the screw is an application of the inclined plane.

The screw is really an application of the inclined plane. If a triangular piece of paper, as in Fig. 131 be wrapped about a cylinder (a lead pencil, for instance), the hypotenuse of the triangle will trace out a spiral like the thread of a screw.

Examples of the screw are seen in the letter press (Fig. 132), and the vice (Fig. 133).

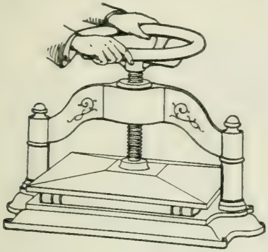


Fig. 132.—The letter press.

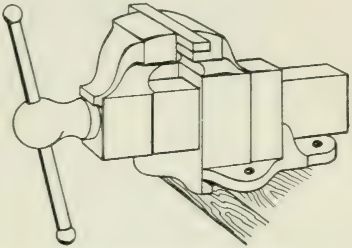


Fig. 133.—The mechanic's vice.

**ILLUSTRATIVE PROBLEMS**

1. Why should shears for cutting metal have short blades and long handles?

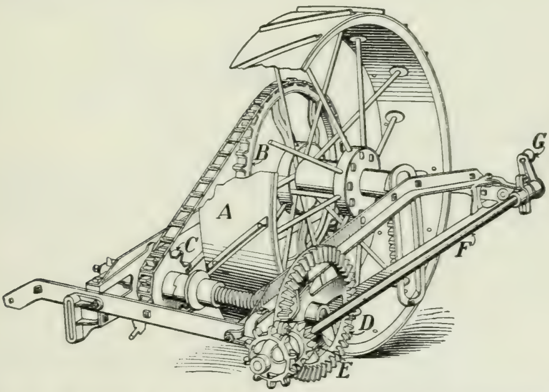



Fig. 134.—The driving part of a self-binder. The driving-wheel $A$ is drawn forward by the horses. On its axis is the sprocket-wheel $B$, and this, by means of the chain drives the sprocket-wheel $C$. The latter drives the cog-wheel $D$ which, again, drives the cog-wheel $E$, and this causes the shaft $F$ with the crank $G$ on its end to rotate.

2. In the driving mechanism of a self-binder, shown in Fig. 134, the driving-wheel $A$ has a diameter of 3 feet, the sprocket-wheels $B$ and $C$ have 40 teeth and 10 teeth, respectively. The large gear-wheel $D$ has 37 teeth and the small one $E$ has 12 teeth, and the crank $G$ is 3 in. long. Neglecting friction, what pull on the driving-wheel will be required to exert a force of 10 pounds on the crank $G$?

3. Explain the action of the levers in the scale shown in Fig. **135.**

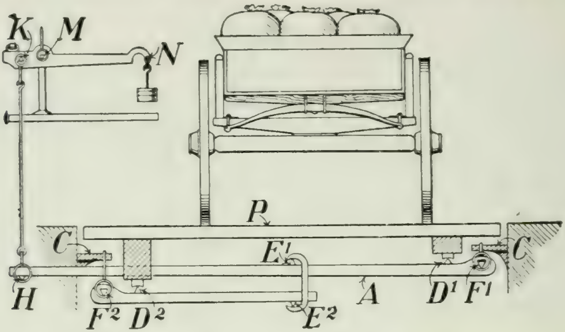



FIG. 135.—Diagram of multiplying levers in a scale for weighing hay, coal and other heavy loads. In the figure is shown one-half of the system of levers, as seen from one end. The platform $P$ rests on knife-edges $D^1$, $D^2$, the former of which is on a long lever, the latter on a short one. The knife-edges $F^1$, $F^2$ at the end of these levers are supported by suspension from the brackets $C$, $C$ which are rigidly connected with the earth.

If $HF^1 = 12$ ft., $F^1D^1 = 4$ inches, $MN = 36$ inches, $KM = 3$ inches, what weight on $N$ would balance 2000 pounds of a load (wagon and contents)? In the scale $E^1F^1 = E^2F^2$, and $F^1D^1 = F^2D^2$, so the load is simply divided equally between the two levers.

4. In the bicycles used before about 1880 there was a large wheel in front and a small one behind. The crank, with the pedals, was attached to the hub of the large wheel which the rider drove directly from his seat above it. One revolution of the pedals drove the bicycle forward a distance equal to the circumference of the large wheel. The rating of the gear of our present bicycles is made by comparison with the old ones. A "gear of 70" means that one revolution of the pedals drives the bicycle forward as far as an old style bicycle in one revolution, if the diameter of its large wheel was 70 inches.

($a$) The wheels of a bicycle are 28 in. in diameter. The large sprocket-wheel, to which the crank is attached, has 21 teeth, and the small one, which is attached to the hub of the rear wheel, has 8 teeth. Find the gear of the bicycle.

If the large sprocket-wheel has 24 and the small one 7 teeth, find the gear.

($b$) If the crank arm is 7 in. long, find the tangential force exerted on the road by the tire when the rider pushes downwards with a force of 50 pd. upon the arm when it is horizontal, the numbers of the teeth on the sprocket-wheels being 20 and 7, respectively. Find the force when the arm makes 30° with the horizontal.

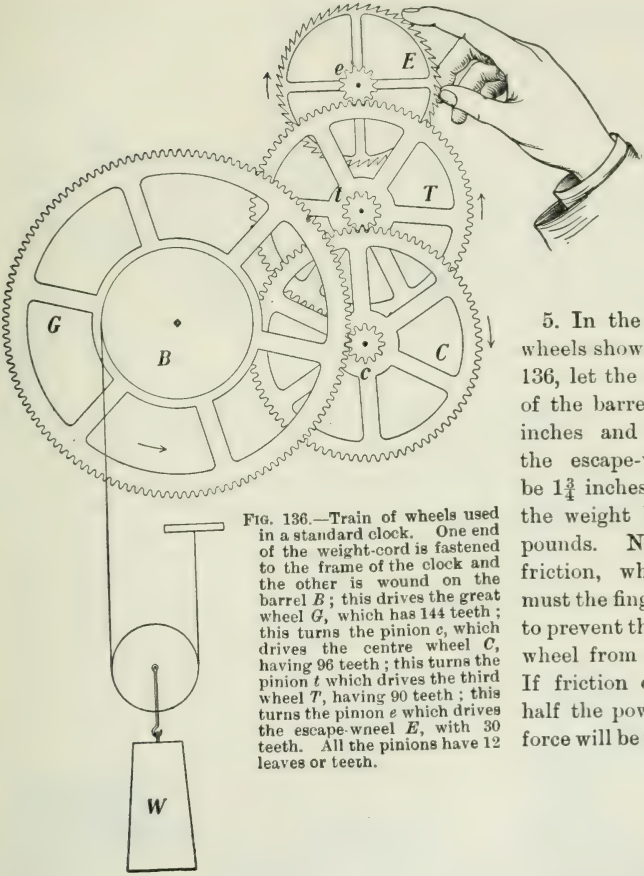



FIG. 136.—Train of wheels used in a standard clock. One end of the weight-cord is fastened to the frame of the clock and the other is wound on the barrel $B$; this drives the great wheel $G$, which has 144 teeth; this turns the pinion $c$, which drives the centre wheel $C$, having 96 teeth; this turns the pinion $t$ which drives the third wheel $T$, having 90 teeth; this turns the pinion $e$ which drives the escape-wheel $E$, with 30 teeth. All the pinions have 12 leaves or teeth.

5. In the train of wheels shown in Fig. 136, let the diameter of the barrel $B$ be 2 inches and that of the escape-wheel $E$ be $1\frac{3}{4}$ inches, and let the weight $W$ be 10 pounds. Neglecting friction, what force must the fingers exert to prevent the escape-wheel from turning? If friction consumes half the power, what force will be required?

**121. Automobile Transmission.** The automobile, considered as a whole, is an excellent example of a high-class machine, but some parts of it are especially interesting. Two of these are the 'transmission' and the 'differential.'

By means of the transmission the driver of the car can go forward with different speeds, can go backwards or can stand still while the engine continues to run. There are two main types of transmissions, the *planetary* and the *selective* transmission.

**122. Planetary Transmission.** One style of planetary transmission is illustrated in Figs. 137 and 138. In Fig. 137, the various

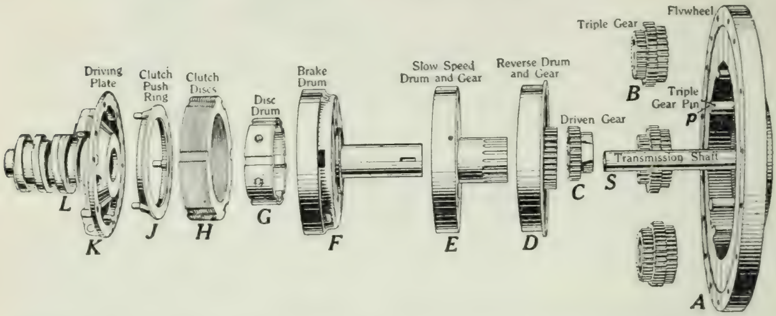
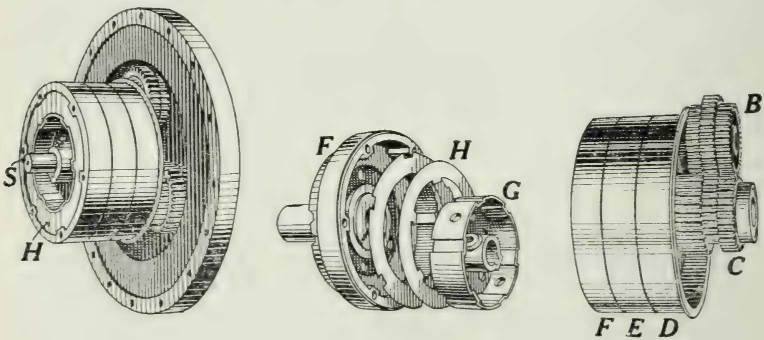



Fig. 137.—The parts of a planetary transmission and a clutch arranged in assembling order.

parts are shown in their relative positions for putting together, with their names beside them. First, the drum $E$ is moved up close to $D$, with its gear passing through a central hole in $D$. Then drum $F$ is fitted into $E$, with its hollow shaft passing through $E$ and $D$, and on the end of the shaft the driven gear $C$ is firmly fixed. Then the triple gears $B$ are placed in position to mesh with the three gears on $D$, $E$, $F$. Only one triple gear is shown in




(c) (b) (a)

Fig. 138.—Showing different parts of the assembling.

place in Fig. 138$a$ in order to exhibit more clearly how the teeth of the gears fit together; but in actual practice three are used so

that too great a strain may not be placed upon a single one. Two of them can be seen in Fig. 138*c*, the third being hidden by the drums. Then the flywheel $A$ is put in place, its shaft $S$ passing completely through the three drums, and the three pins $p$ on it passing through the triple gears. As the flywheel rotates, the triple gears are carried about on these pins, upon which they can also rotate. Thus the triple gears *revolve* about the central shaft $S$ and they also *rotate* on their own axes. Their motions are similar to the motions of a planet about the sun, and they are therefore called planetary gears.

Upon the end of the shaft $S$ the drum $G$ is securely fixed, within the drum $F$. Then a large annular disc (Fig. 138*b*) is slipped over $G$ and within $F$. This disc has notches in its outer edge which exactly fit projections on the inner surface of the drum $F$, so that this disc can turn freely about $G$ but it cannot turn in $F$. Then a small disc is slipped over $G$ and within $F$. It has projections which fit grooves in $G$, and thus it must turn with $G$, but it turns freely within the drum $F$. Then large and small discs are added alternately, ending with a large one (about 25 in all). The assembly thus far is shown in Fig. 138*c*. Observe that the small discs must turn with the drum $G$, and, as this is fast on the shaft S, they must turn with the flywheel, and hence with the engine, since the flywheel is fixed on the engine shaft.

Next, the projections on the ring $J$ fit into holes in the driving plate $K$, which is then securely bolted to the drum $F$. The plate $K$ is connected to the driving shaft which runs to the rear of the car and drives the wheels. Observe now that this shaft is attached to $K$ which is rigidly attached to $F$, and on the hollow shaft of $F$ the driven gear $C$ is firmly fixed. Thus when the gear $C$ is rotated, the rear wheels of the car (or at least one of them) must turn.

By means of the strong spring seen just left of $L$ (Fig. 137) the ring $J$ is pressed against the outer disc and thus all the discs are pressed together. As there are many discs there is a larger surface area in contact and hence much friction is developed when the small discs slide over the large ones. Consider now the condition existing when the spring presses the discs together. The engine turns the flywheel; its shaft $S$ turns the drum $G$, which carries about with it the small discs; these drag the large discs by friction, and hence turn the drum $F$; this is connected to the plate $K$ and the driving shaft, and the car moves forward, the driving shaft turning at the same rate as the flywheel.

By pushing a pedal or pulling a lever conveniently placed, the ring $L$ (Fig. 137) can be pushed backwards, compressing the spring

and relieving the friction between the discs. In this condition the flywheel may rotate and turn the drum $G$ and the small discs with it, while the large discs (and hence the drum $F$ and the driving shaft) do not turn and the car remains standing still. This arrangement of discs is called the *clutch*; when the discs are pressed together the clutch is 'in,' and when the pressure is removed the clutch is 'out.' It is evident, then, that when the clutch is 'in' the power of the engine is transmitted directly to the rear wheels of the car and it goes forward. When the clutch is 'out' the clutch discs slip past each other and the car stands still while the engine continues to run.

**123. How to Reverse or go at Slow Speed.** When the driver of the car wishes to go backwards he pushes a pedal, thereby tightening a brake-band about the drum $D$, and preventing it from turning. Then as the triple gear is made to revolve by the flywheel the gear on $D$, meshing with the smallest gear of the triple gear, makes it rotate on its axis, while the forward gear of the triple gear, meshing with $C$, the driven gear, makes $C$ rotate in the opposite direction.

In order to go forward at slow speed, a brake-band is tightened about $E$, preventing it from turning. Then the middle gear of the triple gear is caused to rotate by meshing with the (smaller) gear on $E$, while the forward gear of the triple gear makes $C$, the driven gear, rotate, but this time in the forward direction with reduced speed.

**124. Explanation of Reversing.** In order to understand how these results can be produced, consider Fig. 139. $A$, $B$, $a$, $b$, are gear wheels; $a$ is fixed on the shaft $E$, $b$ on the shaft $D$, while $A$ and $B$ are fastened together and move as one. $A$ meshes with $a$ and $B$ with $b$. The planetary double gear $A$, $B$ is carried on a pin $p$ in the flywheel $C$, which rotates freely on the shaft $D$. Suppose now the shaft $D$ is prevented from turning, and the flywheel is made to turn about $D$; what will happen to the shaft $E$?

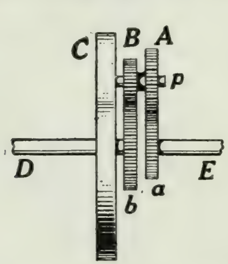


FIG. 139.—Model to illustrate action of planetary gears.

Let the wheels $b$, $B$, $a$, $A$ have 24, 12, 18, 18 teeth, respectively, and suppose the wheel $C$ to be turning so that the top is moving away from the observer. Then Fig. 140 shows the positions of the wheels (as seen from the $E$-shaft side) at three moments,

namely, at the beginning of the motion, when $C$ has turned through 30° and when it has turned 60°. The action of $b$ on $B$

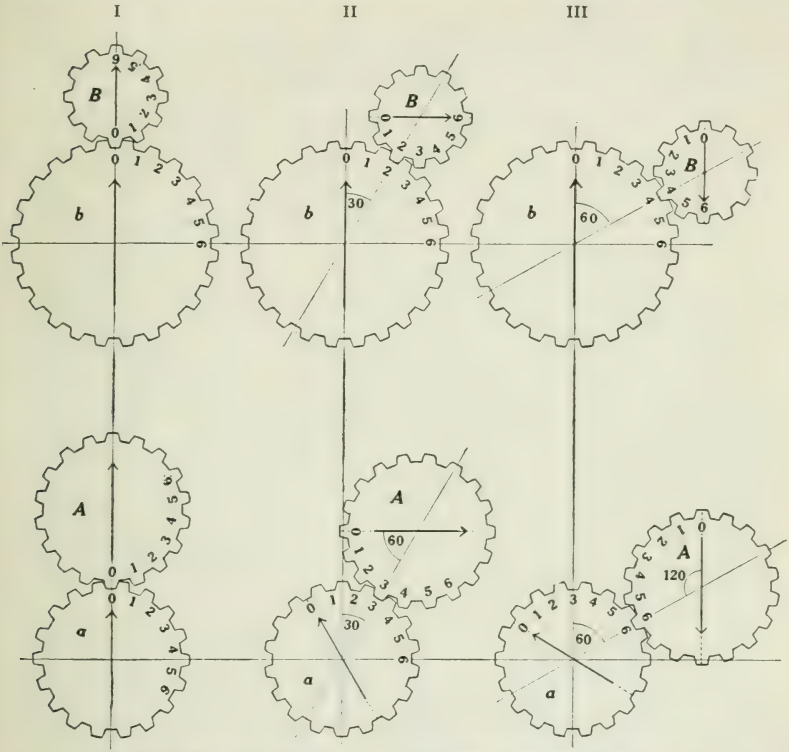



FIG. 140.—Showing how the revolution of $B$ causes a backward rotation in $a$.

is shown in the upper half of the figure and that of $A$ on $a$ in the lower half. It is at once seen that when the planetary gears have revolved through 60° the gear $a$ has rotated through the same angle but *in the opposite direction*; in other words, the shaft $E$ rotates at the same rate as the flywheel $C$, but in the opposite direction.

**EXERCISE**

Cut four circles from thin cardboard with diameters 2, 3, 3, 4 inches, respectively and having teeth as shown in the figure. Mark a diameter on each and number the teeth and the spaces as illustrated. Then draw diagrams to show the position of the wheels at 30-degree intervals of the revolution of the planetary gears.

**125. Method of Calculating.** Next let us establish a method of determining the rotation of $a$ when the number of teeth on each of the gears is given.

When a body rotates so that its motion is in the same direction as the hands of a clock its rotation will be called *forward*; when in the opposite direction, the rotation is *backward*.

First, suppose that the gear $B$ cannot turn on the pin $p$ and that it does not mesh with $b$. It is clear that as it is carried once round by $C$, the arrow on it will continually change its direction, passing through 360° and coming back to its original position. Thus, *while $B$ makes one revolution about the shaft it also makes a rotation.*

Next, consider the effect of its meshing with $b$. There are 24 teeth on $b$ and hence 30° corresponds to 2 teeth; but 2 teeth on $B$ corresponds to 60° at its centre. Hence in position II, $B$ has rotated 30° on account of its revolution and 60° through its meshing with $b$, or 90° in all. In position III, $B$ has rotated 60° on account of its revolution and 120° through its meshing with $b$, or 180° in all. That is, during $\frac{1}{6}$th of a revolution $B$ makes $\frac{1}{2}$ a rotation, and in 1 revolution it will make 3 rotations. Of these three, 1 will be on account of its revolution and the other 2 through its meshing with $b$. These 2 rotations would have been produced if the centre of $B$ were kept fixed and $b$ were made to rotate once in the backward direction. Or we can find the number of rotations of $B$ through meshing with $b$ by dividing the number of teeth on $B$ into the number of teeth on $b$ ($24 \div 12 = 2$).

In the next place, turn to the action of $A$ on $a$. $A$ is rigidly connected with $B$ and must turn with it, that is, during the revolution it has rotated once on account of the revolution and twice besides. From the figure it is seen that if it simply revolved (without having the two additional rotations) it would carry $a$ round once in the forward direction. Again if it rotated twice in the forward direction on the pin $p$, without any revolution, it would cause $a$ to rotate twice in the *backward* direction. The result of the revolution and the rotation taken together will thus be that $a$ will rotate once in the backward direction while the planetary gears (or the flywheel) make one revolution.

**Example:**—Let $B$ have 24 teeth, $b$ have 30, and $A$ and $a$ 27 each. Then during 1 revolution $B$ will make $\frac{30}{24} + 1 = 1\frac{1}{4} + 1$ forward rotations. Also $A$ will make $1\frac{1}{4} + 1$ forward rotations, and hence $a$ will make 1 forward and $1\frac{1}{4}$ backward rotations, or, added together, $\frac{1}{4}$ of a backward rotation. Thus $a$ will rotate in the reverse direction with $\frac{1}{4}$ the rate of the flywheel in the forward direction.

**126. Slow Speed Forward.** Next, suppose that the gears $B$ and $b$ (Fig. 139) are interchanged and that the larger gear revolves about the smaller.

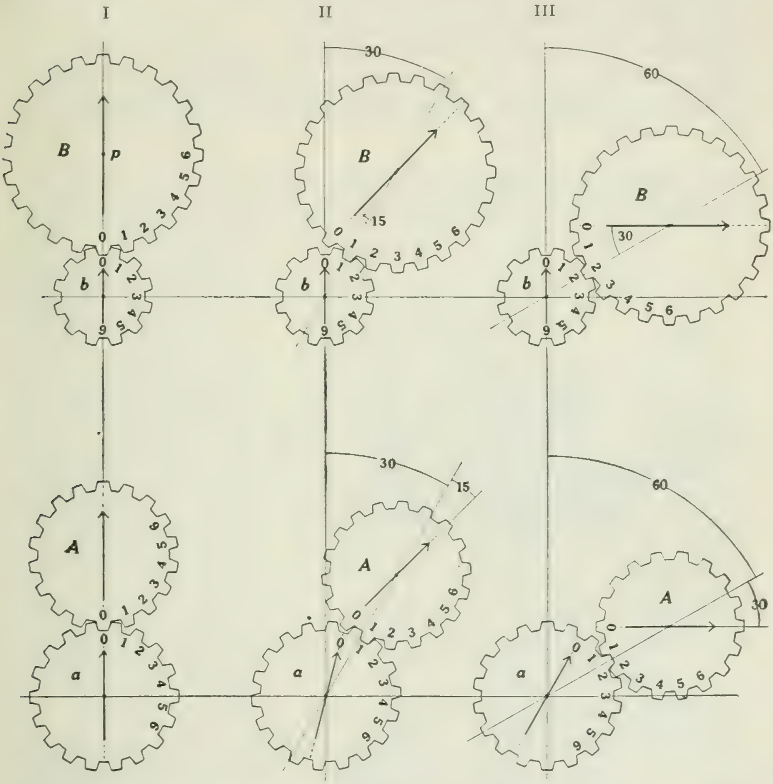



FIG. 141.—Diagram to show how a slow speed forward is obtained.

The result is shown in Fig. 141. While $B$ revolves through 60° it rotates 60° on account of its revolution and 30° through its meshing with $b$, or 90° in all. From the lower half of the figure it is seen that during this 60° of revolution $a$ has rotated *in the same direction* through 30°, that is, the gear $a$ rotates in the forward direction, with one-half the speed of revolution of the flywheel.

Applying our method of calculation :—When $B$ revolves once, it rotates $1 + \frac{12}{24} = 1 + \frac{1}{2}$ times. Now $A$ moves in the same manner

as $B$, and on account of its revolution $a$ will make 1 forward rotation, and through the $\frac{1}{2}$ forward rotation of $A$, $a$ will make $\frac{1}{2}$ of a backward rotation. The result will be that $a$ will rotate $+ 1 - \frac{1}{2} = + \frac{1}{2}$ time during each revolution.

**Example**:—Let $B$ have 33 teeth; $b$, 21; and $A$, $a$ each have 27. Then $a$ will make $+ 1 - \frac{21}{33} = + \frac{4}{11}$ of a rotation during each revolution or the rate of rotation is in the same direction as the flywheel (*i.e.*, forward) and $\frac{4}{11}$ as great.

**127. Selective Type of Transmission.** When it is desired to have three (or more) speeds forward the selective type, illustrated in Fig. 142, is used. Its action is much easier to understand.

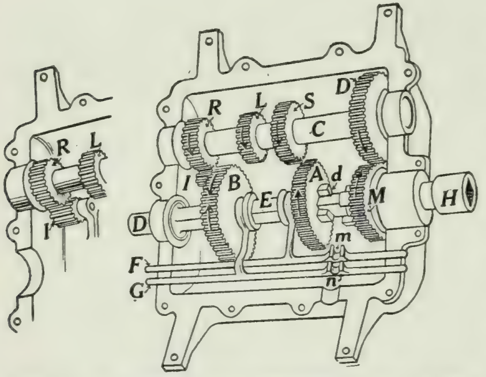


FIG. 142.—The selective type transmission, by which three forward speeds can be obtained.

The shaft which comes from the engine enters the square opening in the shaft $H$, which, beyond the bearing, has the gear $M$ fixed upon it. This shaft terminates just beyond $M$, but in the same line with it is the shaft $E$, which for some distance has a square section and which at $D$ is attached to the driving shaft leading back to the rear wheels.

Mounted parallel to $E$ is the countershaft $C$ on which are 4 gears $D$, $S$, $L$, $R$. The gear $D$ is always in mesh with $M$ and consequently always rotates when the shaft $H$ does.

The gears $A$ and $B$ can be shifted forward or backward on the square shaft $E$, by means of the sliding rods $F$, $G$. In order to obtain first, or low, speed, the gear $B$ is shifted forward until it meshes with $L$. Thus $M$ turns $D$ and $L$ turns $B$ which turns the driving shaft. For second, or intermediate, speed the gear $A$ is shifted backwards until it meshes with $S$. The speed will now be greater since $S$ is larger than $L$, and $A$ is smaller than $B$. For third, or high, speed the gear $A$ is shifted forward until the little projections or 'dogs' $d$ fit between similar dogs on $M$. In this case

the shaft $E$, and hence the driving shaft, turns at the same rate as the shaft $H$.

In order to reverse, the gear $B$ is shifted backwards until it meshes with a small idle gear $I$ (seen better in the left-hand portion of the figure). In this case $M$ turns $D$, $R$ turns $I$, and $I$ turns $B$ in the opposite direction.

The shifting of the gears is accomplished by moving a lever, the lower end of which fits into the notches $m$, $n$ according to the way the lever is moved.

**128. The Differential.** This is placed on the rear axle and its object is to permit the two rear wheels to turn independently. Such an arrangement is very necessary, since in turning around one of the rear wheels moves much farther than the other. Without the deferential it would be almost impossible to turn sharply as one wheel would have to slide on the road.

Fig. 143 gives a view of the differential as seen from above, and facing the front of the car. The pinion $A$ on the end of the driving shaft $S$ meshes with the large gear wheel. Upon this latter wheel is a strong metal case or 'housing' $C$ which rotates with the wheel. In the walls of this are pins $d$, $d$ upon which gears $D$, $D$ can turn, and these mesh with gears $E$, $E$ one of which is fixed on the left axle $L$, the other on the right axle $R$.

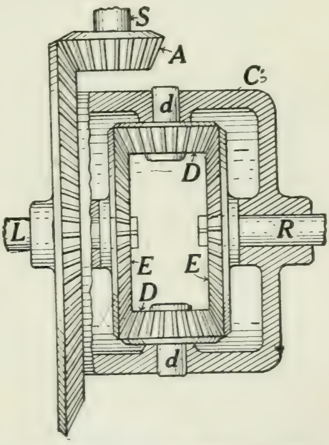


FIG. 143.—The 'differential' of an automobile.

Imagine the housing $C$ to be turning in such a way that the upper part of the figure is moving from the observer. Then the gears $D$, $D$ will drag $E$, $E$ in this direction and the two axles $L$ and $R$ will drive the car forward. $D$, $D$ do not rotate on their axes $d$, $d$ at all.

But suppose the left wheel is fast and cannot move. Then the left gear $E$ does not move, and as $D$, $D$ are carried about by the housing they must rotate on their axes, and this rotation will simply double the rate of rotation of the $R$ axle.

## PROBLEMS

1. If the pinion *A* has 11 teeth and the larger gear into which it meshes has 40 teeth, compare the revolutions per minute of the wheels with those of the driving shaft. (Fig. 143).

2. If the wheels of the car are 30 inches in diameter find the revolutions per minute of the engine when the car is going forward at 30 miles per hour.

# CHAPTER XV

## Pressure and Its Transmission

**129. Pressure: How Measured.** With the idea of pressure we are all familiar. In a pile of books each presses the one below it; or when a piece of wood or metal is held in a vice the jaws of the vice press upon the surface of the object. Many other illustrations could be given, and it is to be observed that in every case *pressure acts upon a surface.*

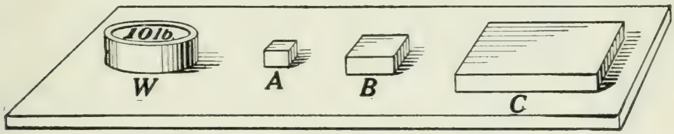



Fig. 144.—Illustrating the measurement of pressure.

Cut from a thin board three square blocks having edges $\frac{1}{2}$ inch, 1 inch and 3 inches, respectively, and lay them on a table ($A$, $B$, $C$, Fig. 144). These are so very light that we may neglect the forces exerted by them upon the table. Now place the 10-pound weight $W$ upon $B$; there is pressure thus produced upon the table and as the area of the surface acted upon is 1 square inch we say the pressure is 10 pounds per square inch.

Next, place the weight upon $C$. The area of the surface now acted upon is 9 square inches, and we say the pressure on it is $1\frac{1}{9}$ pounds per square inch.

Finally place the weight on $A$. In this case the area acted upon is $\frac{1}{4}$ square inch; so the pressure is 10 pounds per $\frac{1}{4}$ square inch or 40 pounds per square inch.

In each case the total force exerted is the same but the pressure, or the force per square inch, differs, being in the three cases in the proportion 1 : 9 : 36.

In specifying a pressure always give the force on unit area; as, pounds per square inch, grams per square centimetre, or tons per square yard.

**130. Pressure of a Fluid.** It is a matter of common experience that a fluid exerts a force upon the surface with

which it is in contact. A wooden tank, such as we often see above buildings for fire-protection purposes, or beside the railway for supplying water to the locomotives, is bound with strong iron bands to prevent the water from pushing the staves outwards. Note, also, that the bands are closer together near the bottom than higher up, indicating that the pressure at the bottom is greater than near the top.

Consider a vessel like that in Fig. 145, having a piston inserted in the bottom. A force must be applied upwards on the piston to prevent the water from pushing the piston out. Let the force upwards required to balance the pressure of the water be 10 pounds, and the area of the piston be 5 square inches. Then the pressure of the water on the piston is 2 pounds per square inch.

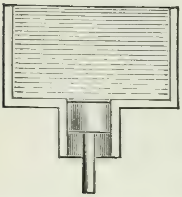


FIG. 145.—Pressure at the bottom of a vessel.

Next consider the case of a piston of the same size inserted in the side of the vessel (Fig. 146). As remarked above, the water exerts a force upon the piston. If we adjust the depth of the water so that, as before, the force required to balance the pressure of the water is 10 pounds, then the *average* pressure of the water on the piston is 2 pounds per square inch. In this case it is necessary to say *average* pressure because of the fact of experience mentioned above, that the pressure depends upon the depth and so is not uniform over the surface of the piston. The manner in which the pressure varies with the depth will be taken up in the next chapter.

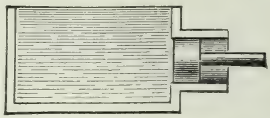


FIG. 146.—Pressure at the side of a vessel.

**131. Pressure at a Point.** We have just seen that pressure in a fluid varies from place to place, and we often use the phrase *pressure at a point.* Let us look into the precise meaning of the phrase.

Consider a *very small* surface of area $A$, containing the point, so small indeed that the pressure upon it may be considered uniform all over it. Let $F$ be the total force exerted upon the area $A$. Then the force $P$ on unit area $= F/A$. This is the pressure at the point.

Suppose we press upon the hand with the flat end of a lead pencil with such a force that the pressure is 5 pounds per square inch. It is easily seen that the force applied is not equal to 5 pounds nor is the surface acted upon equal to a square inch. But if

$F$ = applied force in pounds,

and $A$ = area acted upon in square inches,

Then pressure $= F/A = 5$ pounds per square inch.

**132. Transmission of Pressure by a Fluid.** The subject of pressure in a fluid and its transmission is so important that we must consider it still further.

Take an apparatus like that shown in Fig. 147 and fill it with water. When a force is exerted on the water by means of a piston, it will be seen that the pressure *is transmitted*, not simply in the direction in which the force is applied, but *in all directions*; because jets of water are thrown with velocities which are apparently equal from all the apertures. If the conditions are modified by connecting U-shaped tubes partially filled with mercury with the globe as shown in Fig. 148, it will be found that when the piston is inserted, the change in level of the mercury, caused by the transmitted pressure, is the same in each tube. This would show that the pressure applied to the piston is transmitted *equally* in all directions by the water.

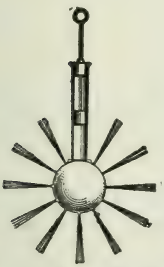


FIG. 147.—Pressure applied to the piston transmitted in all directions by the liquid within the globe.

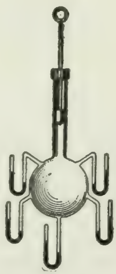


FIG. 148.—Transmission shown to be equal in all directions by pressure gauges.

Again, consider the vertical section of a closed vessel filled with some fluid, say, water, and fitted with pistons of equal area as shown in Fig. 149. Let a force $P$ pounds be applied to the piston $A$. This will produce a force within the water which will be transmitted by the water to every surface with which it is in contact.

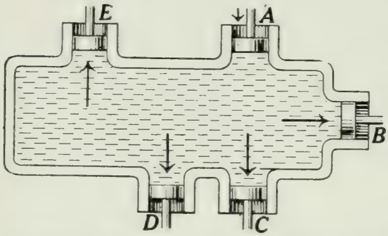


FIG. 149.—Diagram illustrating transmission of pressure.

The piston $A$ will exert a pressure of $P$ pounds upon the surface of the water in contact with it and this pressure will be transmitted not only to $C$, which is directly opposite $A$, but also to $D$, which is alongside $C$, to $B$ which is at one end of the vessel, and to $E$ which is at the top beside $A$. The pressure is transmitted by the fluid *in all directions* and *undiminished in intensity*.

If the pistons $C$ and $D$ were merged into a single one the pressure on it would be 2 $P$ pounds, or the pressure is proportional to the area of the surface.

If the area of the piston $A$ is 5 square inches and the force applied to it be 10 pounds, there will be a pressure of 2 pounds on every square inch of the inner surface of the vessel.

**133. Pressure of a Fluid at Right Angles to the Surface.** From the fact that a fluid transmits pressure perfectly, that is, in every direction and without diminution, we must conclude that its particles are perfectly free to move about amongst themselves, that the slightest force applied to a liquid can displace its particles.

It must follow that when a fluid is at rest its pressure is at right angles to the surface upon which it acts. This can be proved in the following way.

If it were possible, let the pressure at $A$ (Fig. 150) upon the side of the vessel be not at right angles to the surface, but in the direction $R$. Resolve the force into two components, $Q$ at right angles to the surface and $P$ parallel to the surface. The force $Q$ is balanced by the reaction of the side of the vessel, but $P$ is unopposed and it must cause a sliding of the particles of the liquid along the surface in the direction $AB$. But this is impossible as, by hypothesis, the fluid is at rest.

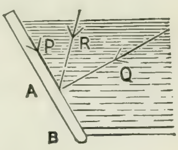


Fig. 150.—Pressure is at right angles to the surface.

We must, therefore, conclude that the pressure of a fluid is at right angles to the surface upon which it acts.

We are now in a position to state the following law:

*Pressure exerted anywhere upon a mass of fluid within a closed vessel is transmitted undiminished in all directions, and acts with the same force on all equal surfaces and in a direction at right angles to them.*

This is known as Pascal's Law or Principle.

**134. Mechanical Application—Hydraulic Press.** The transmission of pressure by a liquid equally in all directions is the principle utilized in the hydraulic press. Fig. 151 illustrates this. $D$ and $E$ are two hollow cylinders connected by a tube $C$ and partly filled with water. $A$ and $B$ are two pistons, fitted into $D$ and $E$ respectively. Any force applied to $A$ is transmitted by the liquid to $B$, and as the pressures on $A$ and $B$ are undiminished in intensity, the total forces exerted by the liquid upon $A$ and $B$ are proportional to their areas. Thus, if the area of $A$ is 1 square inch and that of $B$ is 10 square inches; then a weight of 1 pound on $A$ will sustain a weight of 10 pounds on $B$.

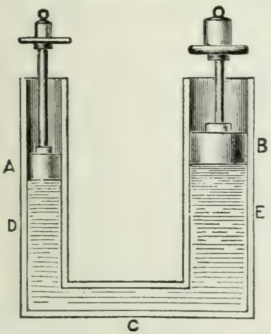


Fig. 151.—Illustrating the hydraulic press.

For a description of Bramah's Press, see Sec. 181.

**135. Pressure at a Point in a Fluid.** Consider a vessel filled with fluid, and let the area of the piston $A$ be one unit, say, 1 square inch. Let $C$ be any point within the mass of the fluid, and imagine it to be at the centre of a circular plane area $mn$ (seen edgewise). Let the area of $mn$ be 1 square inch.

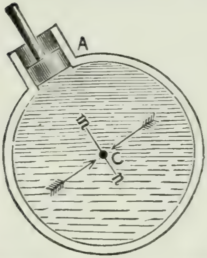



Fig. 152.—Pressure the same in all directions in a fluid.

If the piston is pushed inwards with a force of $P$ pounds, the pressure on every square inch of the inner surface of the vessel will be $P$ pounds; also, each face of $mn$ will be subjected to a force of $P$ pounds.

Now the magnitude of this force does not depend on the direction in which the area $mn$ is turned, that is, the pressure at $C$ is the same in all directions.

#### PROBLEMS AND EXERCISES

1. A fluid pressure of 1,728 pounds is uniformly distributed over a surface whose area is 3 sq. ft. Find the measure of the pressure at a point in the surface (1) when the unit-area is 1 sq. in., (2) when it is 1 sq. ft., the unit of force in each case being 1 pound.

2. The pressure is uniform over the whole of a sq. yard of a plane area in contact with a fluid, and is 7,776 pounds. Find the measure of the pressure at a point (1) when the unit of length is 1 in., (2) when it is 3 in., the unit of force in each case being the pound.

3. The uniform pressure of a fluid over a circular plane, diameter 14 cm., is 770 kg ; find the measure of the pressure at a point (1) when the unit-area is 1 sq. mm., (2) when it is 1 sq. dm.; if the unit of force is the gram.

4. A rectangular surface, length 50 cm. and width 4 cm., is subjected to a uniformly distributed fluid pressure of 4 kg. Find the measure of the pressure at a point (1) when the unit of length is 1 mm., (2) when the unit of length is 2 mm.; if the unit of force is the gram.

5. If the area of a piston inserted in a closed vessel is $3\frac{1}{4}$ sq. in., and if it is pressed with a force of 35 pounds, find the pressure which it will transmit to a surface of $7\frac{2}{7}$ sq. inches.

6. A closed vessel is filled with liquid, and two circular pistons, whose diameters are respectively 2 cm. and 5 cm. inserted. If the pressure on the smaller piston is 50 grams, find the pressure on the larger piston when they balance each other.

7. A closed vessel is filled with fluid and two circular pistons whose diameters are respectively 3 in. and 7 in. inserted ; if the pressure on the larger piston is $a$ pounds, find the pressure on the smaller.

8. The diameter of the large piston of a hydraulic press is 100 cm. and that of the smaller piston 5 cm. What force will be exerted by the press when a force of two kilograms is applied to the small piston ?

9. The diameter of the piston of a hydraulic elevator is 14 inches. Neglecting friction, what load, including the weight of the cage, can be lifted when the pressure of the water in the mains is 75 pounds per sq. inch ?

10. The horizontal cross-section of the neck of a glass bottle, just capable of sustaining a pressure of 11 pounds to the sq. in., is $2\frac{3}{4}$ sq. in. It is filled with a fluid supposed weightless, and a piston is inserted into the neck. What is the least force that must be applied to the piston to break the bottle ?

11. If the diameter of the small piston (Fig. 151) is 5 cm., and that of the larger one 2.5 metres, and if the small piston is pushed with a force of 8 gms., what force will it transmit to the large piston ?

12. In the same machine the horizontal cross-section of the small piston is 3 sq. cm.; with what force must it be pushed that it may sustain a force of 7.25 kg. applied to a piston whose horizontal cross-section is 7 sq. dm. ?

13. Pour a small quantity of mercury into a tube of the form shown in the Fig. 153. Now pour some water into the larger branch.

(1) What changes take place in the levels of the mercury in the two branches ? Why ?

(2) How much water do you suppose must be put into the smaller branch to bring the mercury to the same level in each branch ? Give reasons for your answer. Verify by pouring water into the smaller branch.

(3) How does the weight of the water in the large branch compare with that in the smaller one when the mercury is restored to the same level in each tube ?

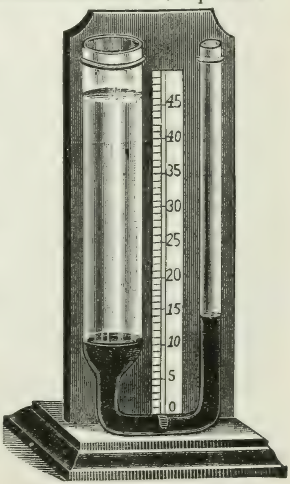



FIG. 153.—Experiment with two liquids.

## CHAPTER XVI

### Equilibrium of Fluids Under Gravity

**136. Liquids are Attracted Towards the Earth.** Anyone who has carried a pail of water need not be told that water is *heavy*. Every particle of it is attracted towards the earth and it is for this reason that liquids must be held in non-porous vessels, though these need not be covered as the liquid keeps to the bottom.

If bricks be laid one upon another there is a pressure upon the surface of any brick produced by those bricks above it. The one at the bottom has to bear the weight of all those above it. So it is in a vessel containing a fluid. The lower layers have to support all the fluid above them and we would expect them to be under pressure. Also there must be pressure upon the bottom of the vessel and, on account of the nature of the fluid, upon its sides as well. In the previous chapter we learned that within a fluid the pressure at a point is the same in all directions.

**137. Force Within the Liquid.** If we pierce a hole in the side or bottom of a vessel filled with water, the water rushes out and the farther the hole is below the surface the more quickly does the liquid escape. It is an old camper's experiment to obtain cold water from the bottom of a lake by lowering a bottle closed by a cork and so arranged that the water will force it into the bottle when it gets low enough down. These results show in a general way that the pressure depends upon the depth.

**138. Pressure Proportional to Depth.** A simple experimental demonstration that the pressure within a liquid

increases with the depth can be made with the apparatus shown in Fig. 154.

Over the mouth of a thistle-tube *A* is stretched a thin rubber membrane (such as is used in toy ballons) and by means of a rubber tube the thistle-tube is connected with a U-tube *B*, partially filled with water. On thrusting the finger against the membrane it bends inwards and the pressure upon the air in the connecting tube is transmitted to the surface of the water and causes the water to fall in one arm of the U-tube and rise in the other.

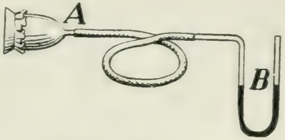


FIG. 154.—Pressure gauge.

Now place *A* in a vessel of water which has been allowed to reach the temperature of the room, and gradually push it downward (Fig. 155). The lower it goes the greater becomes the difference of level in the gauge *B*, thus showing that the pressure increases with the depth.

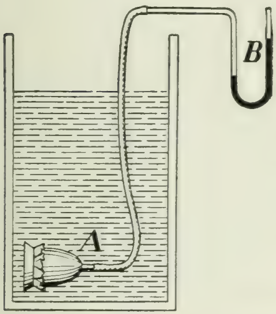


FIG. 155.—Investigation of pressure within the mass of a liquid by a pressure gauge.

By means of suitable apparatus, careful experiments show that the pressure within a liquid increases from the surface downward *in direct proportion to the depth.*

The same apparatus can be used to show that at any depth the pressure is the same in all directions. Being careful to keep the centre of the membrane at the same depth, turn *A* to face downward, upward or in any direction. The pressure indicated in *B* is the same in every case.

**139. Pressure Independent of Shape of Containing Vessel.** In our proof of the law that the pressure within a liquid varies with the depth nothing was said about the shape of the containing vessel or the quantity of liquid present. It will be useful to investigate if this matter should be taken into consideration.

In the case of a vessel with vertical sides the pressure on the bottom is obviously the weight of the liquid. But what if the sides are not vertical? We can settle the question by means of the apparatus shown in Fig. 156. *A*, *B*, *C*, *D* are tubes of different shapes but made to fit into a common base. *E* is a detachable bottom held in position by a lever and weight.

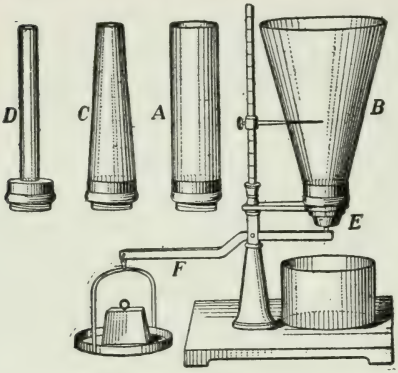


Fig. 156.—Pressures on the bottoms of vessels of different shapes and capacities.

First, screw the cylindrical tube *A* in position and place a suitable weight on the scale-pan. Then let us pour water into the vessel until at last the pressure due to the water pushes the bottom down and allows the water to escape. With the pointer mark the height reached by the water when this happens. Now repeat the experiment using *B*, *C*, *D* in succession. We observe that the height of the water when the bottom drops down is the same for all the various vessels.

Thus we see that the pressure on the bottom of a vessel produced by liquid in it depends only on the depth of the liquid, not at all upon the shape of the vessel or the amount of liquid in it.

**140. Surface of Liquid in Connecting Vessels.** Pour water into a series of vessels, *A*, *B*, *C*, *D*, *E*, of different shapes, connected together so the liquid can pass freely from one to another. The water will rise in them so that all their surfaces will be in the same horizontal plane.

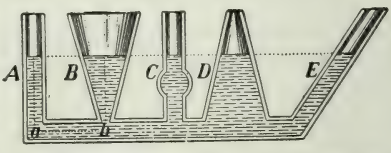


Fig. 157.—Surface of a liquid in connecting tubes in the same horizontal plane.

The reason for this can easily be given. Consider the vessels $A$ and $B$, and let $a$ and $b$ be two points in the liquid on the same horizontal plane. Now by hypothesis the liquid is at rest and therefore the pressure at $a$ toward $b$ is equal to that at $b$ towards $a$ since there is no motion from one to the other; but these pressures are equal only when $a$ and $b$ are at the same depth below the surface of the liquid. But the line $ab$ is horizontal, and hence *the surface of a liquid at rest is horizontal.*

It may be well, however, to consider this matter a little further. If it were possible, suppose the surface of the liquid in a vessel not horizontal but inclined as in Fig. 158; and let $a$ and $b$ be on the same horizontal, $b$ being on the surface and $a$ below it. At $a$ there is a downward pressure proportional to the depth $ac$, which pressure is transmitted in all directions. At $b$ there is no fluid pressure; consequently the water particles will be forced toward $b$ and the surface there will rise while that at $c$ will sink until they all reach the same level.

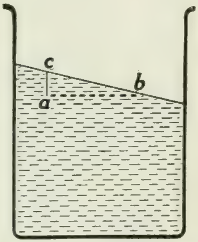



FIG. 158.—Diagram to show that the surface is horizontal.

**141. "Water Seeks its Own Level."** This is a familiar statement, and the cause of the water seeking its own level is the *force of gravity*. The water moves until its surface is at right angles to the direction of the force of gravity. Over a small area we consider the force of gravity to act in parallel vertical lines and the surface then would be a horizontal plane. But the force of gravity is directed towards the centre of the earth, and over an area of the earth's surface of considerable size, the radii of the sphere cannot be taken as parallel. As the surface of the water is at right angles to the directions of the force, that is, the earth's radii, the free surface of the liquid must be spherical. But the curvature is so slight that

we do not notice it in the case of a pail of water, while when the body is a large one, like a great lake or the ocean, the curvature is evident enough.

**142. Calculation of Pressure: Examples.** (1) What is the pressure at a point, (*a*) 2 metres below, (*b*) 30 feet below, the surface of water?

(*a*) Consider 1 sq. cm. of horizontal area at a depth of 2 metres. Then the pressure upon this surface is equal to the weight of a vertical column of water standing upon it and reaching to the surface. Its volume = 200 c.c., and its weight = 200 grams. Hence the pressure = 200 grams per sq. cm.

(*b*) Taking 1 sq. ft. of horizontal area at depth of 30 ft., the volume of the vertical column upon this = 30 cu. ft., and its weight = 30 × 62.5 pounds = 1875 pounds. Hence the pressure is 1875 pounds per sq. ft.

(2) A tube of 10 metres long and 1 sq. cm. in cross-section. One end is screwed into the upper face of a cylindrical vessel of radius 7 cm. and height 2 cm. The tube and vessel are filled with water. Find the weight of the water; also the pressure per sq. cm. as well as the whole force downwards upon the bottom.

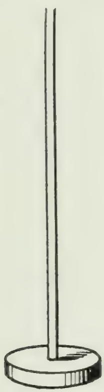


Fig. 159.—Pressure, depth, and volume.

The volume of the cylinder = $\frac{22}{7} \times 7^2 \times 2 = 308$ c.c. The volume of the tube = 1000 c.c. Total volume = 1308 c.c. and the weight of the water = 1308 grams or 1.308 kg.

The bottom is 1002 cm. below the surface of the water. Consider a column standing on 1 sq. cm. and reaching upward 1002 cm.; its volume = 1002 c.c. and weight 1002 grams. Hence the pressure = 1002 grams per sq.cm. The area of the base = $\frac{22}{7} \times 7^2 = 154$ sq. cm. Hence whole force = 154 × 1002 = 154,308 grams or 154.308 kg.

We can solve the problem in a slightly different way—The pressure on the bottom will be equal to the weight of a column of water 154 sq. cm. in cross-section and 1002 cm. high or 154,308 c.c. Hence the whole force = 154,308 grams. The pressure = 154,308 ÷ 154 = 1002 grams per sq. cm., as before.

**143. Hydrostatic Paradox.** The result just given illustrates the peculiar fact that by means of a small amount of a liquid

we can obtain a very great pressure. In the case considered, if the liquid were conceived to become solid and to stand on the bottom of the vessel, the whole force downwards on the bottom would be its weight, or 1.308 kg.; but if the same matter is in the form of a liquid, the force downwards on the bottom is 154.308 kg. This result appears paradoxical, that is, seemingly absurd or contradictory.

Fig. 160 will help us to explain it. Consider a vessel of the shape shown, filled with water. At depth $AB$ there is a certain pressure in the water, which pressure is the same all over the horizontal plane $CKBE$. This pressure acts equally in all directions, and hence acts upwards upon the surfaces $CK$, $BE$. These surfaces react and exert a pressure downwards upon the water. These forces of reaction together with the weight of the water produce the total force on the base $DF$.

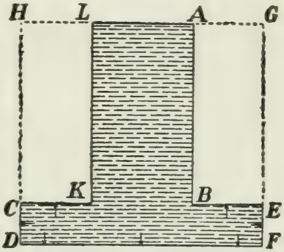


FIG. 160.—Explanation of the hydrostatic paradox.

Now the forces upwards upon $CK$, $BE$, due to the liquid, would be just balanced by the weight of liquid in the spaces $HK$ and $AE$ respectively.

Consequently the whole force on the bottom is equal to the weight of liquid which would just fill the entire space $HDFG$, that is, the weight of a column of liquid standing on $DF$ and reaching to the surface.

#### PROBLEMS

1. If the pressure of a liquid at a depth of 14 ft. 3 in. is 6 pounds to the sq. in., find the pressure at a depth of 21 ft. 8 in.

2. If the pressure at a depth of 5.6 metres is 2.8 gm., what is the pressure at a depth of 7.5 cm. ?

3. If the pressure on a sq. in. at a depth of 40 cm. is 10 pd., find the pressure 6 cm. lower down.

4. What is the pressure in grams per sq. cm. at a depth of 100 metres in water? (Density of water one gram per c.c.).

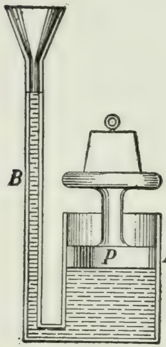


FIG. 161.

5. The area of the cross-section of the piston $P$ (Fig. 161), is 120 sq. cm. What weight must be on it to maintain equilibrium when the water in the pipe $B$ stands at a height of 3 metres above the height of the water in $A$?

6. The water pressure at a faucet in a house supplied with water by pipes connected with a distant reservoir is 80 pounds per sq. inch when the water in the system is at rest. What is the vertical height of the surface of the water in the reservoir above the faucet? (1 lb. water = 27.73 c. in.).

7. Find the measure in pounds per square inch of the pressure at a point 72 ft. below the surface of a pool of water. (Density of water $62\frac{1}{2}$ lbs. per cubic foot).

8. A reservoir of water is 100 metres above the level of the ground-floor of a house. Find the pressure in grams per square centimetre of the water at a point in a water-pipe at a height of 10 metres above the ground-floor.

9. The pressure at a point within a body of water under the action of gravity is 100 pd. per square inch. If the weight of a cu. foot of water is 1000 oz., find the depth of the point below the surface.

10. The water in a canal lock rises to a height of 10 ft. against one side of a vertical flood-gate whose breadth is 12 ft. Find the total force against it.

(In this case the pressure varies directly with the depth and hence the average pressure is equal to the pressure half-way down, that is, 5 ft. below the surface. The width is uniform and consequently the total force is equal to the area × average pressure.)

11. A rectangular box 2 cm. long, 1.5 cm. wide, and 8 mm. deep, is filled with water. Find the total force on (1) the bottom, (2) a side, (3) an end.

12. A rectangular vessel 80 cm. long, 20 cm. wide, and 60 cm. deep, supposed weightless, is placed on a horizontal table. Into its upper face is let perpendicularly a straight tube which rises to a height of 2 metres above this face, the internal cross-section of the tube being 1 sq. cm. The vessel and the tube are filled with water. Find the total force on (1) the bottom of vessel, (2) a side, (3) an end, (4) the upper surface, (5) the table.

# CHAPTER XVII

## BUOYANCY; ARCHIMEDES' PRINCIPLE

**144. Buoyant Force of a Liquid.** We know very well that a liquid exerts an upward force upon a body which is either partially or completely immersed in it. A cork or a sea-gull bobs about on the surface of a lake, a heavy log floats on the river and is towed to the saw-mill, and even the great ship of ten thousand tons is supported by the water. In all these cases the object *floats*, its weight is *entirely* overcome by the upward force due to the water.

But an upward force is exerted also when the body is fully immersed. An expert swimmer can keep a drowning person from sinking, though out of the water he might not be able to lift the body at all. Sometimes in fishing, a heavy stone is attached to a rope and let down as an anchor. On pulling it up, to go to another place, comparatively little effort is needed as long as it is in the water, but it becomes decidedly heavier as soon as it comes to the surface.

Let us find out by experiment just how much of a body's weight is apparently lost when it is immersed in a liquid.

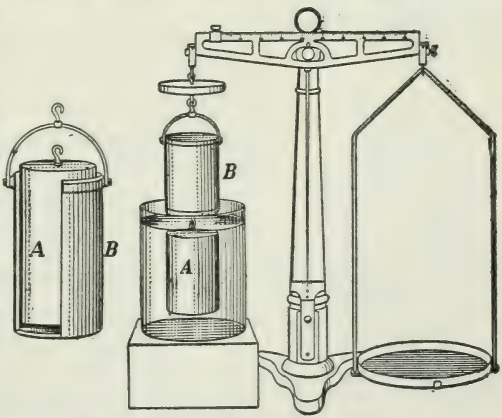


FIG. 162.—Determination of buoyant force.

**145. The Principle of Archimedes.** A suitable form of apparatus for the purpose is shown in Fig. 162. $A$ is a

metal cylinder, closed at both ends, which fits exactly into a hollow socket *B*. Hook the cylinder to the bottom of the socket, suspend them from one end of the beam of a balance, and add weights to the other end to bring the balance to equilibrium. Next, surround *A* with water, as shown in the figure. The buoyancy of the water upon *A* destroys the equilibrium. Now carefully pour water in the socket *B*. It will be found that when *B* is just filled, equilibrium will be restored. The buoyant force is equal to the weight of the water displaced by the body.

This result has been obtained with water, but we might have used any other liquid, and it must also hold for a gas.

If a balance such as that shown in Fig. 162 is not available, any ordinary good balance may be used. Take a cylindrical body, and having measured its diameter and length calculate its volume in c.c. The weight of a volume of water equal to it follows at once since 1 c.c. of water = 1 gram.

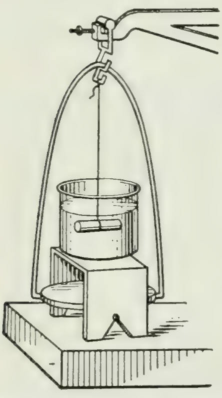


FIG. 163.—Finding the apparent loss in weight when a body is immersed in a liquid.

Next, hang it by a fine thread from one end of a balance arm (Fig. 163) and weigh it, first, when in air and, secondly, when immersed in water. The difference between the two weights should equal the weight found from measuring the volume of the body.

We, therefore, conclude that: *The buoyant force exerted by a fluid upon a body immersed in it is equal to the weight of the fluid displaced by the body;* or, in slightly different words,

*A body when weighed in a fluid loses in apparent weight an amount equal to the weight of the fluid which it displaces.*

This is known as the PRINCIPLE OF ARCHIMEDES.

It is stated that King Hiero of Syracuse, Sicily, suspected that his crown was not made of pure gold but contained some silver, and he asked the great scientist Archimedes (287-212 B.C.) to determine if such was the case. It is evident that if the volume of the crown were known, the weight, if of pure gold, could easily be calculated. If its actual weight was less, some lighter substance than gold must be combined with it. The solution of the problem was suggested to Archimedes by the buoyant action of the water when he was in a bath. According to tradition, he leaped from the bath and rushed through the streets crying, "Eureka! Eureka!" (I have found it! I have found it!)*

**146. Theoretical Proof by Calculation.** Archimedes' principle is so important that a simple proof by calculation will be given. Consider a solid in the form of a cube to be immersed in water with its upper face horizontal.

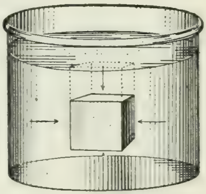


FIG. 164.—Buoyant force of a liquid on a solid.

Let the edge of the cube be 3 cm. in length and the upper face be 2 cm. below the surface (Fig. 164).

Evidently the pressures on the vertical sides balance, and the resultant vertical force due to the water will be equal to the difference between the pressures on the bottom and the top.

Now the pressure on the top is equal to the weight of a column of water standing on 9 sq. cm. and reaching to the surface, that is, having a height of 2 cm. The volume is 18 c.c. and the weight is 18 grams.

The pressure on the bottom (upwards) is equal to the weight of a column of water standing on 9 sq. cm. and reaching to the surface, that is, having a height of 5 cm. The volume is 45 c.c. and the weight is 45 grams.

*See "A Short History of Science" by Sedgwick and Tyler (N.Y., 1918), page 113.

Resultant pressure $= 45 - 18 = 27$ grams upwards.

But the volume of the cube is 27 c.c. and the weight of the water displaced by it $= 27$ grams.

**147. Principle of Flotation.** It is obvious that if the weight of a body immersed in a liquid is greater than the weight of the liquid displaced by it, the body will sink; but if less, the body will rise until it reaches the surface. Here it will come to rest when it has risen so much above the surface that the weight of the water then displaced is equal to the weight of the body.

It should also be observed that two parallel forces cannot be in equilibrium unless they act in the same straight line but in opposite directions. In the case of the floating body the weight acts downwards through the centre of gravity of the body, while the buoyant force acts through the centre of buoyancy (that is, the centre of gravity of the fluid displaced). Consequently for flotation the centre of gravity of the body must be vertically below the centre of buoyancy.

### PROBLEMS AND QUESTIONS

(Take 1 cu. ft. water = 62.5 lb.)

1. A cubic foot of marble which weighs 160 pounds is immersed in water. Find (1) the buoyant force of the water on it, (2) the weight of the marble in water.

2. Twelve cubic inches of a metal weigh 5 pounds in air. What is the weight when immersed in water?

3. If 3500 c.c. of a substance weigh 6 kg., what is the weight when immersed in water?

4. A piece of aluminium whose volume is 6.8 c.c. weighs 18.5 grams. Find the weight when immersed in a liquid twice as heavy as water.

5. A body whose volume is $2\frac{2}{5}$ cu. ft. weighs 420 pounds. Find its weight when $\frac{5}{6}$ of its volume is immersed in water.

6. A substance whose volume is $3\frac{1}{5}$ c.dm. weighs $7\frac{3}{5}$ kg. Find its weight when $\frac{3}{8}$ of its volume is immersed in a liquid one-half as heavy as water.

7. One cubic decimetre of wood floats with $\frac{3}{5}$ of its volume immersed in water. What is the weight of the cube?

8. A cubic centimetre of cork weighs 250 mg. What part of its volume will be immersed if it is allowed to float in water?

9. A cubic in. of pine floats with $\frac{5}{7}$ of its volume in water. Find its weight.

10. A c.c. of poplar floats with $\frac{m}{n}$ of its volume out of water. Find its weight.

11. The weight of $2\frac{1}{4}$ cu. feet of elm is 124 pounds. What part of its volume will be immersed if it is allowed to float in water?

12. The weight of $6\frac{2}{3}$ c.dm. of cork is $1\frac{2}{3}$ kg. If it is allowed to float in water, how many c.dm. will remain above the surface?

13. A piece of wood weighing 100 pounds floats in water with $\frac{2}{3}$ of its volume above the surface. Find its volume.

14. What is the least force which must be applied to a cu. ft. of larch which weighs 30 pounds that it may be wholly immersed in water?

15. A c.dm. of cork, weighing 480 grams, floats just immersed in water, when prevented from rising by a string attached to the bottom of the vessel containing the water. Find the tension of the string.

16. A cylindrical cup weighs 35 grams, its external radius being $1\frac{3}{4}$ cm., and its height 8 cm. If it be allowed to float in water with its axis vertical, what additional weight must be placed in it that it may sink?

17. A cylinder of wood, 8 in. long and weighing 15 pounds, floats vertically in water with 3 in. of its length above the surface. What is the tension of the string which will hold it just immersed in water?

18. The cross-section of a boat at the water-line is 150 sq. ft. What additional load will sink it 2 inches?

19. A piece of wood whose mass is 100 grams floats in water with $\frac{3}{4}$ of its volume immersed. What is its volume?

20. Why will an iron ship float on water, while a piece of the iron of which it is made sinks?

21. A vessel of water is on one scale-pan of a balance and counterpoised. Will the equilibrium be disturbed if a person dips his fingers into the water without touching the sides of the vessel? Explain.

22. A piece of coal is placed in one scale-pan of a balance and iron weights are placed in the other scale-pan to balance it. How would the equilibrium be affected if the balance, coal and weights were now placed under water? Why?

23. A block of wood 1 inch square and 6 inches long is tied at one end to the bottom of a tank on the inside. Mercury is poured into the tank until the block, when standing vertically, is just half immersed; then water is poured in until the block is entirely covered.

(*a*) Does the tension of the string that holds the block down change as the water is being poured in? Give reason for the answer.

(*b*) Would the tension of the string have been different had mercury been used instead of water? Why?

## CHAPTER XVIII

### Density and Specific Gravity

**148. Mass per Unit Volume.** Obtain cylindrical pieces of brass, iron and wood, having flat ends; also a cylindrical vessel such as a tin can.

Measure the diameters and lengths of the cylinders in cm. and calculate their volumes. Also measure the internal diameter of the can and its height up to a mark, in inches, and calculate its volume.

Weigh the cylinders in grams and calculate the mass per c.c. of each of the substances.

Weigh the vessel in pounds; then fill with water up to the mark and weigh again. Calculate the mass of the water per cu. in.

| | | | |
|---|---|---|---|
| **Examples**:—(*a*) *Cylinder of iron.* | Diam. | = | 2.34 cm. |
| | Length | = | 8.42 " |
| | By calculation, volume | = | 3.621 c.c. |
| | By weight, mass | = | 273.03 grams. |
| | Whence mass of 1 c.c. of iron | = | 7.54 grams |
| (*b*) *Tin vessel.* | Diam. | = | 3.12 in. |
| | Height | = | 4.00 in. |
| | By calculation, volume | = | 30.58 c. in. |
| | By weight, mass | = | 1.10 lb. |
| | Whence mass of 1 c. in. of water | = | 0.036 " |
| | and " of 1 c. ft. " " | = | 62.2 " |

The mass per unit volume of a substance is its *density.* Thus, the density of the iron used is 7.54 grams per c.c.; that of water is 0.036 lb. per cu. in. or 62.2 lb. per cu. ft.*

**149. Density and Specific Gravity.** As we have just seen, the *density* of a body is its *mass per unit volume.*

The *specific gravity* of a substance is the *ratio* which the weight of a given volume of it bears to the weight of an equal volume of water.

*More accurately, 62.4 lb. per cu. ft. at 4° C. It is usual, however, to take 1 cu. ft. of water as 62.5 lb. or 1000 oz.

As this is simply a ratio it is expressed by a simple number, and is independent of any system of units; but it is related to density in the following way:

Let $W$ pounds = wt. of a given volume (say 1 cu. ft.) of the substance
and $w$ " = " the same volume of water

$$\text{Then sp. gr.} = \frac{W}{w},$$

$$= \frac{\text{density of substance}}{\text{density of water}}.$$

If now we use the C.G.S. system of units the density of water = 1 gm. per c.c.; and the number which expresses the specific gravity will also be the measure of the density. This, however, will not be the case if we use the F.P.S. units, as then the density will be the number of pounds per cu. ft., while the specific gravity will be the same as before.

**Example**:—Suppose the volume of a piece of cast-iron is 50 c.c. and that its weight is 361 grams. Find its specific gravity and its density.

The weight of 50 c.c. of water = 50 grams.

Therefore the sp. gr. of the iron = $\frac{361}{50}$ = 7.22, which is the measure of the weight in grams of 1 c.c. of iron, or its density.

In the F.P.S. system the sp. gr. is the same, but the density = 62.5 × 7.22 = 451.25 pounds per cu. ft.

**PROBLEMS**

1. Find the mass of 140 c.c. of silver if its density is 10.5 gm. per c.c.

2. The specific gravity of sulphuric acid is 1.85. How many c.c. must one take to weigh 100 gm.?

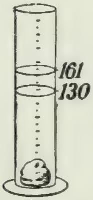


FIG. 165.

3. A piece of granite weighs 83.7 gm. On dropping it into the water in a graduated vessel, the water rises from 130 c.c. to 161 c.c. (Fig. 165). Find the density of the granite.

4. A tank 50 cm. long, 20 cm. wide and 15 cm. deep is filled with alcohol of density 0.8. Find the weight of the alcohol.

5. A rectangular block of wood 5 x 10 x 20 cm. in dimensions weighs 770 grams. Find the density.

**150. Specific Gravity of a Solid Heavier than Water.** The specific gravity of a solid body can easily be found by an application of Archimedes' principle.

The balance as shown in Fig. 166 is especially adapted for this experiment. Remove the cylinder and socket and in their place suspend by means of a fine thread the solid body and find how much it weighs in air. Then bring the vessel containing water under the body and raise it until the body is fully immersed. Weigh again.

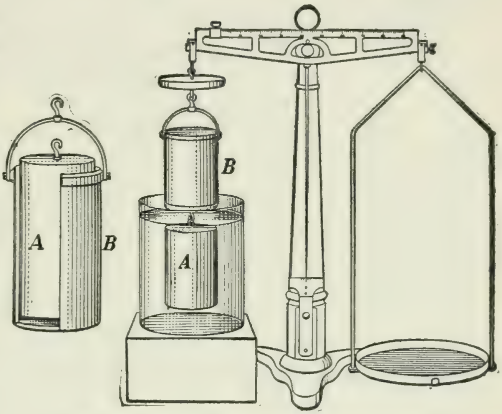



Fig. 166.—Balance for finding specific gravity.

Let $m$ grams = wt. of the body in air,

and $m_1$ " = " " " water.

Then $m - m_1$ " = loss of weight in water

= wt. of an equal volume of water.

$$\text{Hence, sp. gr.} = \frac{m}{m - m_1}$$

= density in grams per c.c.

**Example**:—A piece of iron weighed in air 263.5 grams, and in water 226.4 grams. Find its specific gravity.

Here loss of weight in water = 37.1 grams

= wt. of equal vol. of water.

$$\text{Hence, sp. gr.} = \frac{263.5}{37.1} = 7.10,$$

and the density of the iron = 7.10 grams per c.c.

**151. Specific Gravity of a Solid Lighter than Water.** Attach a heavy body which will cause the light body to sink in the water, and then proceed as follows:

1st. Weigh the body in air. Let the weight $= m$ grams.

2nd. Attach the sinker to hang below the body. Weigh both *with the sinker only* in the water. Let this weight $= m_1$ grams.

3rd. Weigh them when *both* are in the water. Let the weight $= m_2$ grams.

Now the only difference between the 2nd and 3rd operations is that in the former the body is weighed in air, in the latter in the water. The sinker is in the water in both cases.

Hence, $m_1 - m_2 =$ buoyancy of the water on the body $=$ wt. of the water displaced by the body,

and the sp. gr. $= \dfrac{m}{m_1 - m_2}$.

This experiment may be performed in a slightly different way.

| | | | |
|---|---|---|---|
| 1st. | Weigh the body in air | $= m$ | grams |
| 2nd. | Weigh the sinker in air | $= m_1$ | " |
| 3rd. | Weigh the sinker in water | $= m_2$ | " |
| 4th. | Weigh the body and sinker in water | $= m_3$ | " |

Now $m_1 - m_2 =$ loss of wt. of sinker in water $=$ wt. of a vol. of water equal to that of sinker.

Hence, volume of sinker $= m_1—m_2$ c.c.

Again, $m + m_1 =$ wt. of body and sinker in air,
and $m_3 =$ wt. of body and sinker in water.

Hence, $m + m_1 - m_3 =$ loss of wt. of body and sinker in water
and volume of body and sinker $= m + m_1 - m_3$ c.c.

But volume of sinker $= m_1 - m_2$ c.c.
and therefore volume of body $= m + m_1 - m_3 - (m_1 - m_2)$ c.c.
$= m + m_2 - m_3$ c.c.

Its weight in air $= m$ grams.

Hence its density $= \dfrac{m}{m + m_2 - m_3}$ grams per c.c.

**152. The Specific Gravity Bottle.** The specific gravity bottle, one form of which is shown in Fig. 167, is specially adapted for finding the sp. gr. of liquids. The procedure is as follows:

FIG. 167.—Specific gravity bottle.

1st. Weigh the bottle empty $= m$ grams.

2nd. Weigh it filled with water $= m_1$ "

3rd. Weigh it filled with the liquid $= m_2$ "

Then water which fills the bottle

weighs $m_1 - m$ grams,

and liquid which fills it weighs $m_2 - m$ "

Hence, sp. gr. of the liquid $= \dfrac{m_2 - m}{m_1 - m}$.

**Example:**—A bottle empty weighed 21.10 grams; when filled with water, 71.22 grams; when filled with alcohol, 61.73 grams. Find the sp. gr. of the alcohol.

Weight of water filling bottle $= 50.12$ grams.

Weight of alcohol filling bottle $= 40.63$ "

Hence, sp. gr. $= \dfrac{40.63}{50.12} = 0.81$

**153. Specific Gravity of a Liquid by Archimedes' Principle.** In finding the specific gravity of a liquid by Archimedes' principle take a heavy body (say, a glass stopper) and weigh it in air, when immersed in water and when immersed in the liquid.

Let wt. of sinker in air $= m$ grams

" " " " water $= m_1$ "

" " " " liquid $= m_2$ "

Then wt. of water displaced by sinker $= m - m_1$ grams

and " " liquid " " " $= m - m_2$ "

Hence, sp. gr. of liquid $= \dfrac{m - m_2}{m - m_1}$.

This also expresses the density of the liquid in grams per c.c.

**154. The Hydrometer.** The approximate sp. gr. of a liquid is a quantity which it is often necessary to determine quickly and for this purpose an instrument known as a *hydrometer* has been devised.

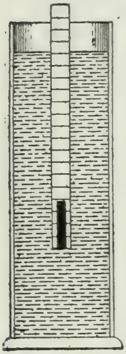


FIG. 168.—Illustration of the principle of the hydrometer.

The principle underlying its action may be illustrated as follows. Take a straight rod of wood, of cross-section 1 sq. cm. and (say) 25 cm. long, and bore a hole in one end. After inserting enough shot to make the rod float upright in water plug up the hole and after marking on one of the long faces a centimetre scale, dip the rod in hot paraffin to render it impervious to water. Now place the rod in water and suppose it to sink to a depth of 16 cm. Then the weight of the rod = weight of water displaced = 16 grams.

Again place it in a liquid whose density is to be determined, and suppose it to sink 20 cm.

Then the volume of the liquid displaced = 20 c.c., and this = wt. of the rod = 16 grams.

Hence, density of liquid = $\frac{16}{20}$ = .80 gram per c.c.

It is evident, also, that the rod could be marked so as to indicate the sp. gr. directly. Thus,

| | | | |
|---|---|---|---|
| for readings | 12, | 16, | 20 cms. |
| the sp. gr. is | 1.25 | 1.00 | .80 |

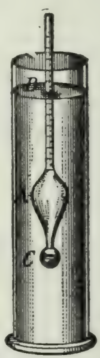



FIG. 169.—The hydrometer.

For commercial purposes the hydrometer is usually constructed in the form shown in Fig. 169. At the end of a slender stem $B$ is a float $A$, and a little chamber $C$ which contains mercury and makes the instrument take an upright position when in a liquid. The graduations are either on the outside of the stem or on a paper within it. The weight and volume are so adjusted that the instrument sinks to the mark at the lower

end of the stem when in the densest liquid to be tested, and to the mark at the upper end when in the least dense liquid. The scale is marked so as to indicate directly any density between the limits chosen. By making the float $A$ much larger than the stem the instrument is sensitive.

As the range of an instrument of this class is necessarily limited, special instruments are constructed for use with different liquids. Thus one instrument is used for testing the density of milk, another for the acid in a storage battery, and so on.

### PROBLEMS

1. A body whose mass is 6 grams has a sinker attached to it and the two together weigh 16 grams in water. The sinker alone weighs 24 grams in water. What is the density of the body ?

2. A body whose mass is 12 grams has a sinker attached to it and the two together displace when submerged 60 c.c. of water. The sinker alone displaces 12 c.c. What is the density of the body ?

3. A body whose mass is 60 grams is dropped into a graduated tube containing 150 c.c. of water. If the body sinks to the bottom and the water rises to the 200 c.c. mark, what is the density of the body ?

4. If a body when floating in water displaces 12 c.c., what is the density of a liquid in which when floating it displaces 18 c.c. ?

5. A piece of metal whose mass is 120 grams weighs 100 grams in water and 104 grams in alcohol. Find the volume and density of the metal, and the density of the alcohol.

6. A hydrometer floats with $\frac{2}{3}$ of its volume submerged when floating in water, and $\frac{3}{4}$ of its volume submerged when floating in another liquid. What is the density of the other liquid ?

7. A cylinder of wood 8 inches long floats vertically in water with 5 inches submerged. (*a*) What is the specific gravity of the wood ? (*b*) What is the specific gravity of the liquid in which it will float with 6 inches submerged ? (*c*) To what depth will it sink in alcohol whose density is 0.8 ?

8. The specific gravity of pure milk is 1.086. What is the density of a mixture containing 500 c.c. of pure milk and 100 c.c. of water ?

9. How much silver is contained in a gold and silver crown whose mass is 407.44 grams, if it weighs 385.44 grams in water? (Density of gold 19 32 and of silver 10.52 grams per c.c.).

10. The mass of a piece of limestone (sp. gr. = 2.637) is 256.34 gm. What is its apparent weight in water?

11. The apparent weight of a mineral when weighed in water is 195.46 gm. If its specific gravity is 2.678, what is its mass?

12. Find the apparent weight of 5 c.c. of gold (sp. gr. = 19.3) in mercury (sp. gr. = 13.6.)

13. What is the least weight that must be placed upon a cu. ft. of cork (sp. gr. = .25) that it may float totally immersed in a liquid whose specific gravity is .9?

14. What is the least weight that must be placed upon a piece of wood weighing 20 pounds and floating with $\frac{2}{3}$ of its volume immersed in a liquid whose specific gravity is 1.5 that it may be totally immersed?

15. A cylinder of cork weighs 10 grams, and its specific gravity is .25. Find the least force that will immerse it (1) in water, (2) in a liquid whose specific gravity is .75.

16. A body (sp. gr. = .5) floats on water. If the weight of the body is 1 kg., find the number of cubic centimetres of it above the surface of the water.

17. A body floats in a fluid (sp. gr. = .9) with as much of its volume out of the fluid as would be immersed if it floated in a fluid (sp. gr. 1.2). Find the specific gravity of the body.

18. A cubical block of wood (sp. gr. = .6) whose edge is 1 foot floats, with two faces horizontal, down a fresh water river out to sea, where a fall of snow takes place, causing the block to sink to the same depth as the river. If the specific gravity of the sea water is 1.025, find the weight of the snow on the block.

19. A ship, of mass 1000 tons, goes from fresh water to salt water. If the area of the section of the ship at the water-line is 15,000 sq. ft., and her sides vertical where they cut the water, find how much she will rise, taking the specific gravity of sea water as 1.026.

20. A beaker partly full of water is balanced accurately on the scales; then a piece of lead (sp. gr. = 11) weighing 66 grams, held in the hand at the end of a fine thread, is lowered into the water without touching the glass. What weight must be added to the opposite side to restore equilibrium?

21. $A$, $B$, and $C$ are three beakers filled to the top with water :

(1) A block of wood weighing 30 grams (sp. gr. = 0.4) is placed in $A$.

(2) A piece of lead measuring 3 c.c. (sp. gr. = 11) rests at the bottom of $B$.

(3) The lead and wood are fastened together and placed in $C$.

Find the change (if any) that has taken place in the weight of each beaker, giving full explanation in each case.

**155. Liquids in a Bent Tube.** It is possible to find the density of one liquid with respect to another by means of a bent tube, provided the liquids do not mix.

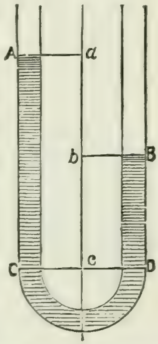


FIG. 170.—Liquids of unequal density in a bent tube.

Pour the liquids in and allow them to come to equilibrium (Fig. 170). Let $A$ and $B$ be their free surfaces and $C$ their common surface. It is evident that the liquid $AC$ is not so dense as the other.

Let $d_1$, $d_2$ be the densities, and $ac$, $bc$ be the heights of the free surfaces above the horizontal plane $CD$ drawn through their common surface.

Since the liquids are in equilibrium, it is evident that the pressure at $C$ = the pressure at $D$.

The pressure per sq. cm. at $C$ = the weight of a column of liquid of density $d_1$, having a cross-section 1 sq. cm. and height $ac$ cm.

$$= d_1 \times ac \text{ grams per sq. cm.}$$

Similarly the pressure per sq. cm. at $D$

$$= d_2 \times bc \text{ grams per sq. cm.}$$

Hence, $\quad d_1 \times ac = d_2 \times bc,$

or $\quad d_1/d_2 = bc/ac.$

Hence, when the liquids are in equilibrium their densities are inversely as the heights of their free surfaces above their common surface.

**Example** :—Let the column $AC$ be oil and the rest water. By measurement, $ac = 20$ cm., $bc = 18$ cm.

Hence, density of oil, $d_1 = \frac{18}{20}$ of water

$= 0.9$ grams per c.c.

**PROBLEMS**

1. Two liquids which do not mix are contained in a bent tube. If their specific gravities are 1.2 and 1.8 respectively, and the height of the first above their common surface is 15 inches, find the height of the other.

2. In a bent tube a column of mercury (sp. gr. 13.6) is balanced by a column of alcohol (sp. gr. .8). If the height of the former is (1) 4 cm., (2) 10 cm., (3) 15 cm., what in each case is the height of the latter?

3. Two tanks are connected by a pipe. Into one tank is poured salt water (sp. gr. 1.03), and into the other a very light oil (sp. gr. .5). The oil is found to be 5 ft. above their common surface. Find the height of the water.

4. Mercury and ether are poured into a bent tube. The mercury stands 5.25 cm. when ether stands 100 cm. above their common surface. If the density of the ether is .715 grams per cubic centimetre, what is the density of the mercury?

5. Two liquids that do not mix are contained in a bent tube. The difference of their levels is 40 cm. and the height of the denser above their common surface is 70 cm. Compare their densities.

6. If water and a denser liquid which does not mix with it are placed in a U-tube, the internal cross-section of which is 1 sq. cm., the difference of their levels is found to be 5 cm., and the height of the liquid above their common surface is 10 cm. What is the height of the water?

## CHAPTER XIX

### Pressure of the Air—The Barometer

**156. Air has Weight.** Though we cannot *see* the air, we are fully convinced that it is a substance which actually exists and is quite as real as the solid soil or the water of the ocean. The air offers a resistance to the rapidly moving automobile or railway train; and were it not a real substance the airplane could not soar upon it or drive itself forward by its propeller. Sometimes great trees are blown over or mountainous waves are raised upon the sea. These disturbances are not due to some imaginary force but are caused by real masses of matter sweeping forward over the surface of the earth.

However, the air is so thin and fluid that we might almost expect it to escape the laws of weight. We speak of a thing being as "light as air"; but it is not difficult to demonstrate that air has weight and that its weight is not so small as many people seem to think.

From an ordinary (not gas-filled) electric light bulb the air has been carefully removed and the space within is almost a perfect vacuum. Take one of these bulbs (one with a broken filament—do not waste a good lamp) and, having heated the butt-end in a flame, remove the brass plug. Then, by means of a delicate balance, weigh the glass portion which is left. Next make a scratch on the sharp glass tip with a file and then by a smart tap break off the tip. This will make a hole in the bulb and the air will rush in and fill it. Now weigh the bulb, including the tip, again. The weight will be distinctly greater than before.

**Example**:—The following measurements were made:

| | | |
|---|---|---|
| Weight of bulb at first | 24.572 | grams. |
| Weight of bulb + air | 24.755 | " |
| Increase in wt. | 0.183 | " |

An attempt was also made to measure the capacity of the bulb and then to calculate the weight of 1 litre of air.

On forcing the bulb down into water in a graduate the entire volume was 160 c.c. Now the density of glass is about 2.5 grams per c.c. and the mass of the bulb is 24.572 grams; hence, the volume occupied by the glass = 24.572 ÷ 2.5 = 10 c.c. (approx.).

Consequently the capacity of the bulb = 160 − 10 = 150 c.c.

Hence, 150 c.c. of air weighed 0.183 grams.

The temperature was 19.5° C. and the barometer read 73 cm.

Using the laws of expansion of a gas, we find that the weight of 1 litre at 0° C. and 76 cm.

$$= \frac{1000 \times .183}{150} \times \frac{292.5}{273} \times \frac{76}{73} = 1.36 \text{ grams.}$$

The experiment to determine the weight of a given volume of air can be performed more satisfactorily with the gas flask shown in Fig. 171. It can be connected to an air-pump for removing the air from the globe, or to one for forcing the air into it; and a stop-cock allows the vessel to be made air tight.

FIG. 171.—Globe for weighing air.

First, let the air be removed from the vessel and the stop-cock closed, and then let it be weighed. Then admit air and weigh again. Finally, cautiously force air into it and weigh a third time. The third weight will be greater than the second and the second greater than the first. The capacity of the flask can be determined by filling it with water and weighing it.

By observing the temperature of the gas, the pressure to which it is subjected and the barometric pressure, the weight of the gas at standard temperature and pressure can be determined. Careful experiments show that

1 litre of air at 0° C. and 76 cm. pressure = 1.293 grams.

From this we find that 1 cu. ft. = 1.28 ounces or 12 cu. ft. = 1 lb. (approximately). Consequently the air in a room 20 × 24 × 15 ft. weighs 600 lbs. It is not so light after all.

**157. Pressure of the Atmosphere.** The reason why the air is heavy is because it is attracted to the earth by the force of gravity, and just as liquids exert pressure upon all surfaces with which they are in contact, so must the air do the same. The bed of the ocean is subjected to enormous pressure by the water above it, and in the same way the surface of the earth must sustain a pressure from the the aerial ocean which rests upon it. As we have seen, the pressure in the water is directly proportional to its depth; in the atmosphere the pressure becomes less the higher above the earth one goes. Thus the pressure at sea-level at Halifax, N.S. or Victoria, B.C., is greater than in the Rocky Mountains.

That the atmosphere exerts pressure can be demonstrated by many simple experiments. Tie a piece of thin sheet rubber over the mouth of a thistle-tube (Fig. 172) and exhaust the air from the bulb by suction or by means of an air-pump. As the air is exhausted the rubber is pushed inward by the pressure of the outside air. Again, fill a bottle with water, and place a sheet of writing paper over its mouth. Then, holding the paper in position with the palm of the hand, invert the bottle (Fig. 173). The pressure of the atmosphere against the paper prevents the water from running out. Numerous other experiments can be performed to show the same effect.

FIG. 172.—Rubber membrane forced inwards by pressure of the air.

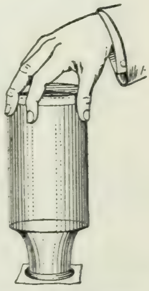


FIG. 173.—Demonstrating atmospheric pressure.

If one end of a tube is thrust into water and the air is withdrawn from it by suction, the water rises in the tube.

This phenomenon was known for ages and was accounted for by the simple statement that *nature abhors a vacuum.* In 1640 the Grand Duke of Tuscany dug a deep well, but found that the water could not be raised more than 32 feet above the level in the well. He applied to the aged scientist Galileo for an explanation, and though he had proved that air had weight he did not connect that fact with the problem. He simply inferred that the horror felt by nature had its limitations. After his death the problem was solved by his pupil Torricelli, who was made, by the Grand Duke, professor of mathematics in the Academy of Florence, in succession to Galileo. Torricelli showed definitely that the reason why water rose in the pump only to a height of 32 feet was because the pressure of the atmosphere was not able to push it up any higher.

#### PROBLEMS AND EXERCISES

1. Fill a tumbler and hold it inverted in a dish of water as shown in Fig. 174. Why does the water not run out of the tumbler into the dish?

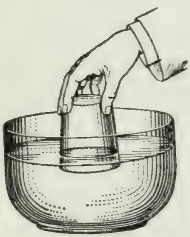


FIG. 174.

2. Take a bent glass tube of the form shown in Fig. 175. The upper end of it is closed, the lower open. Fill the tube with water. Why does the water not run out when it is held in a vertical position?

FIG. 175.

3. Why must an opening be made in the upper part of a vessel filled with a liquid to secure a proper flow at a faucet inserted at the bottom?

4. Fill a narrow-necked bottle with water and hold it mouth downward. Explain the action of the water.

5. On the tin top upon a pot of jam is sometimes seen the instruction :—"To open, puncture and push up at edge." Give the reason for this.

6. A flask weighs 280.60 gm. when empty, 284.19 gm. when filled with air and 3060.60 gm. when filled with water. Find the weight of 1 litre of air.

**158. Torricelli's Experiment.** Torricelli reasoned that since a water column rises to a height of 32 ft., and since mercury is about 14 times as heavy as water, the atmosphere would be able to support a mercury column only about $\frac{1}{14}$th of 32 ft., or approximately 28 in. in height. Under his direction Vincenzo Viviani, one of his pupils, performed an experiment similar to the following:

Take a glass tube about 1 metre (39 inches) long (Fig. 176), closed at one end, and fill it with mercury. Then, stopping the open end with the finger, invert it and place it in a vertical position, with the open end under the surface of the mercury in a bowl. Remove the finger. The mercury will fall in the tube, and, after oscillating up and down, will come to rest with the surface of the mercury in the tube between 28 and 30 inches (71 and 76 cm.) above the surface of the mercury in the bowl.

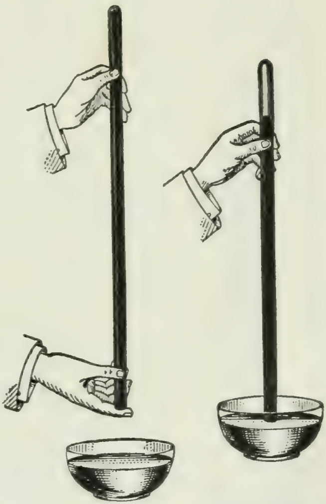


FIG. 176.—Mercury column sustained by the pressure of the air.

The experiment resulted precisely as Torricelli expected, and conclusively showed that the column of mercury was sustained by the pressure of the atmosphere upon the surface of the mercury in the bowl. The empty space above the mercury is called a Torricellian vacuum.

When a report of this experiment reached France it created a sensation among the scientists there, but it was not repeated by them until 1646, as no suitable tubes were available before that date. In that year the experiment was performed by Pierre Petit, of Rouen, in conjunction with the great Pascal,

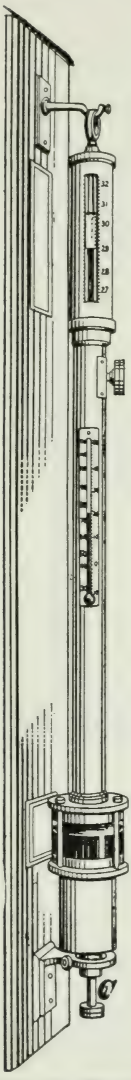


FIG. 177.—The cistern barometer.

who concluded "that the vacuum is not impossible in nature, and that she does not shun it with so great a horror as many imagine." Pascal reasoned that if the mercury column is simply held up by the pressure of the air the column should be shorter at a higher altitude. He asked his brother-in-law, Périer, who resided at Clermont, in the south of France, to test it on the Puy-de-Dôme, a near-by mountain over 1000 yards high. Using a tube about 4 ft. long, which had been filled with mercury and then inverted in a vessel containing mercury, Périer found that the column fell over 3 inches (8 cm.) on going to the summit. Later Pascal tried the experiment at the base and the top of the tower of Saint-Jacques-de-la-Boucherie, in Paris, which is about 150 feet high. There was a difference of 2 *lines* (about 0.5 cm.).

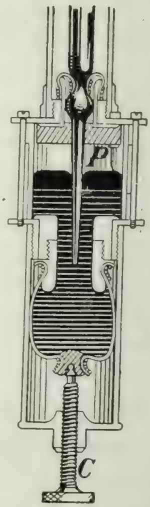



FIG. 178.—Section of the cistern.

**159. The Barometer.** In his experiments Torricelli says he aimed "not simply to produce a vacuum, but to make an instrument which shows the mutations of the air, now heavier and dense, now lighter and thin"; and the modern mercury barometer, which is designed to measure the pressure of the atmosphere, is similar in principle to that constructed by Torricelli. Two forms of the instrument are in common use.

**160. The Cistern Barometer.** This is simply a convenient

arrangement of the original Torricellian experiment. The bowl, or cistern, and the tube are permanently mounted on a board, and a scale, engraved on the metal case protecting the glass tube, shows the height of the mercury in the tube above the surface of the mercury in the cistern.

A common form of the instrument is shown in Figs. 177, 178. The cistern has a flexible leather bottom which can be raised or lowered by a screw *C*, in order to adjust the level of the mercury. Before taking the reading, the screw is turned until the tip of the pointer *P* (which is the zero of the scale on the case) just touches the mercury. To do this, the level is slowly changed until the image of the tip just reaches the tip itself. The height of the column is then read directly from the scale on the case, the reading being made with accuracy by the assistance of a vernier.

In constructing a barometer of this kind the mercury must be very pure, since impure mercury has a different specific gravity and besides it adheres to the glass. Also. all bubbles of air and of moisture must be carefully removed. In order to do this the mercury is boiled. First, the tube is filled about one-third full of mercury which is then boiled over a charcoal fire or a large gas flame. Then more is added and the boiling continued, until at last the whole is thoroughly boiled. The temperature of boiling is so high (350° C.) that all the air and the moisture are completely removed. The operation is sometimes shortened and made easier by a suitable arrangement whereby an air-pump removes much of the air and the moisture, and also causes the mercury to boil at a lower temperature.

**161. The Siphon Barometer.** This barometer consists of a tube of sufficient length, sealed at one end and bent into U-shape at the other (Fig. 179). When filled with mercury and held in an upright position the mercury in the long closed tube falls until the atmospheric pressure on the open end is just sufficient to balance a column of mercury extending from the level in the open tube to the level in the closed tube. A scale is attached to or engraved upon each branch. The upper scale gives the height of the mercury in the closed branch above a fixed point and the lower scale gives the distance of the mercury in the open branch below the same point. The sum of the two readings is the height of the barometric column.

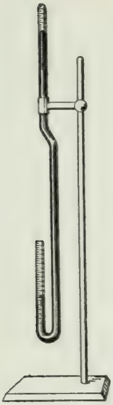


Fig. 179.—Siphon barometer.

**162. The Aneroid* Barometer.** In this barometer no liquid is used. The air presses upon the flexible corrugated cover of

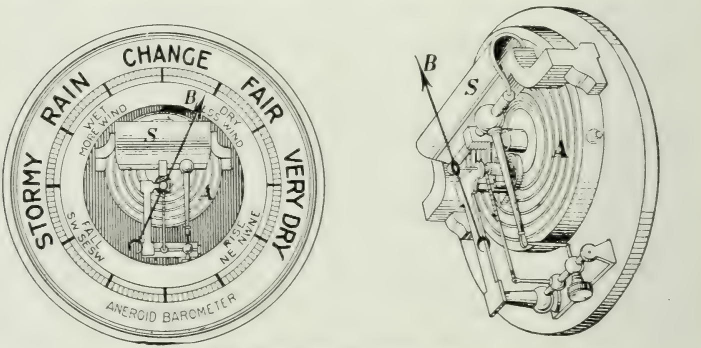



Fig. 180.—Aneroid barometer.

a circular, air-tight metal box $A$ (Fig. 180) from which the air has been partially exhausted. The cover, which is usually supported by a spring $S$, responds to the pressure of the

* Aneroid, from Greek *a* = not, *nēros* = wet.

atmosphere, being forced in when the pressure is increased and coming outwards when it is decreased. The movement of the cover is very small but it is multiplied and transmitted to an index band *B* by a system of delicate levers and a chain, or by gear wheels. The circular scale is graduated by comparison with a mercury barometer.

The aneroid is not so accurate as the mercury barometer, but it is very portable and very sensitive and is in very common use It is specially serviceable in determining differences of level. A good aneroid will indicate a fall in pressure in going from the cellar to the attic of a house.

On the face of the aneroid barometer is often seen the words, "stormy, rain, change, fair, very dry." They have little meaning, and the barometer by itself cannot indicate with certainty the nature of the coming weather. However, there are some laws which have been found to hold. If the barometer falls rapidly we may expect strong winds; and if it is low rain or snow is likely to fall. If it is rising fine weather is probably coming, and if it stays high and steady

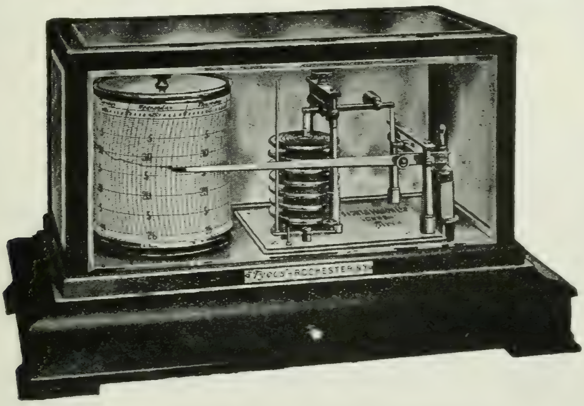



FIG. 181.—Barograph or self-recording barometer.

fine weather is likely to continue. The barometer is highest in calm, clear, cold winter weather. Barographs, or self-registering

barometers, have been devised on the principle of the aneroid. In these an index carries a pen which makes a continuous record upon a strip of paper on a revolving drum. (Fig. 181).

**163. Calculation of Atmospheric Pressure.** If we know the barometric height at a given place we can calculate the pressure of the atmosphere there. For example, suppose the barometer stands at 76 cm. Then to find the atmospheric pressure in grams per sq. cm. we have only to find the weight of a column of mercury 1 sq. cm. in section and 76 cm. in height, that is, the weight of 76 c.c. of mercury.

Now mercury expands as its temperature rises, and consequently the weight of 1 c.c. of mercury depends on the temperature. The following table gives the values for three temperatures:

| Temperature. | | Wt. of 1 c.c. | Wt. of 76 c.c. |
|---|---|---|---|
| $-2°$ C. | = 28°.4 F. | 13.600 grams. | 1033.600 grams. |
| 0 | 32 | 13.596 " | 1033.296 " |
| 25 | 72 | 13.534 " | 1028.584 " |

In a barometer like that illustrated in Fig. 177 the scale is engraved on the brass case. Now this case increases in length as the temperature rises, and if we desire to determine accurately the pressure of the atmosphere we must make allowance for this too. It is usual to read the height of the mercury and also the temperature indicated by the thermometer attached to the case, and then to reduce the reading to zero, that is, to determine what the reading would be if the temperature fell to zero. If, further, it is desired to compare the atmospheric pressures at various places, as is done in the Meteorological Service, it is usual also to reduce the readings to sea-level, that is, to determine what the readings would be if the barometers were lowered down deep holes until they reached the level of the sea. In order to facilitate these reductions, tables have been prepared from which the corrections to be applied for various temperatures and altitudes may be found without much labour.

To illustrate the amount of these corrections the following example is given:

Temperature 72° F. = 25° C.
Altitude 1000 ft. = 304.8 metres.

| | in. | mm. |
|---|---|---|
| Reading of barometer | 28.900 | 735.6 |
| Correction for temperature | −.113 | −3.0 |
| | 28.787 | 732.6 |
| Correction for altitude | +1.02 | +25.8 |
| Reading at freezing point and sea-level .......... | 29.807 | 758.4 |

### PROBLEMS

NOTE.—In the following questions the density of mercury is to be taken as 13.6 grams per c.c., or as 7.858 oz. per cu. in. = 848 lb. per cu. ft.

1. Find the atmospheric pressure per sq. inch when the mercury barometer stands at 30 inches.

2. Find the pressure of the atmosphere on a square centimetre when the mercury barometer stands at 76 cm.

3. Three barometers are constructed to use liquids whose specific gravities are respectively 7.2, 2.9, and 11.8. Find the atmospheric pressure on a sq. inch (1) when the first barometer stands at 4.8 ft., (2) when the second stands at 11.52 ft., (3) when the third stands at 5.76 cm.

4. Three barometers are constructed to use liquids whose specific gravities are respectively 13.6, 5.17, and 2.06. Find the atmospheric pressure on 1 sq. cm., (1) when the first barometer stands at 70 cm., (2) when the second stands at 2 metres, (3) when the third stands at 5 metres.

5. If in ascending a mountain the barometer falls from 30 in. to 20 in., find the decrease in the atmospheric pressure on an area of 10 sq. ft.

6. The density of mercury being 13.6 grams per c.c., find the pressure of the atmosphere in dynes per square centimetre when the barometer stands at 75 cm.

**164. Determination of Heights by the Barometer.** Since the pressure of the atmosphere decreases with the elevation above sea-level it is evident that the barometer may be used to measure the difference between the altitudes of two places. If the density of the air was uniform, its pressure,

like that of liquids, would vary directly with its depth; but the air is very compressible and the lower layers are much denser than those above them. As a consequence the relation between barometric height and altitude is somewhat complicated. It has been found that for small elevations a fall of 1 inch in the mercury column corresponds to a rise of 900 ft. in elevation.

For heights less than 1000 metres (3280 ft.) the following formulas have been found to hold:

Let $H$, $h$ be the barometric heights at the lower and upper stations, and $T$, $t$ be the temperatures at the lower and upper stations.

Then, if $T$ is in degrees Fahr.,

$$\text{Difference in height} = 52{,}494 \left(\frac{H - h}{H + h}\right)\left(1 + \frac{T + t - 64}{900}\right) \text{ feet.}$$

If $T$ is in degrees centigrade,

$$\text{Difference in height} = 16{,}000 \left(\frac{H - h}{H + h}\right)\left(1 + \frac{2(T + t)}{1000}\right) \text{ metres.}$$

**165. Buoyancy of Gases.** It is evident that Archimedes' principle applies to gases as well as to liquids. A simple experiment to demonstrate the buoyant force of air is illustrated in Fig. 182. A hollow metal or glass globe $A$ is suspended from one end of a short balance beam and is counterpoised by a small weight $B$. If the air exerts a buoyant force, as in a liquid, the force upward on $A$ must be greater than that on $B$, and if the air be removed from about the balance the globe $A$ should sink. On putting the apparatus under the receiver of an air-pump and exhausting the air, the globe at once sinks.

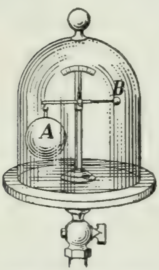



FIG. 182.—Buoyancy of air.

A gas exerts upon a body immersed in it a buoyant force which is equal to the weight of the gas displaced by the body; and, of course, if this buoyant force is greater than the weight of the body, the body will rise in the air.

This makes possible the operation of a balloon. It must be filled with a gas lighter than air and there are several to choose from. Hydrogen is the lightest and so is most efficient as far as lifting power is concerned. But illuminating gas is more conveniently obtained and is often used, though it is 8 times as heavy as hydrogen. Now both of these are very inflammable and consequently there is always great danger from fire, especially if used in warfare. On this account great efforts were made during the recent war to obtain the gas helium in sufficiently large quantities, and with considerable success. Helium is twice as heavy as hydrogen but it will not ignite.

It will be interesting to compare the lifting powers of a balloon filled with these gases. Let its capacity be 80,000 cu. ft. (1699 cu. metres). If it were spherical the diameter would be 48.6 ft. Balloons have ordinarily been nearly spherical in shape, though some recent ones are in the form of a 'sausage.'

The weight of 1 cu. metre of hydrogen = .09 kg.; of helium, .18 kg ; of illuminating gas, .75 kg.; of air, 1.29 kg. (at standard pressure and temperature).

Hence the weight of 1699 cu. m. of hydrogen = 152.9 kg.; of helium, 303.8 kg.; of illuminating gas, 1274.3 kg.; and the same volume of air weighs 2191.7 kg., which is the buoyant force of the air (neglecting the volume of the material of the balloon and its basket).

The lifting force, therefore, if the balloon is filled with

| | | | | |
|---|---|---|---|---|
| Hydrogen | = 2191.7 − | 152.9 = | 1938.8 | kg. |
| Helium | = 2191.7 − | 303.8 = | 1887.9 | " |
| Illuminating gas | = 2191.7 − | 1274.3 = | 917.4 | " |

Thus the lifting power of helium is about $\frac{11}{12}$, while that of illuminating gas is $\frac{4}{7}$ that of hydrogen.

### QUESTIONS AND PROBLEMS

1. Why should the gas-bag be subject to an increased strain from the pressure of the gas within as the balloon ascends?

2. Aeronauts report that balloons have greater buoyancy during the day when the sun is shining upon them than at night when it is cold. Account for this fact.

3. If the volume of a balloon remains constant, where should its buoyancy be the greater, near the earth's surface or in the upper strata of the air? Give reasons for your answer.

4. The volume of a balloon is 2,500 cu. m. and the weight of the gas-bag and car is 100 kg.; find its lifting power when filled with hydrogen gas, the density of which is 0.0000895 grams per c.c. while that of air is 0.001293 grams per c.c.

**166. Height of the Atmosphere.** There are several ways of obtaining an estimate of the height of the atmosphere, but no means of determining that height accurately. From twilight effects a height of about 40 miles has been calculated. It would seem that above this height the air ceases to reflect light, but other evidence shows that it extends beyond this. Meteors, or shooting stars, which consist of small masses of matter made incandescent by the heat produced as they rush through the atmosphere, have been observed at heights of over 100 miles. The aurora borealis, or northern lights, is probably a phenomenon in our atmosphere, and measurements of brilliant displays seen in the north of Norway show that it usually attains a height of 110 km. (70 ml.) and sometimes 210 km. (130 ml.).

# CHAPTER XX

## Boyle's Law and The Kinetic Theory of Gases

**167. Compressibility and Expansibility of Gases.** We have already referred to the fact that gases can be compressed and that they will expand if allowed to do so. Indeed this is the distinguishing feature of a gas. A solid has both a definite volume and a definite shape; a liquid has a definite volume but no definite shape,—it will take the shape of the vessel which holds it; but a gas has neither definite volume nor definite shape. If a quantity of gas is introduced into a closed vessel it will spread out and go into every corner of it, no matter what the shape may be. If the stopper be removed from a bottle containing ammonia we soon smell the pungent odour of ammonia gas everywhere in the room; or if hydrogen sulphide be introduced into a building (for instance, in natural gas) it before long reveals its undesired presence in all parts of the house.

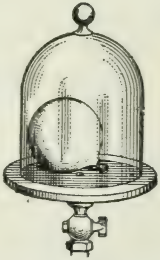


Fig. 183.—When the air is removed from the receiver the toy balloon expands.

In its efforts to escape, the gas exerts a pressure against the walls of the vessel enclosing it. This can be illustrated in the following way. Place a toy balloon or a half-inflated rubber from a football under the receiver of an air-pump and operate the pump. (Fig. 183.) As the air about the bag is continually removed, the bag expands; and when the air is admitted again the bag resumes its former volume.

To account for this we imagine the bag to be the seat of two contending factions,—the troops of molecules within endeavouring to keep back the invading hosts of molecules

without. Incessantly they rush back and forth, perpetually striking against the surface of the bag. As the enemies are withdrawn by the action of the pump, the defenders within gain the advantage and, pushing forward, enlarge their boundary which at last, however, becomes so great that the outsiders can again hold it in check.

The never-ceasing impact of the molecules of the gas against a surface produces the pressure exerted by the gas. This view of a gas is known as the Kinetic Theory of Gases.

#### QUESTIONS AND EXERCISES

1. Arrange apparatus as shown in Fig. 184*a*. By suction remove a portion of the air from the flask, and keeping the rubber tube closed by pressure, place the open end in a dish of water. Now open the tube. Explain the action of the water.

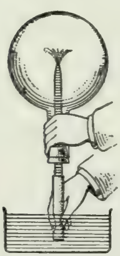


FIG. 184*a*.

FIG. 184*b*.

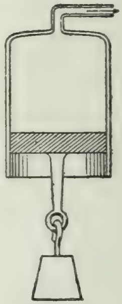


FIG. 185.

2. Guericke took a pair of hemispherical cups (Fig. 184*b*) about 1.2 ft. in diameter, so constructed that they formed a hollow air-tight sphere when their lips were placed in contact; and at a test at Regensburg before the Emperor Ferdinand III and the Reichstag in 1654 showed that it required sixteen horses (four pairs on each hemisphere), to pull the hemispheres apart when the air was exhausted by his air-pump Account for this.

3. If an air-tight piston is inserted into a cylindrical vessel and the air exhausted through the tube (Fig. 185), a heavy weight may be lifted as the piston rises. Explain this action.

4. A rubber tube with thin walls collapses when used for connecting an air-pump with a vessel from which the air is being withdrawn. Explain.

**168. Effect of a Rise in Temperature.** If we place the rubber bag used in the last experiment in an oven, it expands, showing that the pressure upon the inner surface of the bag

has increased. Now there are the very same molecules within—no increase in the number—and we must conclude that a rise in temperature causes the molecules to move with greater speeds, and this produces the increased pressure.

**160. Relation Between Volume and Pressure of a Gas—Boyle's Law.** It is a matter of importance to know the change produced in the volume of a given mass of gas when it is subjected to different pressures. This relation was first determined experimentally in 1660 by the distinguished Irish chemist, Robert Boyle (1627-1691).

The apparatus shown in Fig. 186 is suitable to investigate this relation.

Two glass tubes, *A* and *B*, are supported in such a way that either may be raised or lowered. The upper end of *A* is closed, that of *B* is open, and their lower ends are joined by a heavy rubber tube. The rubber tube and part of *A* and *B* are filled with mercury. The tube *A* is of uniform bore and the volume of the air may be taken proportional to the length of the tube occupied by it, this being obtained from the scale against which it is placed.

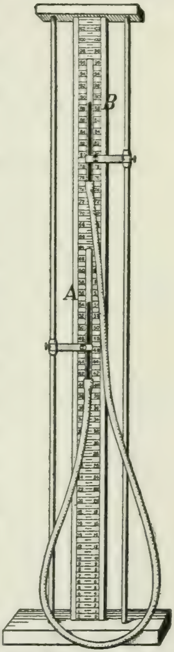



Fig. 186.—Boyle's Law apparatus. *A*, closed tube; *B*, open tube.

First, adjust the amount of mercury so that when it is at the same level in both glass tubes *A* is about half-full of air, which also should be dry.

Read the barometer and record its height in cm. of mercury. When the mercury is at the same level in both tubes the air is under the pressure of one atmosphere, *i.e.*, the pressure shown by the barometer.

Now lower *B* as far as it will go. Do this rather slowly. The temperature of the gas should remain the same, and a

rapid change in volume produces a change in the temperature. Then read the levels of the mercury in the two tubes. The pressure to which the imprisoned gas is subjected is now 1 atmosphere *minus* the difference in the levels of the mercury. Raise $B$ a few cms. and take the readings again. Continue this until $B$ is as high as it can go. When the level of $B$ is above that of $A$ the pressure on the imprisoned air is 1 atmosphere *plus* the difference in level.

The tube $A$ should not be handled for fear of raising the temperature of the inclosed air; and, as has already been remarked, the air should not be compressed or expanded quickly.

The following results were obtained with such an apparatus:

| Level of Mercury in | | Difference between the Levels. | Height of Barometer. | Total Pressure in cm. of Mercury = $P$. | Length of Air in Tube = $V$. | Product $P \times V$. |
|---|---|---|---|---|---|---|
| Closed Tube. | Open Tube. | | | | | |
| 46.2 | 7.6 | −38.6 | 74.6 | 36.0 | 30.1 | 1083.6 |
| 51.6 | 21.0 | −30.6 | " | 44.0 | 24.7 | 1086.8 |
| 55.3 | 32.35 | −23.0 | " | 51.6 | 21.0 | 1085.7 |
| 59.2 | 48.1 | −11.1 | " | 63.5 | 17·1 | 1085.8 |
| 61.7 | 61.7 | 0.0 | " | 74.6 | 14.6 | 1089.2 |
| 63.8 | 76.3 | +12.5 | " | 87.1 | 12.5 | 1088.7 |
| 65.5 | 91.9 | +26.4 | " | 101.0 | 10.8 | 1090.8 |
| 66.9 | 107.7 | +40.8 | " | 115.4 | 9.4 | 1084.7 |
| 68.0 | 124.8 | +56.8 | " | 131.4 | 8.3 | 1090.6 |
| 68.5 | 132.6 | +64.2 | " | 138.8 | 7·8 | 1082.6 |

Reading of top of closed tube, 76.3 cm.

From this experiment we learn that the pressure and volume vary in such a way that the product $P \times V$ is constant. If the pressure is doubled the volume becomes half as great, if the pressure is multiplied threefold, the volume becomes one-third, etc. In other words,

*If the temperature is kept constant, the volume of a given mass of air varies inversely as the pressure to which it is subjected.*

This relation is generally known as BOYLE'S LAW. In France it is called Mariotte's Law, because it was independently discovered by a French physicist named Mariotte (1620-1684), fourteen years after Boyle's publication of it in England.

### PROBLEMS

1. A tank whose capacity is 2 cu. ft. has gas forced into it until the pressure is 250 pounds to the sq. inch. What volume would the gas occupy at a pressure of 75 pounds to the sq. inch ?

2. A gas-holder contains 22.4 litres of gas when the barometer stands at 760 mm. What will be the volume of the gas when the barometer stands at 745 mm. ?

3. A cylinder whose internal dimensions are : length 36 in., diameter 14 in., is filled with gas at a pressure of 200 pounds to the sq. inch. What volume would the gas occupy if allowed to escape into the air when the barometer stands at 30 in.? (For density of mercury see Sec. 163).

4. Twenty-five cu. ft. of gas, measured at a pressure of 29 in. of mercury, is compressed into a vessel whose capacity is $1\frac{1}{2}$ cu. ft. What is the pressure of the gas?

5. A mass of air whose volume is 150 c.c. when the barometer stands at 750 mm. has a volume of 200 c.c. when carried up to a certain height in a balloon. What is the reading of the barometer at that height?

6. A piston is inserted into a cylindrical vessel 12 in. long, and forced down within 2 in. of the bottom. What is the pressure of the inclosed air if the barometer stands at 29 in.?

7. The density of the air in a gas-bag is 0.0001293 grams per c.c. when the barometer stands at 760 mm. ; find its density when the barometric height is 740 mm.

8. An open vessel contains 100 grams of air when the barometer stands at 745 mm. What mass of air does it contain when the barometer stands at 755 mm.?

9. Oxygen gas, used for the 'lime-light,' is stored in steel tanks. The volume of a tank is 6 cu. ft., and the pressure of the gas at first was 15 atmospheres. After some had been used the pressure was 5 atmospheres. If the gas is sold at 6 cents a cu. ft., measured at atmospheric pressure, what should be charged for the amount consumed ?

**170. Explanation of Boyle's Law.** This law naturally follows from the kinetic theory of gases. Suppose a certain

quantity of a gas is in a cylinder closed by a piston, and let the gas at first occupy any definite volume ($A$, Fig. 187). The molecules dart about in all directions and maintain a pressure upon the inner surface, which exactly balances the downward push of the piston. Next, let the piston be thrust down until the volume is one-half as great ($B$, Fig. 187). Then the number of molecules within this space is twice as great as before and the blows delivered against its sides are twice as numerous, and consequently the pressure exerted by the gas is twice as great. In the same way, if the volume is reduced to $\frac{1}{n}th$ part, the pressure exerted will be $n$ times as great.

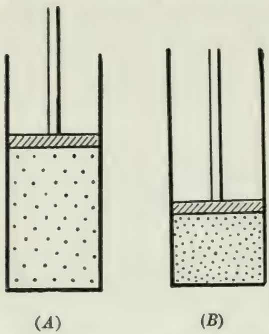


FIG. 187.—Pressure in $B$ is twice that in $A$.

Let successive volumes be $v, \frac{1}{2}v, \frac{1}{3}v, \ldots\ldots \frac{1}{n}v.$

Then corresponding pressures are $p, 2p, 3p, \ldots\ldots n\,p.$

$$\text{Now } p \times v = 2p \times \tfrac{1}{2}v = 3p \times \tfrac{1}{3}v = \ldots\ldots n\,p \times \frac{1}{n}v,$$

that is, the pressure $\times$ the volume is constant $= k$ (say).

$$\text{Then } p = k\,\frac{1}{v}, \text{ or } p \text{ varies inversely as } v.$$

Now if the volume of a gas is reduced to $\frac{1}{2}$, its density becomes 2 times as great; if to $\frac{1}{3}$, its density is 3 times as great; if to $\frac{1}{n}th$, its density is $n$ times as great. From this we see that the density varies inversely as the volume. Consequently we say that *the pressure exerted by a gas is directly proportional to its density.*

This is only another statement of Boyle's Law.

**171. The Speed of the Molecules.** The average speed of the molecules of a gas at a given temperature may be

calculated in the following way. Consider a cubical vessel, 1 cm. to the edge, containing a certain quantity of gas which, of course, exerts equal pressures on all the surfaces. Though the motions of the particles are indiscriminately in all directions, striking one surface and rebounding from it to strike another or perhaps to collide with another molecule; yet it seems reasonable to assume that the pressures on the six sides of the cube would be maintained if the molecules were divided into three equal sets; one set moving continually back and forth parallel to $AB$ and keeping up a bombardment against the two sides perpendicular to $AB$; the second moving parallel to $AC$ and bombarding the two sides perpendicular to $AC$; the third moving parallel to $AD$ and bombarding the two sides perpendicular to $AD$; and the molecules all moving with a speed which is the average of all the speeds.

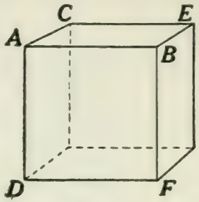


FIG. 188.—Speed of molecules of gas in a cubical vessel.

Let us consider the first set, namely those moving parallel to $AB$, and taking $n$ to be the total number of molecules in the cube, the number of the first set will be $\frac{1}{3}n$.

Let the speed of the molecules be $V$ cm. per sec., and the mass of each be $m$ grams. Each moving molecule will possess a momentum $mV$.

Suppose a molecule to strike against the side $CD$ with speed $V$; we assume that it rebounds with the same speed. In this way a momentum $mV$ in one direction on impact is changed into one of equal amount in the opposite direction, or the *change of momentum* at one impact $= 2mV$. Now the speed is $V$ cm. per sec., and the molecule will travel across the cube and back, a distance of 2 cm., in the $\frac{2}{V}th$ part of a sec. In 1 sec. it will do this $\frac{V}{2}$ times, that is, each molecule will make $\frac{V}{2}$ impacts against a side every second, and as in each impact there is a change of momentum of $2mV$, it is

clear that in 1 sec. upon each side 1 sq. cm. in area there will be produced a change of momentum,

$$\tfrac{1}{3} n \times \frac{V}{2} \times 2 m V = \tfrac{1}{3} n m V^2.$$

This gives rise to a pressure of $p$ (say) upon the side.

According to Newton's Second Law (Secs. 38, 39)

Force $\times$ time = momentum generated.

In the present case, force $= p$ (dynes per sq. cm.)
time $= 1$ sec.
momentum $= \tfrac{1}{3} n m V^2$ units.

Hence, $p = \tfrac{1}{3} n m V^2$.

Now $n m$ = entire mass of the molecules in 1 c.c.,
$= \rho$, the density of the gas.

$$\text{Hence, } p = \tfrac{1}{3} \rho V^2, \text{ or } V = \sqrt{\frac{3p}{\rho}}.$$

This velocity is not strictly the average of all the velocities but is the square root of the mean square velocity. Observe also that $p$ varies as $\rho$ (Boyle's Law).

Let us now calculate the velocity for a gas,—for example, hydrogen, under standard pressure and temperature.

For hydrogen, $\rho = .0000895$ gm. per cc.

Also $p = 1033.296 \times 981$ dynes per sq. cm. (Sec. 163),

$$\text{and } V = \sqrt{\frac{3 \times 1033.296 \times 981}{.0000895}} = 184{,}300 \text{ cm. per sec.}$$

In the same way the velocity for any other gas may be calculated.

TABLE OF MOLECULAR VELOCITIES

| Gas. | Velocity. |
|---|---|
| Hydrogen.................. | 1843 m. = 6046 ft. per sec. |
| Nitrogen ................ | 493 " = 1618 " " " |
| Oxygen.................. | 462 " = 1517 " " " |
| Carbon Dioxide .......... | 393 " = 1291 " " " |

For a fuller treatment of the kinetic theory of gases see Maxwell's "Theory of Heat," Chapter XXII, or Edser's "Heat for Advanced Students," Chapter XIII.

# CHAPTER XXI

## Air-Pumps and Water-Pumps

**172. Air-Pump.** In Fig. 189 is shown the construction of a common type of pump used for removing the air from a vessel.

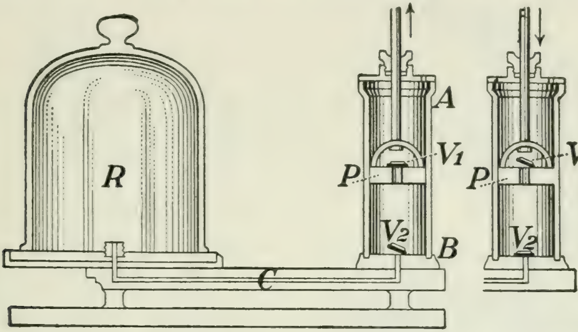



Fig. 189.—Common form of air-pump. $AB$, cylindrical barrel of pump; $R$, receiver from which air is to be exhausted; $C$, pipe connecting barrel with receiver; $P$, piston of pump; $V_1$ and $V_2$, valves opening upwards.

Its operation is as follows:—When the piston $P$ is raised, the valve $V_1$, in it, remains closed, due to its own weight and the pressure of the air above it. The air in $R$ expands, and some of it passes by way of the pipe $C$, into the lower portion of the barrel, lifting the valve $V_2$ in doing so. When the piston descends, the valve $V_2$ remains closed, and the air in the barrel passes up through the valve $V_1$ and escapes outside. Thus at each up-and-down stroke a fraction of the air is removed from the receiver, $R$. The pump will continue to act until the air on expanding from $R$ is no longer able to lift the valve $V_2$, or when the pressure of the air below the piston is insufficient to raise the valve $V_1$. It is evident, therefore, that only a partial vacuum can be obtained with a pump of this kind. To secure more complete exhaustion, pumps have been constructed in which the valves are opened and closed automatically as the piston moves, but even with these the air cannot all be removed from the receiver, since at each double-stroke the air in it is reduced only by a fraction of itself.

**173. The Geryk or Oil Air-Pump.** This pump is much more efficient than that just described. Its action is as follows:—The piston $J$ (Fig. 191), made air-tight by the leather washer $C$ and by being covered with oil, moves up and down in the cylinder. The tube $A$, opening into the chamber $B$ surrounding the cylinder, is connected to the vessel from which the air is to be removed. On rising, the piston pushes before it the air in the cylinder, and on reaching the top it pushes up $G$ about $\frac{1}{4}$ inch, thus allowing the imprisoned air to escape through the oil into the upper part of the cylinder, from which it passes out by the tube $D$.

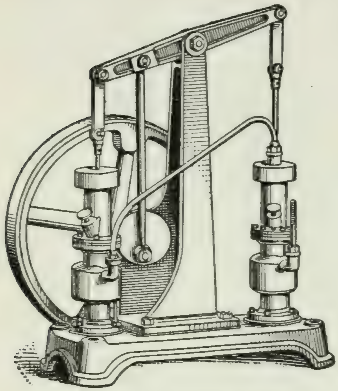


FIG. 190.—An oil air-pump with two cylinders.

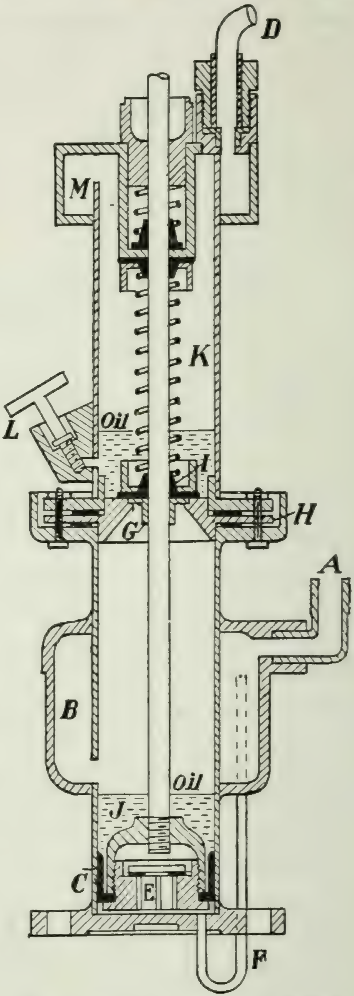



FIG. 191.—Vertical section of a cylinder of an oil air-pump.

When the piston descends the spring $K$, acting upon the packing $I$, closes the upper part of the cylinder, and the piston on reaching the bottom drives whatever oil or air is beneath out through the tube $F$, or allows it to go up through the valve $E$, into the space above the piston.

Oil is introduced into the cylinder at $L$. When the pump has two cylinders they are connected as shown in Fig. 190. With one cylinder the pressure of the air can be reduced to $\frac{1}{4}$ mm. of mercury, while with two a reduction to $\frac{1}{500}$ mm., it is claimed, can be quickly obtained.

**174. Mercury Air-Pump.** When a very high vacuum is required a mercury pump of some sort is generally used. There are various forms. The principle of that devised by Sprengel may be explained by Fig. 192. Here $R$ is the vessel from which the air is to be removed; it is fused on the pump. Mercury is poured into the reservoir $A$ and as it falls in a broken stream from the nozzle $N$ into the tube $B$ (which is of small bore and about a metre long) it carries air with it, each pellet of mercury acting as an air-tight piston and bearing a small portion of air before it. The mercury which overflows into $D$ is poured back into $A$. The air is gradually removed from $C$ and $R$, and if the operation is continued long enough an almost perfect vacuum can be obtained.

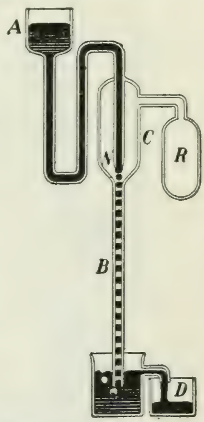


FIG. 192. — Sprengel air-pump. $A$, reservoir into which mercury is poured. $B$, glass tube of small bore, about one metre long; $R$, vessel from which air is to be drawn.

Another form of mercury pump is illustrated in Fig. 193. The vessel containing the air to be exhausted is connected to $C$, and a tube runs down from it to the lower end of the chamber $A$, while to the upper end of $A$ is attached a long tube $F$ whose open end dips into a vessel of mercury $G$. A mercury reservoir $B$ is connected by means of a strong

flexible rubber tube to a tube projecting downward from *A*. On raising *B* the mercury flows into *A* and closes the connection of *D* with *A*, and as *B* is raised still higher the mercury fills *A*, driving the gas which was in it, by way of *F*, out through *G*. When *B* is lowered, mercury rises in *F* and prevents the air from entering through *F*, and the gas in the vessel to be exhausted expands into *A*. On raising *B* more gas is expelled, and so on. The side tube *H* prevents the sudden inrush of gas into the bottom of *A* as *B* is lowered and thus avoids danger of breakage.

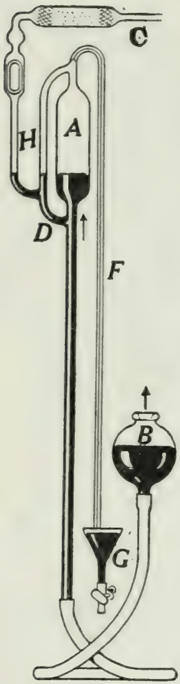


FIG. 193.—Principle of Toepler mercury pump.

The above mercury pumps are somewhat slow in action, although mechanical devices have been introduced to make the operation easier. In quite recent years rotary mercury pumps have been constructed which act rapidly and which are also very efficient. Their operation is too complicated to be explained here.

**175. The 'Condensation' Vacuum Pump.** The wonderful uses made of highly-exhausted glass bulbs in investigations into the nature of matter, in the production of X-rays, in the wireless telegraph and telephone, and for other purposes, has led to the invention of several kinds of high-vacuum pumps. A very recent and very rapid type, known as Langmuir's condensation pump, is constructed on a new principle, which may be explained with the help of Fig. 194.

A metal cylinder *A* is provided with two openings, *B* and *C*. The latter is connected to the vessel to be exhausted and the former is joined to an auxiliary pump which, itself, must be

able to produce a vacuum of about 0.1 mm. Within the cylinder is a funnel-shaped tube $F$ which rests on the bottom of $A$. Suspended from the top of $A$ is a cup $E$, inverted over the upper end of $F$. A water-jacket $J$ surrounds the wall of $A$ from the level of $B$ to a height

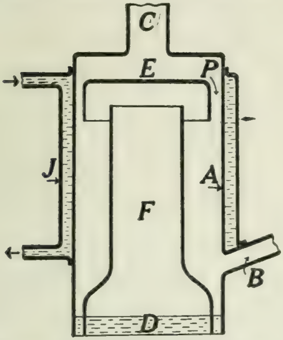



**Fig. 194.**—Diagram to explain Langmuir's condensation pump.

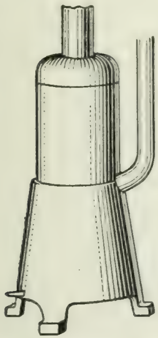


Fig. 196.—Outer view of the condensation pump.

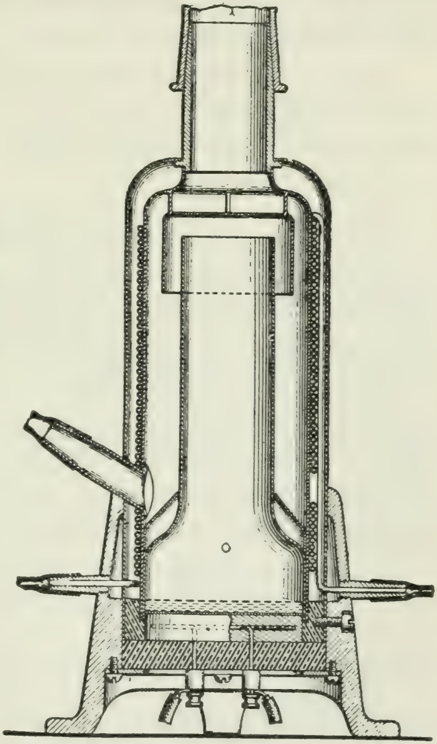


Fig. 195.—Showing actual construction of the condensation pump.

somewhat above the lower edge of the cup $E$. Mercury is placed in the cylinder, as indicated at $D$.

By applying heat to the bottom of the cylinder the mercury is caused to evaporate. The vapour passes up through $F$, and being deflected by $E$, is directed downwards and outwards

against the water-cooled wall of $A$. The gas as it comes from the vessel which is being exhausted enters the pump at $C$, passes down between $E$ and $A$ and at $P$ meets the stream of mercury vapour which forces it down along the wall of $A$ and out of the tube $B$ where the auxiliary pump takes it and removes it. The mercury which condenses on the water-cooled wall falls downwards and returns to $D$, ready to be vaporized again.

A detailed drawing of the pump as actually constructed is given in Fig. 195. In the base is a simple electric heater which produces the mercury vapour. Around the wall of the pump and just inside the outer casing is a coiled tube through which water is kept running. The ends of this tube are seen projecting outwards near the base. The tube leading to the auxiliary pump is higher up on the left, while the tube from the vessel which is being exhausted is at the top. The outer appearance of the pump is shown in Fig. 196.

The pump as just described is constructed of metal but it may also be made of glass. It is very rapid in its action and there is no lower limit to which the pressure may be reduced. Pressures lower than $\frac{1}{100,000}$ of a dyne per sq. cm., or 0.000,000,007,5 mm. of mercury have been produced.

**176. Air-Compressors.** The simplest compression pump is that used for inflating rubber tires. Its construction is seen in Fig. 197. When the piston $P$ is pushed down, the air in the cylinder is forced through the valve $v$ into the inner tube of the tire $T$, the valve immediately closing to check the air from going back. On lifting the piston a partial vacuum is produced in the cylinder

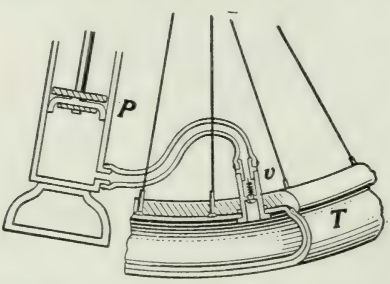



Fig. 197.—Air-compressor for pneumatic tire.

and the air from outside enters, going past the soft cup-shaped leather forming a part of the piston. When the piston is moving downward this leather is pressed against the inside of the cylinder, thus preventing the air from escaping. Each downward stroke forces more air into the tire until at last it becomes sufficiently hard. In a bicycle tire the pressure seldom exceeds 45 pounds per square inch, while in automobile tires the pressures run from 70 to 95 pounds.

Another form of air-compressor or condenser is illustrated in Fig. 198. Here the piston is solid and there is an inlet, as well as an outlet, valve. When the piston is raised the inlet valve $V_1$ opens and the barrel is filled with air from the outside, and when the piston is pushed down the inlet valve is closed and the air is forced into the tank through the outlet valve $V_2$, which closes on the up-stroke and thus retains the air within the tank. Hence at each double stroke a cylinder-full of air is forced into the tank. For rapid compression a double-action pump of the form shown in Fig. 202 is used.

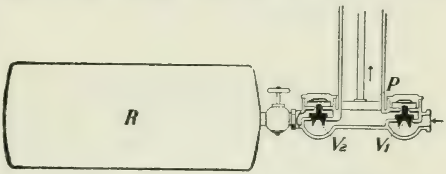


FIG. 198.—Air-compressor. *P*, piston; *R*, tank or receiver; $V_1$, inlet valve; $V_2$, outlet valve.

### PROBLEMS

1. The capacity of the receiver of an air-pump is twice that of the barrel; what fractional part of the original air will be left in the receiver after (*a*) the first stroke, (*b*) the third stroke?

2. The capacity of the barrel of an air-pump is one-fourth that of the receiver; compare the density of the air in the receiver after the first stroke with the density at first.

3. The capacity of the receiver of an air-compressor is ten times that of the barrel; compare the density of the air in the receiver after the fifth stroke with its density at first.

4. The capacity of the barrel of an air-pump used to exhaust a litre flask is 250 c.c.; compare the density of the air in the flask after the second stroke with its original density.

5. The capacity of the barrel of an air-compressor used to force air into a tank, whose capacity is one litre, is 200 c.c.; compare the density of the air in the tank after the fifth stroke with its density at first.

**177. Water Pumps.** From very early times pumps were employed for raising water from reservoirs, or for forcing it through tubes. It is certain that the suction pump was in use in the time of Aristotle (born 384 B.C.). The force-pump was probably the invention of Ctesibius, a mechanician who flourished in Alexandria in the second century B.C. To Ctesibius is also attributed the ancient fire-engine, which consisted of two connected force-pumps, spraying alternately.

**178. Suction or Lift-Pump.** The construction of the common suction-pump is shown in Fig. 199. During the first strokes the suction-pump acts as an air-pump, withdrawing the air from the suction pipe *BC*. As the air below the piston is removed its pressure is lessened, and the pressure of the air on the surface of the water outside forces the water up the suction pipe, and through the valve $V_1$ into the barrel. On the down-stroke the water held in the barrel by the valve $V_1$ passes up through the valve $V_2$, and on the next up-stroke it is lifted up and discharged through the spout *G*, while more water is forced up through the valve $V_1$ into the barrel by the external pressure of the atmosphere. It is evident that the maximum height to which water, under perfect conditions, is raised by the pressure of the atmosphere cannot be

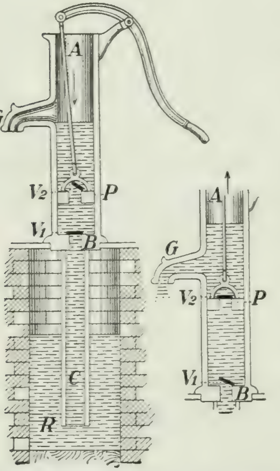



FIG. 199.—Suction-pump. *AB*, cylindrical barrel; *BC*, suction-pipe; *P*, piston; $V_1$ and $V_2$, valves opening upwards; *R*, reservoir from which water is to be lifted.

greater than the height of the water column which the air will support. Taking the relative density of mercury as 13.6 and the height of the mercury barometer as 30 inches, this height would be $\frac{30}{12} \times 13.6 = 34$ feet. To this height, then, above the level of the water in the well, the atmosphere can raise the water, and, of course, for the pump to lift the water higher its piston must be immersed in this water column. Consequently the pump rod must extend downwards within 34 feet of the level of the water in the well. As a matter of fact, on account of the air within the water and the vapour from the water, the piston should be within 25 feet of the surface of the water in the well.

## PROBLEMS AND EXERCISES

1. What is the greatest height to which water can be raised by a common pump when the mercury barometer stands at 76 cm., the sp. gr. being 13.6?

2. How high can sulphuric acid be raised by a common pump when the mercury barometer stands at 27 in., the sp. gr. of sulphuric acid being 1.8 and that of mercury being 13.6?

3. How high can alcohol be raised by a lift-pump when the mercury barometer stands at 760 mm. if the relative densities of alcohol and mercury are 0.8 and 13.6 respectively?

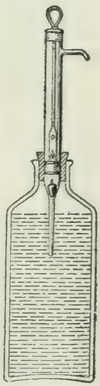


Fig. 200.

4. Connect a glass model pump with a flask, as shown in Fig. 200. Fill the flask (*a*) full, (*b*) partially full of water, and endeavour to pump the water. Account for the result in each case.

**179. Force-Pump.** When it is necessary to raise water to a considerable height, or to drive it with force through a nozzle, as for extinguishing fire, a force-pump is used. Fig. 201 shows the most common form of its construction. On the up-stroke a partial vacuum is formed in the barrel, and the air in the suction tube expands and passes up through the valve $V_1$. As the plunger is pushed down, the air is forced out through the valve $V_2$. The pump, therefore, during the first strokes acts as an air-pump. As in the suction-pump, the water is forced up into the suction pipe by the pressure of the air on the surface of the water in the reservoir. When it enters the barrel it is forced by the plunger at each down-stroke through the valve $V_2$ into the discharge pipe. The flow will obviously be intermittent, as the outflow takes place only as the plunger is descending. To produce a continuous stream, and to lessen the shock on the pipe, an air chamber, *F*, is often inserted in the discharge pipe. When the water enters this chamber it rises above the outlet, *G*, which is somewhat smaller than the inlet, and compresses the air in the chamber. As the plunger is ascending, the pressure of the inclosed air forces the water out of the chamber in a continuous stream.

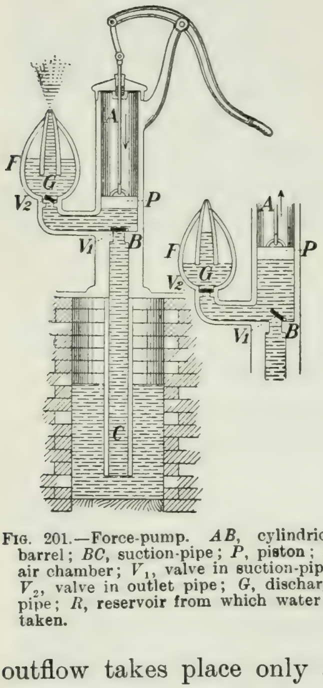



FIG. 201.—Force-pump. *AB*, cylindrical barrel; *BC*, suction-pipe; *P*, piston; *F*, air chamber; $V_1$, valve in suction-pipe; $V_2$, valve in outlet pipe; *G*, discharge pipe; *R*, reservoir from which water is taken.

**180. Double-Action Force-Pump.** In Fig. 202 is shown the construction of the double-action force-pump. When the

piston is moved forward in the direction of the arrow, water is drawn into the back of the cylinder through the valve $V_1$, while the water in front of the piston is forced out through the valve $V_3$. On the backward stroke water is drawn in through the valve $V_2$ and is forced out through the valve $V_4$. Pumps of this type are used as fire engines, or for any purposes for which a large continuous stream of water is required. They are usually worked by steam or other motive power.

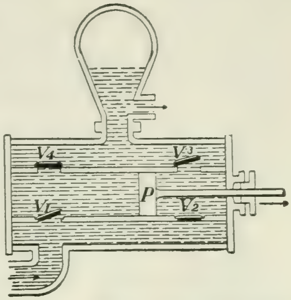


FIG. 202.—Double-action force-pump. $P$, piston; $V_1$, $V_2$, inlet valves; $V_3$, $V_4$, outlet valves.

**181. Hydraulic Press.** This machine is ordinarily used whenever great force is to be exerted through short distances, as in pressing goods into bales, extracting oils from seeds, making dies, testing the strength of materials, etc. Its construction is shown in Fig. 203. $A$ and $B$ are two cylinders connected with each other and with a water cistern by pipes closed by valves $V_1$ and $V_2$. In these cylinders work pistons $P_1$ and $P_2$ through water-tight collars, $P_1$ being moved by a lever. The bodies to be pressed are held between plates $C$ and $D$. When $P_1$ is raised by the lever, water flows up from the cistern through the value $V_1$ and fills the cylinder $A$. On the down-stroke the valve $V_1$ is closed and the water is forced through the valve $V_2$ into the cylinder $B$, thus

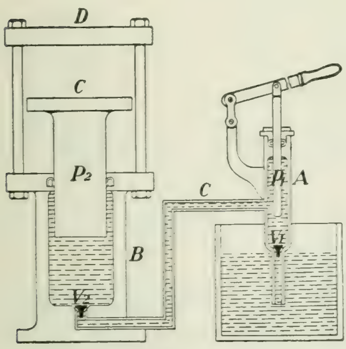


FIG. 203.—Bramah's hydraulic press.

exerting a force on the piston $P_2$, which will be as many times that applied to $P_1$ as the area of the cross-section of $P_2$ is that of the cross-section of $P_1$. It is evident that by decreasing the size of $P_1$, and increasing that of $P_2$, an immense force may be developed by the machine.

**182. Siphon.** If a bent tube is filled with water, and placed in a vessel of water and the ends unstopped, the water will flow freely from the tube, so long as there is a difference in level in the water in the two vessels. A bent tube of this kind, used to transfer a liquid from one vessel to another at a lower level, is called a siphon.

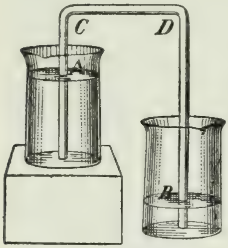


FIG. 204.—The siphon.

To understand the cause of the flow consider Fig. 204.

The pressure at $A$ tending to move the water in the siphon in the direction $AC$

= the atmospheric pressure − the pressure due to the weight of the water in $AC$;

and the pressure at $B$ tending to move the water in the siphon in the direction $BD$

= the atmospheric pressure − the pressure due to the weight of the water in $BD$.

But since the atmospheric pressure is the same in both cases, and the pressure due to the weight of the water in $AC$ is less than that due to the weight of the water in $BD$, the force tending to move the water in the direction $AC$ is greater than the force tending to move it in the direction $BD$; consequently a flow takes place in the direction $ACDB$. This will continue until the vessel from which the water flows is empty or until the water comes to the same level in each vessel.

**183. The Aspirating Siphon.** When the liquid to be transferred is dangerous to handle, as in the case of some acids, an aspirating siphon is used. This consists of an ordinary siphon to which is attached an offset tube and stopcock, as shown in Fig. 205, to facilitate the process of filling. The end $B$ is closed by the stopcock and the liquid is drawn into the siphon by suction at the mouth-piece $A$. The stopcock is then opened and the flow begins.

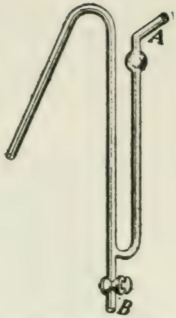


Fig. 205.—The aspirating siphon.

**PROBLEMS AND EXERCISES**

1. Upon what does the limit of the height to which a liquid can be raised in a siphon depend?

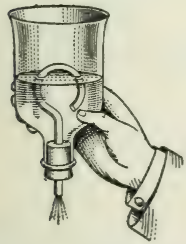


Fig. 206.—Intermittent siphon or Tantalus cup.

2. Over what height can (*a*) mercury, (*b*) water, be made to flow in a siphon?

3. How high can sulphuric acid be raised in a siphon when the mercury barometer stands at 29 in., taking the relative densities of sulphuric acid and mercury as 1.8 and 13.6 respectively?

4. Upon what does the rapidity of flow in the siphon depend?

5. Arrange apparatus as shown in Fig. 206. Let water from a tap run *slowly* into the bottle. What takes place? Explain.

6. Natural reservoirs are sometimes found in the earth, from which the water can run by natural siphons faster than it flows into them from above (Fig. 207). Explain why the discharge through the siphon is intermittent.*

*Such intermittent springs exist near Atkins, in the mountain region of southwestern Virginia; near Giggleswick, in Yorkshire, England; and in Germany. (See "Scientific American," August 17, 1918).

7. Arrange apparatus as shown in Fig. 208. Fill the flask $A$ partly full of water, insert the cork, and then invert, placing the short tube in water. Explain the cause of the phenomenon observed.

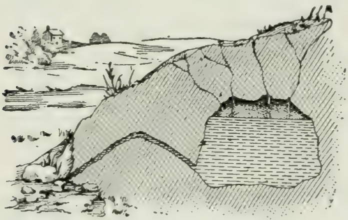


FIG. 207.—An intermittent spring.

8. A boat on the beach is full of water. How could you empty it with the help of a suitable length of rubber hose? Could you use the same method to get the bilge water out of a boat floating on water? Explain.

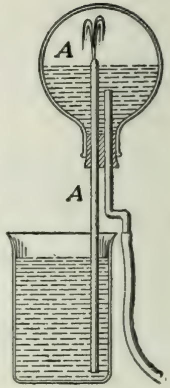



FIG. 208.

9. Find the greatest height over which a liquid of density $\rho_1$ can be carried by a siphon when the height of the barometer is $h$, the density of the liquid used in the barometer being $\rho$.

10. What would be the effect when the siphon is working of making a hole in it (Fig. 204), (1) at $C$, (2) between $A$ and $C$, (3) at $D$, (4) between $C$ and $D$, (5) between $D$ and $B$?

## CHAPTER XXII

### Uses of Compressed Air

**184. Air-Brakes.** One of the many uses of compressed air is to set the brakes on railway trains. Fig. 209 illustrates the principal working parts of the Westinghouse air-brakes in common use in this country. A steam-driven air-compressor pump $A$ and a tank $B$ for compressed air are attached to the locomotive. The equipment on each car consists of (i) a cylinder $C$ in which works a piston $P$, directly connected, by a piston-rod $D$ and a system of levers, with the brake-shoe,

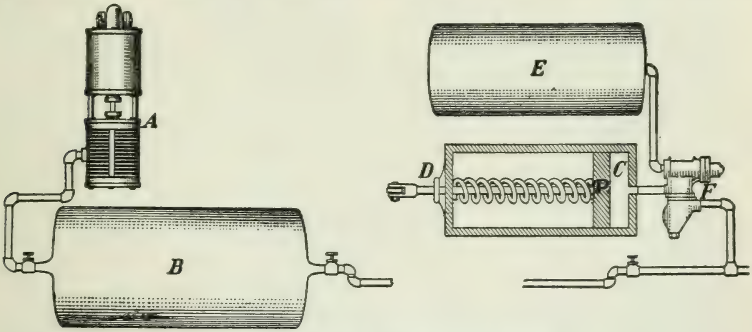



Fig. 209.—Air-brakes in use on railway trains.

(ii) a secondary tank $E$, and (iii) a system of connecting pipes and a special valve $F$ which automatically connects $B$ with $E$ when the air from $B$ is admitted to the pipes, but which connects $E$ with the cylinder $C$ when the pressure of the air is removed.

When the train is running, pressure is maintained in the pipes and in the tank $E$, and the brakes are free; but when the pressure is decreased, either by the engineer or by the accidental breaking of a connection, the inrush of air from $E$

to $C$ forces the piston $P$ forward and sets the brakes against the wheels. To take off the brakes, the air is again turned into the pipes, the valve $F$ then connects $B$ with $E$ and the air in $C$ is allowed to escape, while the piston $P$ is forced into its original position by a spring.

**185. Diving-Bell.** This is made of steel or iron, heavy enough to sink in the water when the open side is downwards, and large enough to accommodate two or more workmen. In

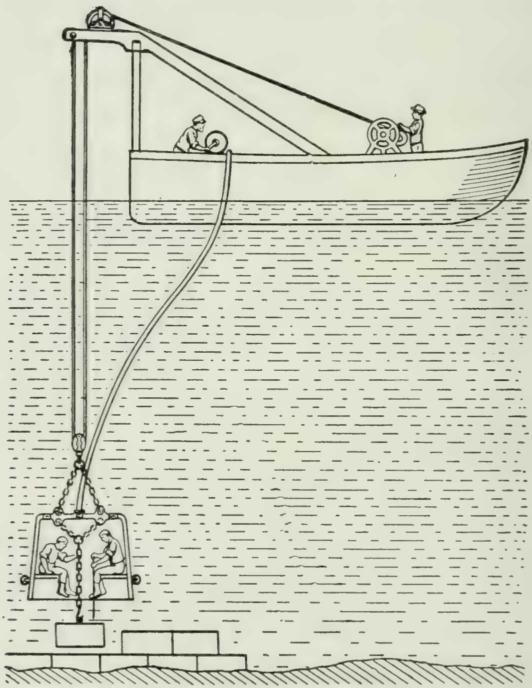


FIG. 210.—Laying a stone foundation with a diving-bell.

Fig. 210 is shown how blocks of stone or cement are placed when a pier of a bridge is being constructed. The bell is let down from a boat or from a wooden staging built over the

water. From an air-compressor on the boat air is forced into the bell, thus preventing the water from entering it and also supplying the men with fresh air to breathe. Surplus air escapes at the lower edge of the bell.

**186. Pneumatic Caisson.** The pneumatic caissons used in laying the foundations of bridges, piers, elevators, etc., are operated on the same principle as the diving bell. A section of a typical caisson is shown in Fig. 211. The sides of the caisson are extended upward and are strongly braced to keep back the water. Masonry, or concrete, $C$, $D$, placed on top of the caisson, presses it down upon the bottom, while compressed air, forced through a pipe $P$ drives the water from the working chamber and also sustains the men. To leave the caisson the workman climbs up and passes through the open door $B$ into the air-lock $L$. The door $B$ is then closed and the air is allowed to escape from $L$ until it is at atmospheric pressure. Then door $A$ is opened and the workman climbs out. In order to enter, this procedure is reversed. Material is hoisted out in the same way or is sucked out by a mud-pump. As the earth is removed, the caisson sinks, until at last the solid rock is reached. The entire caisson is then

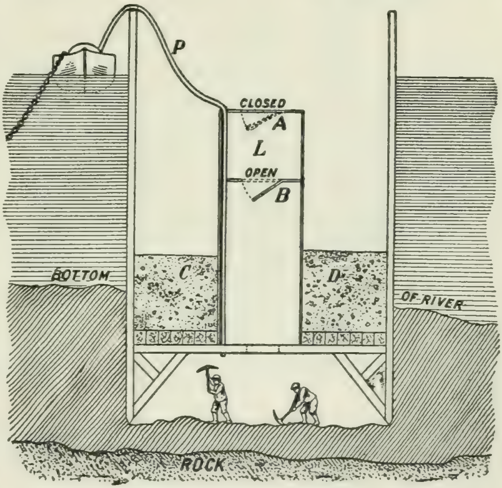



FIG. 211.—Section of a pneumatic caisson.

filled with solid concrete, and a permanent foundation for a dock or bridge is thus obtained.

**187. Diving Suit.** The modern diver is incased in an air-tight weighted suit. (Fig. 212). He is supplied with air from above, through pipes or from a compressed-air reservoir attached to his suit. The air escapes through a valve into the water.

FIG. 212.—Diver's suit.

Manifestly the pressure of the air used by a diver or a workman in a caisson must balance the pressure of the outside air, and the pressure of the water at his depth. The deeper he descends, therefore, the greater the pressure to which he is subjected. The ordinary limit of safety is about 80 feet; but divers have gone much deeper than this. In March, 1915, a United States submarine sank in the harbour of Honolulu. A diver went 288 feet under water and walked along the top of the ship, and in the course of salvaging it he made 5 descents to a depth of 306 feet.

**188. Some Other Uses of Compressed Air.** Another useful application is the pneumatic drill, used chiefly for boring holes in rock for blasting. In it the steel drill is held in the end of a cylinder within which a piston is made to move back and forth by allowing compressed air to act alternately on its two end faces. Each time the piston moves forward it delivers a vigorous blow upon the end of the drill, and as it does this several times per second the drill enters the rock quite rapidly. The pneumatic hammer, which is similar in principle, is used for riveting and in general foundry work. Steam could be used in place of air, but the pipes conveying it would be hot, and water would be formed from it.

By means of a blast of sand, projected by a jet of air, castings and also discoloured stone and brick walls are cleaned. Figures on glass are engraved in the same way. Tubes for transmitting letters or telegrams, or for carrying cash in our large retail stores, are operated by compressed air. Many other applications cannot be mentioned here.

# CHAPTER XXIII

## Surface Tension

**189. Surface Tension Met With Everywhere.** In studying the behaviour of liquids of all sorts, whether contained in ordinary vessels, or in the body of an animal or a plant, whether at a low or a high temperature, we continually meet with a peculiar phenomenon which has been found to be of great importance in the various processes of nature. It is especially prominent when the quantity of liquid is small.

Numerous experiments, many of them easily performed, illustrate the effect and we shall examine some of them.

**190. Formation of a Drop.** On slowly forcing water from a medicine dropper, it gradually gathers at the end, becoming more and more globular, and at last breaks off and falls. (Fig. 213.) We can see that the drop is approximately spherical. When mercury falls on the floor it breaks up into a multitude of shining globules which retain their shape indefinitely. Why do they not flatten out?

Fig. 213.—A drop of water assumes the globular form.

If melted lead is poured through a sieve at the top of a tower it forms into drops which harden on the way down and which finally appear as solid spheres of shot.

While in some cases we can see the drops growing, the final separation from the mass of liquid is ordinarily so sudden and the subsequent motion is so rapid, that it is impossible to trace the successive stages. The drops, also, are usually quite small. In the following experiment,* however, the process of formation

*Devised by Charles R. Darling, "Nature," Vol. 83, page 37, 1910.

is so slow and the drop is so large that the effect of surface tension can be conveniently observed.

Aniline is an oily liquid which at ordinary temperatures is denser than water. When poured into water it does not mix with it, but falls to the bottom, and the colour assumed by the aniline renders the surface between the water and the aniline clearly visible at a considerable distance. However when heated above 80° C. it rises to the surface of the water.

Into a beaker about 9 inches high and 4½ inches in diameter pour water to the depth of about 7 inches. Then add about 80 c.c. of aniline. Place the beaker above a burner and heat gently until a temperature of about 80° is reached.

The hot aniline now rises to the surface, spreads out, and, coming in contact with the air, is cooled and collects in the form of a drop, an inch or more in diameter, hanging down from the mass at the surface. As the drop grows in size a neck is formed, which, after a while, gets thinner at two places; and when it breaks away the large drop is followed by a small one which is known as Plateau's spherule. If the temperature is maintained at about 80° the drops will continue to be formed.

Observe the oscillations in the form of the drop as it descends.

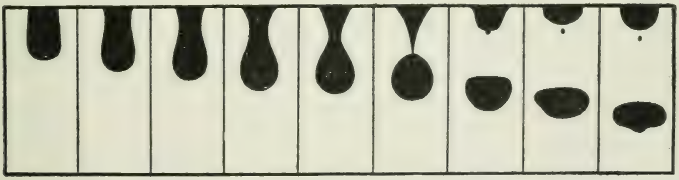


**Fig. 214.—Stages in the development of a drop of water.**
*(From photographs by Boys.)*

Various stages in the development of a drop of water are illustrated in Fig. 214.

Another beautiful experiment, due to the Belgian physicist Plateau, referred to just above, is as follows:

Put water in a beaker and then carefully pour alcohol on top of it. About 40 per cent. of water to 60 per cent. of alcohol is best, but there may be considerable variation from this proportion. Now introduce olive oil into it by means of a pipette* (Fig. 215). If it is of the same density it will neither sink nor rise on account of gravity. It assumes a spherical form *as though an enveloping skin was trying to compress the oil into a smaller space.* For a given volume, a sphere has less surface area than a body of any other form.

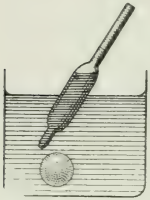


Fig. 215.—A sphere of olive oil in a mixture of water and alcohol.

**191. A 'Skin' on the Surface.** Many other experiments strongly suggest that liquids are enclosed in a thin skin or membrane, which continually tends to contract.

(1) Fill a wine-glass or a small tumbler brimful of water, and then carefully drop into it coins, buttons or other bits of metal. The water slowly rises above the top of the glass, appearing to be restrained within a skin which clings at its edges to the glass. The surface becomes more and more convex until at last the skin breaks and the water runs over the edge.

(2) Place a clean, dry sewing needle on the surface of water by lowering it so that both ends will touch the surface at once. In doing this use a fine wire bent in the form shown in Fig. 216. With a little care it can be done. The surface is made concave (Fig. 217) by laying the needle on it, and in the endeavour to contract and smooth out the hollow, sufficient force is exerted to support the needle, though its density is $7\frac{1}{2}$ times that of water. When once

Fig. 216.—Stirrup for placing a needle on the surface of water.

Fig. 217.—Needle on the surface of water kept up by surface tension.

* Full instructions for performing this experiment in the most satisfactory way are given in "Soap Bubbles," by C. V. Boys, page 141.

the water has wet the needle the water rises against the metal and now the tendency of the surface to flatten out will draw the needle downwards.

If the needle is magnetized, it will act when floating like a compass needle, showing the north and south direction.

(3) Dip the upper edge of a rectangular glass vessel (a projection tank) into melted paraffin wax, and then carefully pour in water until its surface curves over at the top of the tank (Fig. 218).

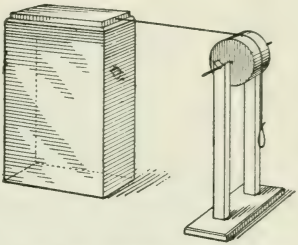


FIG. 218.—Showing tension of the surface film

The surface is rounded at the edges and its tension causes it to shrink to as small an area as possible.

On the surface lay a thin layer of cork, and let a thread, attached to one end of this, pass over a little pulley (made of a pill-box with a needle for axis).

On pulling the string gently the surface is stretched to a greater area, and on letting go it springs back to its original form.

By adding bits of bent wire to the loop on the end of the thread the stretching of the surface can easily be observed, and can be projected on the screen.

A bit of aluminium-leaf or gold-leaf rests quietly on the surface of water, though the former is $2\frac{1}{2}$, and the latter 19, times as dense as the water. In both cases they are not heavy enough to break through the skin on the surface. Remember, however, that this surface layer is not a skin in the ordinary meaning of that term. It is made of liquid, though it is reasonable to suppose that the constitution of the surface layer is somewhat different from that of the rest of the liquid.

**192. Surface Tension in Soap Films.** The surface tension of water is beautifully shown by soap bubbles and films. In these there is very little matter, and the force of gravity does not interfere with our experimenting. It is to be observed,

too, that in the bubbles and films there is an outside and an inside surface, each under tension.

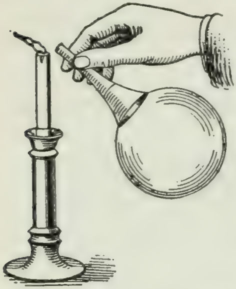


Fig. 219.—Soap-bubble blowing out a candle.

In an inflated toy balloon the rubber is under tension. This is shown by pricking it with a pin or untying the mouth-piece. At once the air is forced out and the balloon becomes flat. A similar effect is obtained with a soap bubble. Let it be blown on a funnel, and the small end be held to a candle flame (Fig. 219). The outrushing air at once blows out the flame, which shows that the bubble behaves like an elastic bag.

There is a difference, however, between the balloon and the bubble. The former will shrink only to a certain size; the latter first shrinks to a film across the mouth of the funnel and then runs up the funnel handle ever trying to reach a smaller area.

Again, take a ring of wire about 2 inches in diameter, with a handle on it (Fig. 220). To two points on the ring tie a fine thread with a loop in it. Dip the ring in a soap solution,* and obtain a film across it with the loop resting on the film. This film is a thin layer of water bounded by two surfaces, the soap making it more permanent. Now puncture the film within the loop. The film which is left contracts, becomes as small as possible and thus draws the loop into a circle, since the area of a circle is greater than that of any other surface having an equal perimeter.

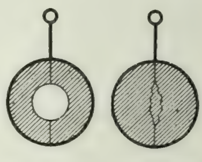


Fig. 220.—A loop of thread on a soap-film.

As in the bubble, the surface acts like a stretched sheet of india-rubber, but there is a further difference between them. The tension in the sheet of rubber depends on the amount of stretching, and may be greater in one direction than in another; whereas the tension in the soap-film remains the same however much the film is extended, and the tension at any point is the same in all directions along the film.

* See method of preparation, page 269.

**193. Surface Energy.** To inflate a rubber balloon or a bicycle tire, or to blow a soap-bubble requires an expenditure of work; and when these bodies contract they exert a force and thus can do work.

The fact that a soap-film will contract and exert a force can be well shown as follows:—Bend a wire into a rectangular shape (Fig. 221) and dip it into a soap solution. On taking it out it is covered with a film. Hold it horizontal and across it lay a thin straight wire *mn*; then puncture the side *Q* of the film. The two surfaces (the upper and the lower) of the film *P* which is left exert a tension on the wire in the direction shown by the arrow, and draw the wire over to the end *ab*, thus reducing the area of the film to as small dimensions as possible.

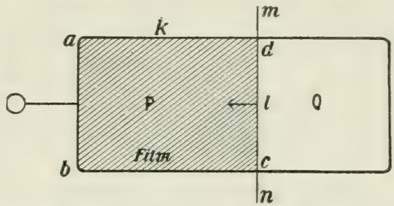


FIG. 221.—The wire *mn* is drawn to the left by the tension of the film.

Tie a thread to the ends of the wire and pull the wire out and release it again and again.

By an experiment somewhat similar to this the magnitude of the surface tension can be roughly determined (see Section 198 below).

It is evident that the greater the width *cd* of the rectangle, the greater will be the entire force drawing the wire in the direction of the arrow, *i.e.*, at right angles to the axis of the wire.

Let the width *cd* of the rectangle be $l$ cm., and let the tension exerted by each surface of the film, on each cm. of the wire, be $T$ dynes. Then the entire tension exerted upon the wire by the two surfaces of the film = $2Tl$ dynes. If the length *ad* of the film is $k$ cm., the work which the film $P$ can do in contracting = $2Tlk$ ergs.

Just as we say that a bent bow or a stretched sheet of rubber possesses potential energy, so we can say that the film possesses potential energy, and its amount is equal to the work which it can do in contracting, that is, $2Tlk$ ergs. But

its area = $2lk$ sq. cm. Hence the potential energy per sq cm. = $2Tlk \div 2lk = T$ ergs.

Again, if one takes hold of the wire and moves it to the right (Fig. 221) a distance $x$ cm., thus increasing the area of the film by $2lx$ sq. cm., the work which one does is $2Tlx$ ergs, and the work done per sq. cm. = $T$ ergs.

Hence we have the relation:—*The measure of the surface tension of a liquid is equal to the measure of its potential energy per sq. cm. of the surface; or it is equal to the measure of the work done in enlarging the surface of the liquid one unit of area.*

Remember, also, that the surface tension is measured in dynes across a linear centimetre.

The quantity $T$ is the SURFACE TENSION.

The question of surface tension arose chiefly through the consideration of the rise of liquids in capillary tubes, *i.e.*, tubes so fine as to admit only a hair (Latin, *capillus*, a hair); but the subject of surface tension is a very broad one with numerous applications. Hence, it is better to use the name *surface tension* than the name *capillarity*, by which it is sometimes known.

**194. Angle of Contact or Capillary Angle.** If a plate of glass is held vertically in water, the liquid in the surface,


(*a*) Water. (*b*) Mercury.

FIG. 222.—Angle of contact of water and mercury.

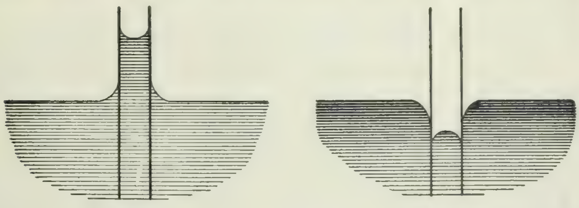
where it touches the glass, is drawn up above the level of the general surface. (*a*, Fig. 222). If the glass be lifted from

the water some water will cling to it. The water is said to wet the glass. If the glass be held in mercury the liquid surface in contact with the glass is depressed (*b*, Fig. 222), and if the glass be removed from the mercury none of the mercury will adhere to it. Mercury does not wet glass.

The angle which the tangent to the liquid surface where it meets the surface of the solid makes with the common surface of the liquid and the solid is called the *angle of contact* or the *capillary angle* (Fig. 222).

The size of this angle depends on the third medium, above the liquid. Thus if oil is used instead of air the angle is much altered. It also depends very materially on the condition of the surfaces. The slightest contamination on the surface of water or on the solid will alter the angle considerably. Figure 222*a* illustrates the usual condition for water and glass. With perfectly clean water and glass the angle of contact $BAC$ is very small, probably zero, but with slight contamination it may reach 90°, *i.e.*, it does not rise on the surface of the glass at all. Figure 222*b* illustrates the effect with mercury and glass. Here the angle of contact is obtuse, varying from 129° to 143°.

**195. Level of Liquids in Fine Tubes.** If a small glass tube is held upright in water the liquid rises within the tube and, both inside and outside, the surface curves upwards where it



Water. Mercury.

FIG. 223.—Level of liquid in a fine tube.

touches the glass. (Fig. 223). This effect can be observed more easily if a little colouring matter (magenta, for example)

is added to the water. If mercury is used instead of water, the liquid within the tube takes a lower level than that outside, and the surface at the glass curves downwards instead of upwards. In these experiments the glass should be perfectly clean.

It is interesting to observe the effect with tubes of various sizes. Take a set having internal diameters ranging from (say) 2 millimetres to the finest obtainable, and hold them in water (Fig. 224). It will be seen that in each of them the level is above that of the water in the vessel, and that the

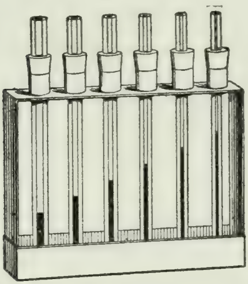


**Fig. 224.—Showing the elevation of water in capillary tubes.**

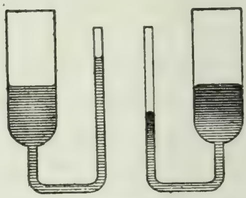


**Fig. 225.—Contrasting the behaviour of water (left) and mercury (right).**

finer the tube the higher is the level. With alcohol the liquid rises, though not so much, but with mercury the liquid is depressed. The behaviour of mercury can conveniently be shown in a U-tube as in Fig. 225.

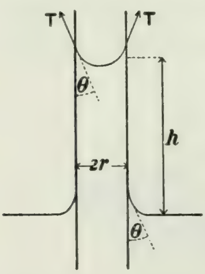



**Fig. 226.—Surface tension supports the column in the tube.**

**196. Calculation of the Rise of Liquid in a Tube.** Consider a tube held vertically in a liquid which wets it. The liquid rises on the outside slightly, but on the inside to a considerable height (Fig. 226).

The phenomenon is "explained" by stating that the attraction of the molecules of the liquid for those of the glass is greater than the attraction of the molecules of the liquid

for each other. The surface of the liquid meets the glass along an inner circumference of the tube, and the attraction exerted, across this line, between the surface molecules of the liquid and those of the glass, is sufficient to support the raised column.

Let $T$ denote the surface tension in dynes per cm.
$r$ " " radius of tube in cm.
$h$ " " mean height of column in cm.
$\rho$ " " density of the liquid in gm. per c.c.
$\theta$ " " angle of contact.

The force $T$ acts in a direction making an angle $\theta$ with the vertical; hence its component in the vertical is $T\cos\theta$.

The surface of the liquid pulls the inner surface of the tube inwards and downwards, acting in the direction of the tangent to the liquid surface where it touches the tube, and the reaction of the tube lifts the liquid upwards.

The length of the line of contact of the liquid and inner surface of the tube $= 2\pi r$ cm., and hence the total force upwards in the direction of the axis of the tube

$$= 2\pi r T\cos\theta \text{ dynes.}$$

This balances the weight of the raised column of liquid, which $= \pi r^2 h\rho g$ dynes.

Equating the total force upwards to the total force downwards, we have

$$\pi r^2 h\rho g = 2\pi r\, T\cos\theta,$$

$$\text{and } T = \frac{h\rho r g}{2\cos\theta}, \text{ or } h = \frac{2T\cos\theta}{\rho r g}.$$

From this we see that $h \propto \dfrac{1}{r}$, or *the height to which the liquid is drawn up is inversely proportional to the radius of the tube.* With a very small tube the rise of the liquid may be considerable.

In a glass tube of radius 1 mm. the water rises about 1.4 cm. Hence in one of radius $\frac{1}{1000}$ mm. the rise would be 14 metres.

It has been surmised that the distribution of sap in plants is partially due to capillary action, but this will not account for the *rate* at which water rises in trees.

The tube of a barometer should be large, otherwise a correction for capillarity is necessary. If the tube has a diameter of 2 mm. the mercury is depressed 4.6 mm., but if it is 2 cm. (about 0.8 inch) or greater, the correction for depression is so small that it may be neglected.

**197. Attraction and Repulsion between Bodies on the Surface of Water.** It has often been noticed that bubbles, small sticks and straws floating on still water appear to attract each other. They gather in groups or become attached to the edge of the containing vessel. This effect can be easily illustrated by means of two discs sliced off a cork, placed on the surface of the water. When they get within a certain distance (about 1 cm.) they run together. If the water does not wet either body they will still attract each other; but when two bodies, one of which is wet and the other is not, are brought near together they will appear to repel each other.

These actions can be explained in the following way. Let $P$ and $Q$ be two plates suspended by threads near together in a liquid.

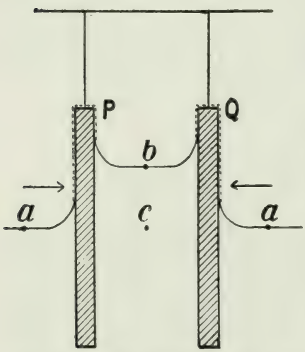


Fig. 227.—If both plates are wet they are attracted.

First, let the liquid wet both plates (Fig. 227). Let $a$, $a$ be points on the surface of the liquid at its ordinary level, away from the plates, and $c$ be a point on the same level in the liquid between the plates. As the liquid is in hydrostatic equilibrium the pressures exerted by the liquid at these three points must be equal, each being equal to that of the atmosphere. If one ascends from $c$

towards $b$ the pressure diminishes, while if one descends below $c$ the pressure increases. Consequently the pressure of the liquid between the plates is less than that of the atmosphere which presses on the outer surface of the plates, and the plates will be pushed together, as indicated by the arrows.

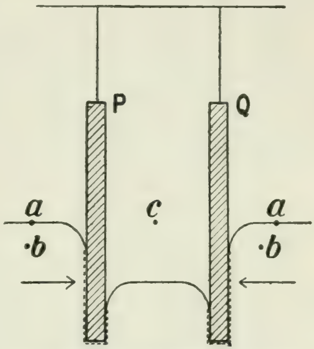


FIG. 228.—If both plates are not wet they are attracted.

Next, take two plates which are not wet by the liquid (Fig. 228). These may be plates of glass, or aluminium, covered with paraffin. Here the pressures at $a$, $a$, as also at $c$ between the plates, are all equal, each being that of one atmosphere. Hence the pressures at $b$, $b$ in the outer liquid are greater than the pressure on the same level between the plates, and the plates will consequently be pushed together as before.

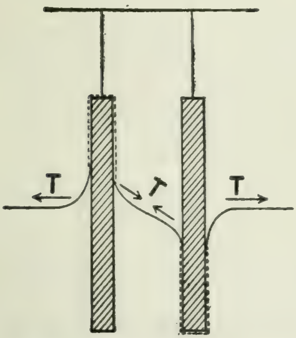


FIG. 229.—If one plate is wet and the other is not they are repelled.

Finally, let the liquid wet one plate but not the other.

When the plates come sufficiently near together the surface of the liquid between the plates assumes the form shown in Fig. 229. It then has no level portion.

The tension of the surface on the outside pulls the plates with the force $T$ in the horizontal plane. This tends to draw the plates apart. The tension of the surface between the plates exerts an equal force, but in a direction making, let us say, the angle $a$ with the horizontal. Resolving this in the horizontal plane, the force drawing the

two plates together is $T \cos \alpha$, and as this is smaller than $T$ acting in the opposite direction, the plates will be drawn apart.

The above results can be neatly illustrated in the following way:

Obtain two hollow glass balls about 2 cm. in diameter and cover one with paraffin. Attach a weight to each (with wax or otherwise) so that they may float rather more than half immersed. They will appear to repel each other. If both are clean glass or both paraffined they will attract each other.

All the above results can be deduced at once from the principle that the potential energy of a system of bodies tends to a minimum.

When a clean plate is wet by a liquid in which it is held a film of the liquid spreads all over it, not just up a short distance. In Fig. 227 the entire surface of the liquid consists of the surface as ordinarily considered, and in addition a thin film extending over the plates as indicated by the dotted lines. The plate acts just as if a layer of paper or cloth covered its surface. In Figs. 228, 229 the entire surfaces are also shown. When the plates approach, the liquid rises in Fig. 227 and falls in Fig. 228. When they separate the portion between in Fig. 229 becomes horizontal. In each case there is a reduction in area, and therefore in potential energy.

**198. Measurement of Surface Tension.** Perhaps the simplest way to obtain an approximate value of the surface tension for water is to use a bent wire of the form shown in Fig. 230, with a wire $mn$ laid across it. Dip it in a soap solution, and when a film adheres, tilt the wire until its plane is as nearly vertical as possible. The wire will run down until the tension of the film on it just balances its weight. If it is not heavy enough, a small weight may be attached to the middle of it by means of a fine thread.

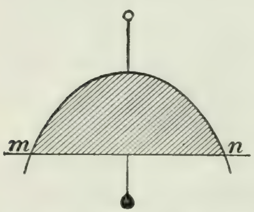


FIG. 230.—Surface tension supports the wire and the weight.

Let its weight be $w$ grams and the length of the part between the two points of the bent wire on which it rests be $l$ cm. Then, as there are two surfaces exerting a tension, each surface supports $\frac{1}{2}w$ grams. Hence

the tension of the surface per cm. = $\frac{1}{2}\frac{w}{l}$ grams, or = $\frac{1}{2}\frac{wg}{l}$ dynes (where $g = 980$).

A more common, as well as more accurate, method is to observe the rise of the liquid in a capillary tube and then use the formula given above (Sec. 196). This requires the angle of contact to be known. For water and other pure liquids it may be taken as zero, in which case $T = \frac{1}{2}\, h\rho r g$. Other methods have been devised but there is not space in this book to describe them.

The values of the surface tensions of various liquids when in contact with air, water or mercury are given in the following table:

TABLE OF SURFACE TENSIONS AT 20° C. (In Dynes per cm.)

| Liquid. | Density. | Tension of Surface Separating the Liquid from | | |
|---|---|---|---|---|
| | | Air. | Water. | Mercury. |
| Water | 1 | 81 | .. | 418 |
| Mercury | 13.6 | 540 | 418 | ... |
| Carbon Bisulphide | 1.27 | 32 | 42 | 372 |
| Chloroform | 1.49 | 31 | 30 | 399 |
| Alcohol | .79 | 26 | .. | 399 |
| Olive Oil | .91 | 37 | 21 | 335 |
| Turpentine | .89 | 30 | 12 | 250 |
| Petroleum | .80 | 32 | 28 | 284 |

**199. Small and Large Bubbles.** The following experiment illustrates the fact that the pressure within a small bubble is greater than that within a larger one.

Take a capillary tube bent in the form of a U, and fill it by drawing water through it. Then put a large drop of water on one end of the tube and a small drop on the other.

Which gains in size? Why?

In place of the bent tube two straight pieces may be joined with a piece of rubber tubing.

**200. Experimental Illustrations of Surface Tension.** (1) If small fragments of camphor are placed upon the surface of clean water they at once move about almost as if alive. The camphor dissolves slowly in the water, and the surface tension of a solution of camphor is smaller than that of pure water. Consequently if the camphor dissolves more rapidly at one side of the fragment than at the other, the surface tension on the first side will be diminished and the greater surface tension on the other side of the fragment will draw the fragment away.

This can be easily shown by rinsing a glass at the tap, filling it with water, and then scraping with a pen-knife small fragments of camphor which are allowed to fall upon the surface. They dart about, but if the surface of the water be touched with the finger the movements will likely cease, being arrested by the grease from the finger communicated to the water. Very little grease is required. Lord Rayleigh found that 0.8 milligram of olive oil on a circular surface 84 cm. in diameter was sufficient. From this he calculated that an oily film 2 millionths of a millimetre in thickness is sufficient to arrest the camphor movements.

(2) The surface tension of alcohol is much smaller than that of water (see Table above). Scatter lycopodium powder over the surface of a thin layer of water, and then place a drop of alcohol on the surface. At the place where the alcohol is, the tension is immediately reduced, equlibrium is destroyed and the superficial film of the liquid is set in motion. This will be shown by the lycopodium powder. If the water is very shallow this motion will drag the water away from the place where the alcohol is, and will lay bare the bottom of the vessel.

(3) Rinse a glass under the tap and fill it with water, and scatter lycopodium powder as in the last experiment. Now touch the middle of the surface with a finger which has been rubbed against the hair. Enough grease will come off the finger to contaminate the water, and reduce its surface tension, and the surface layer will be drawn away from the place where the finger touched the surface. A patch will be entirely free from the powder.

(4) Hold a drop of ether close to the surface of water. The vapour of the ether condenses on the surface, reduces the surface tension and causes an outward motion, producing a dimple on the surface.

(5) Pour clean water on a level board so as to form a shallow pool 2 inches wide and 2 or 3 feet long. Near its middle lay a scrap of paper and on one end place a cake of soap (Fig. 231).

The paper is soon seen to move along the surface away from the soap.

Here the soap in dissolving weakens the surface film, and the tension in the other portion draws the surface layer away from the soap.

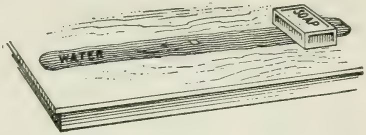



FIG. 231.—Soap reduces the surface tension.

(6) Cut a piece of paper into the shape of a fish (Fig. 232). On its tail put a drop of amyl alcohol (or of fusel oil) and place it on the surface of clean water. The fish swims about in a very interesting way. Why does it stop at last?

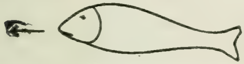


FIG. 232.—The paper fish moves to the left.

Cut the shape of an "S" from paper (Fig. 233), and put a drop of amyl alcohol on each end of it. It spins about like a pin-wheel.

By using a shallow dish and a vertical attachment these motions can be projected on the screen.

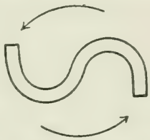


FIG. 233.—The paper spins about.

Where the amyl alcohol is placed, the surface film is weakened, and the tension in the other parts of the surfac draw the surface film away from these places, causing the motion of the pieces of paper.

(7) *A* is a glass bulb, with a smaller one beneath it, on the end of a small glass tube (Fig. 234). Mercury in the lower bulb makes the tube float upright in water. At *d* is a piece of wire gauze attached (by wax) to the small tube.

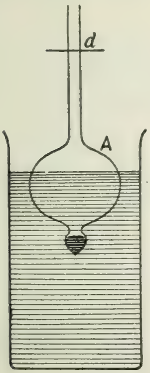



FIG. 234.—Surface tension upon *d* can hold the bulb down.

When floating in water a considerable part of the large bulb *A* is above the surface, and it requires quite a force to push it down.

Now press it down until the gauze touches the surface. The water wets it and clings to each wire. This tension will be sufficient to hold the tube down in the water.

While down put a few drops of ether (or alcohol) on the surface of the water. At once the gauze breaks away and rises as shown in the figure. Adding the ether weakens the surface tension.

The size of the piece of gauze will depend on the size of the bulb and the amount of mercury holding it down; but it will be easy to find suitable dimensions.

FIG. 235.—Simpler form of apparatus, but not so satisfactory.

A simpler form of the apparatus is shown in Fig. 235. It consists of a hat-pin through a cork, with a piece of lead to keep it upright. In place of the wire gauze a cardboard disc *d* may be used. This can be pared down until it is just able to hold the cork down.

In this case the disc is held by the tension exerted only around its edge, while with the gauze the surface clings to each wire and so the total tension is greater.

(8) Two simple methods of removing grease from cloth are based on surface tension. The fatty oils have a greater surface tension than benzine. Hence if one side of a grease-spot on a piece of cloth is wetted with benzine the tension is greatest on the side of the grease. Consequently the portions consisting of a mixture of grease and benzine will be drawn towards the grease and away from the benzine.

In order to cleanse the grease-spot, first apply the benzine in a ring all round the spot, and gradually bring it nearer to the centre of the spot. The grease will be chased to the middle of the spot and if a fibrous substance such as blotting-paper is placed in contact with the cloth, the grease will escape into it. If the benzine had been applied to the centre of the spot the grease would have been spread out into the cloth.

The second method is to apply a hot iron to one side of the cloth and blotting-paper to the other. The surface tension diminishes as the temperature rises. Hence the grease draws away from the hot iron and escapes into the blotting-paper.

Try these two methods.

## PROBLEMS

1. Explain why the end of a stick of sealing-wax when held in a flame becomes rounded.

2. Why are small drops of mercury resting on a horizontal surface more nearly spherical than larger ones?

3. Calculate the work done in blowing a soap bubble 10 cm. in diameter.

4. Two drops of mercury 1 mm. and 2 mm. in diameter, respectively, coalesce. Compare the pressure within the liquid due to surface tension in the two original drops and in the one formed by their union.

5. When a soap bubble bursts the water from it is thrown in every direction. Account for this.

6. Calculate the heights to which pure water, alcohol and turpentine will rise in capillary glass tubes 1 mm. in diameter. (For surface tensions and densities, see Table in Sec. 198).

7. One soap bubble, 8 cm. in diameter, is on one end of a U-tube, and another, 3 cm. in diameter, is on the other end. If there is a free passage from one to the other, which one will increase in size?

8. Two parallel plates, separated by a space $d$, stand vertically in a liquid, having density $\rho$, surface tension $T$ and angle of contact $\theta$. Show that the height $h$ to which the liquid will rise is

$$h = \frac{2T \cos \theta}{g\rho d}.$$

Compare this with the height in a cylindrical tube whose diameter is equal to the distance between the plates.

(Consider the equilibrium of a portion of the liquid between the plates 1 cm. in length.)

# CHAPTER XXIV

## Some Applications of Surface Tension

### I.—*In Agriculture*

**201. Action of the Water in the Soil.** Surface tension undoubtedly plays an important part in supplying moisture to the soil. If a lump of loaf-sugar is placed with one corner in water, the liquid gradually rises and spreads until it completely permeates the lump. The soil behaves similarly. Like the sugar, it is composed of small particles with spaces between them. If water falls upon it, some will pass down through it and run away, but a considerable amount will cling to the surfaces of the particles and gather in the spaces between them. If water is supplied at the side or underneath, as sometimes in irrigation, the water spreads upwards and throughout the mass and much of it remains there on account of surface tension.

**202. Evaporation at the Surface.** The water at the upper surface evaporates, and its place is supplied, as far as possible, by water drawn up by surface tension. The depth from which water can be raised by capillary action differs in different soils and for different conditions of the soil. The finer the texture is, the higher the possible rise.

Experiment has shown that capillary movement can take place through a column 5 feet in height. In this case the soil must be moist to begin with. On the other hand, if the soil is well dried the capillary rise may be less than 1 foot.

It has been shown, also, that evaporation from soil takes place entirely from the layers very near the surface.

**203. Retaining the Moisture in the Soil.** The problem of preventing the rise of the water to the surface and its loss by evaporation is a very important one, especially in those countries where there is no rainfall for months in succession or where the entire yearly rainfall is small, not more than ten or fifteen inches.

It has been found that if a soil after a rain is exposed to very arid conditions, with a high surface temperature and a hot dry wind, the soil at the surface will lose water much faster than it can be brought up from below by capillary action, and a layer of dry soil may be formed on the surface which will be so dry that it will act as a protecting covering.

One of the most effective means of conserving soil moisture, however, is by "mulching," *i.e.*, by covering the surface of the soil with some loosely packed material, such as straw, leaves or stable manure. The spaces between the parts of such substances are too large to admit of capillary action, and hence the water conveyed to the surface of the soil is prevented from passing upwards any further, except by slow evaporation through the mulching layer. A loose layer of earth spread over the surface of the soil acts in the same way, and the same effect may be attained by hoeing the soil or stirring it to the depth of one or two inches with harrows or other implements.

In the semi-arid regions of the United States, Argentina, the Canadian West and other countries, in which the average rainfall lies between 10 and 20 inches, good crops of selected grain can be grown by proper cultivation.

In some cases only one crop can be grown in alternate years, the year of no crop being used to preserve the moisture in the soil. In our Canadian West during a dry season it is found that land which was "summer fallowed" the year before produces the heaviest crop.

## II.—*In Dyeing*

**204. The Process of Dyeing.** There is great variety both in the materials to be dyed and in the colouring matter to be applied to them, and we are not surprised to find that the phenomena observed in the process of dyeing are very complicated. No single hypothesis as to the nature of the action taking place will account for all the results obtained.

In some cases chemical action undoubtedly takes place; in others the process is probably physical, and there is evidence that capillary action or surface tension is of great importance.

The experiments which follow suggest ways in which the dye is transferred to the fabric:

(1) Into vessel $A$ (Fig. 236) pour clean water, and into vessel $B$ a weak solution of saponine (1 gram of saponine to 500 c.c. of water).

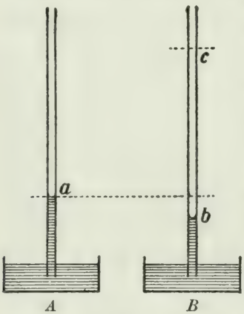


Fig. 236.—Showing concentration in the surface layer.

Hold a capillary tube in $A$. The water rises to a level $a$. Then remove the tube and hold it in $B$. The liquid now rises only to level $b$, considerably below level $a$.

This shows that the saponine solution has a smaller surface tension than clean water has.

Now draw the solution in $B$ up to the level $c$ and let it go suddenly. The column rapidly falls to level $a$ and then settles less rapidly down to $b$.

While falling from $c$ to $a$ the liquid at the surface is being renewed constantly, and so the constitution of the surface layer is very approximately the same as that of the solution generally, which is little different from pure water. However, in a few seconds some of the saponine concentrates at the surface and produces a reduction in the surface tension. This gradual reduction is seen in the slow sinking of the column to its final height.

From this experiment we get a very important result. *When a substance, on being dissolved in water, reduces its*

*surface tension, there is a concentration of the substance in the surface layer.*

This conclusion, indeed, is predicted from theoretical considerations based on the laws of thermo-dynamics, and it can be verified by many other experiments. A system always endeavours to change so as to have the least possible potential energy. When such a substance goes into the surface it reduces the surface energy, thus contributing to a reduction in the total potential energy.

It is to be observed that the surface layer is excessively thin, so that the actual amount of matter concentrated there need not be great.

(2) Stick a pin in a piece of thin cork, tie a thread to it and then lay the cork on the surface of clean water in a large photographic tray. Let the thread pass over a pill-box pulley having a needle for its axis.

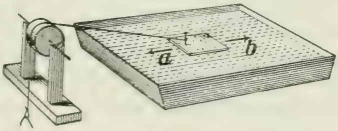


FIG. 237.—Illustrating concentration in the surface layer.

The tension of the surface is the same in all directions in its plane, and hence is the same in the two directions $a$ and $b$ (Fig. 237). The slightest force on the cork will move it in the direction in which the force acts.

Add a small weight (a bit of bent wire) to the end of the thread. If this is sufficient to overcome the friction of the thread on the pulley, the cork will move in the direction $a$ with a certain speed.

Next, in place of the pure water use a solution of methyl violet (1 gram to 4 litres), which reduces the surface tension of the water. Allow the solution to stand for a few minutes before placing the cork on it.

Observe the rate at which the cork is drawn aside. It is not so great as before.

This arises in the following way. When the cork is displaced in the direction $a$ it exposes new surface at $b$. This at first is practically the same as pure water, which has a greater surface tension. Hence the surface tension in pulling the cork in direction $b$, is greater than that in direction $a$, and the motion takes place only as the newly exposed surface becomes concentrated and so is the same as on the other side of the cork.

This explanation can also be verified by removing the weight and then pulling the thread by the hand. On letting go, the water-surface tension at $b$ will draw back the cork in that direction.

Place a match on the surface and drive it endways. It sticks as though there were a scum on the surface.

Finally stir the solution and try the experiment with the cork and the match at once, *i.e.*, before the surface has become concentrated. It acts like that of pure water.

FIG. 238.—Froth of a solution of methyl violet above the liquid.

(3) Make a solution of methyl violet (1 gram to 4 litres of water). About one-third fill a large separating funnel (Fig. 238). Shake vigorously, causing froth to gather above the liquid.

Let it stand 4 or 5 minutes, to allow the liquid between the bubbles to run down. Then drain off all the liquid which has collected. Call this solution $A$.

Next, let it stand for 4 or 5 minutes more, to allow the froth to settle, and then draw off the liquid formed from it. Call this solution $B$.

Now make a solution of 1 c.c. of $A$ to 20 c.c. of water, and pour in one side of a double glass vessel with plane sides (Fig. 239).

Make a solution of 1 c.c. of $B$ to 20 c.c. of water, and pour in the other side of the vessel.

Place the vessel in the lantern, and project on the screen, or hold it in front of a piece of white paper or up to the window.

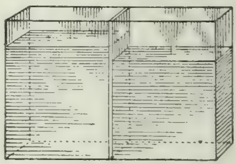


FIG. 239.—The solution from the froth is of a deeper colour than the other.

It will be found that the second solution is of a deeper colour than the first.

This result is explained as follows:—Methyl violet when dissolved in water reduces the surface tension of the water, and any substance which does that concentrates at the surface. The bubbles of froth have much surface compared to their mass, and the methyl violet is concentrated on their surfaces.

Hence, the liquid formed from the bubbles contains more of the dye per c.c. than does the liquid first drained off.

In performing this experiment be sure that the proportions in the two solutions compared are accurately the same as the final difference in colour is only slight. Use a 1 c.c. and a 20 c.c. pipette, previously rinsing out with some of the liquid to be measured.

If froth does not form on the solution, make a new one with fresh water.

(4) Place a drop of a weak solution of red ink on white filter or blotting paper, and observe how it spreads. When the action has ceased it will be found that the red colouring matter has spread a certain distance, but the water in the solution has gone a considerable distance farther.

Again, place drops of a dilute solution of barium hydroxide and an alcoholic solution of phenol phthalein near together on filter paper and allow them to spread into each other. The beautiful pink colour resulting when these two substances combine does not appear at the edge of the drop of barium hydroxide, but some distance within it. The outer portion of the round spot coming from the barium hydroxide is pure water, the solid having been left behind, nearer the centre of the spot.

Many solutions of salts or of dyes exhibit this phenomenon, the water diffusing amongst the fibres of the paper and leaving the dissolved substances behind upon the fibres.

Still more striking and beautiful effects are obtained with solutions of two dyes. Make a dilute solution of picric acid and crocein scarlet, and put several drops on white filter paper (supported on the top of a beaker). When the spreading has ceased there will be seen a large spot of red with a yellow fringe, and this surrounded by clear water. The picric acid diffuses more freely than the scarlet.

A solution of acid magenta and indigo sulphate of soda will give an indigo spot fringed with magenta.

Instead of putting drops on the paper, a strip of filter paper may be suspended with its lower end in the solution. The clear liquid rises highest, and usually one colour higher than the other, if two are present.

These experiments are easy to perform, and the results are beautiful and suggestive.

The action illustrated in the above experiments is almost certainly present in some cases of dyeing. The coloured

solution comes in contact with the surface of the material to be dyed; the tension of the surface there is reduced and the colouring matter concentrates at the surface and is deposited on the material. Probably, with some materials the water of the coloured solution passes freely through the capillary spaces, leaving the particles of the dye behind on the material.

## III.—*In Filtration*

**205. The Action of Filters.** Filters can be divided into two classes.

In filtering solid impurities, or a precipitate, from a liquid, the filtering material (paper, cloth, sand, etc.) has interstices through which the liquid can pass but the solid particles cannot. Surface tension does not enter here.

It has been known for many years that neutral filters, such as sand in layers, will remove colouring matter and, to some extent, salts in solution. This filtering action is undoubtedly intimately connected with the large amount of surface of the particles presented to the liquid, the greater the surface the stronger being the action.

If a dilute solution of acetic acid is filtered through fine white sand, nothing but pure water will percolate through, the whole of the acid being kept back by this action. (Try this). Dilute solutions of various other substances show a similar action.

In experiment 4, above, the red matter in the ink, and the solid in the solution of barium hydroxide were held back while the pure water flowed on.

The action in these cases is certainly a surface phenomenon, probably explainable in the same manner as the phenomena of dyeing just described above.

It may be well to remark, however, that in the purification of water by filtration other considerations enter. For a long

time this was looked upon as a mechanical process of straining out the solid particles and thus rendering turbid water clear. But now it has been shown that in sand-filtration of water on a large scale an essential feature is the presence, in the upper surface layer of the sand, of colonies of bacteria forming jelly-like masses. Not until a fine film of mud and microbes has been formed upon the surface of the sand are the best results obtained.

### *Solution for Soap-Films and Bubbles*

A solution of Castile soap and rainwater, with some Price's glycerine added to make the film last longer, will probably answer all purposes; but for the very best results a specially prepared solution is desirable.

The following is the recipe recommended by Reinold and Rücker and by Boys. Fill a stoppered bottle three-fourths full with distilled water, add one-fortieth by weight of fresh oleate of soda, and leave for a day to dissolve. Nearly fill the bottle with Price's glycerine, and shake well. Leave the bottle a week in a dark place, and then with a siphon draw off the clear liquid from under the scum into a clean bottle, add a drop or two of strong ammonia solution to each pint, and keep carefully in the stoppered bottle in a dark place, filling a small working bottle from it when required, but keeping the stock bottle undisturbed and never putting any back into it. Do not warm or filter the solution and never leave the stopper out or expose the liquid to the air.

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For further information on surface tension consult:

C. V. Boys, *Soap Bubbles.* (Full of fine experiments with instructions for performing them).

Edwin Edser, *General Physics for Students,* Chapters IX and X.

Maxwell, *Theory of Heat*, Chapter XX.

# CHAPTER XXV

## THE FLOW OF FLUIDS

**206. Services Obtained from Flowing Fluids.** From an economic point of view the study of the laws of flowing fluids is of great importance. Immense stores of energy are present in the waters of our rapid rivers, and in order to utilize it we must know the laws according to which they move. In the systems of waterworks in our cities and towns the water is pumped into iron pipes, from which it is drawn for domestic use, for running elevators and water motors, and for other industrial purposes.

Air and steam, forced through pipes, are used for actuating drills, for driving turbine and ordinary engines, for applying the brakes on railway trains and street cars, for heating buildings, and for numerous other purposes.

Again, our winds are currents in the air, their motion being shown in the swaying of trees, and in the sweeping onward of clouds in the sky or of clouds of dust and smoke at the surface of the earth.

It is, therefore, evident that a knowledge of the laws in accordance with which fluids move is of the highest value. The phenomena, however, are very complicated, and the determination of their laws is a matter of difficulty.

**207. Unsteady and Steady Motion.** Consider the water moving forward in a river or flowing in a pipe which has a varying diameter, and which, perhaps, has a varying direction.

If we could colour the particles of water in successive cross-sections, thus rendering it possible to trace their motions, we would probably be surprised to see the extraordinary way in which some of them eddy about instead of simply moving forward. The particles near the shore and bottom of the river, or near the surface of the containing pipe, are continually being thrown into eddies. Such motion is *unsteady*.

But it is evident that if the source of supply is perfectly constant, the flow will be continuously uniform, and the particles in one cross-section will follow approximately the same paths as those in the preceding one. For example, if a vessel is kept constantly full by allowing water to run uniformly into it from a reservoir, and if the water is permitted to escape from an opening anywhere in the vessel, the motion of the particles which pass any fixed point in the vessel will be the same at all times. If a water-sprite could stand in the liquid and mark each particle as it came along in a certain direction to that point, all of these particles would be seen to follow the same curved path.

Motion such as that just described is called *Steady Motion or Simple Flow; and the lines imagined to be drawn in the liquid so as to be at each point in the direction of the flow, or, in other words, the lines along which the particles travel, are called Stream Lines.*

Thus, consider steady motion through a pipe with a

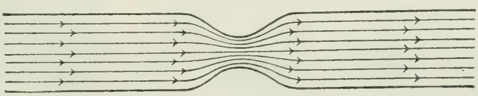

FIG. 240.—Stream lines in a pipe.

contraction, or *throat*, in it (Fig. 240). The fine lines indicate the form of the stream lines.

Let us consider the stream lines drawn through a closed curve $a$ (Fig. 241) in the liquid. A particle of the fluid which commences to move along one of these lines will continue to do so. It is evident that these lines of flow taken together form a tube; it is called a *Tube of Flow.*

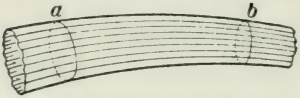


Fig. 241.—A tube of flow.

Since the line of flow, or stream line, passing through a point indicates the direction of flow at that point it is evident that two lines of flow cannot cross each other. If they did the resultant motion at the point of intersection would have two directions, but in steady motion the movement of the particles at a point are continually in a single definite direction.

Such being the case, there can be no flow across the bounding walls of the tube, and the particles which are within the tube at one time will continue within it. The particles composing a cross-sectional layer will continue to be a cross-sectional layer.

**208. Height to Which a Jet will Rise.** *Experiment.*—Arrange apparatus as in Fig. 242. The water escapes from a small orifice. The jet rises nearly to the level of the free surface of the liquid in the vessel, and we suspect at once that if there were no losses through friction the jet would rise exactly to that level.

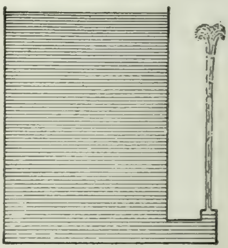

Fig. 242.—Height of a jet of water.

Now if a body falls through a height $h$ it attains a velocity $v$ where $v = \sqrt{2gh}$. In the same way, if the body is thrown upward, and rises to a height $h$ the initial velocity $= \sqrt{2gh}$.

In the case of the jet of liquid, if $h$ is the depth of the orifice below the free surface in the vessel, the velocity of efflux $= \sqrt{2gh}$.

This relation is rigidly true only for a perfect liquid, or one which flows without friction.

**209. Flow of Liquid from an Opening in a Vessel.** The result obtained in the last section can be deduced from the principle of energy.

Let the opening be at a distance $h$ cm. below the surface of the liquid (Fig. 243). Let the density of the liquid be $\rho$, the area of the free surface be $A$ sq. cm., and the velocity of the outflowing liquid be $v$ cm. per sec., that is, a small speck of dust in the liquid at the opening would be carried forward at this rate.

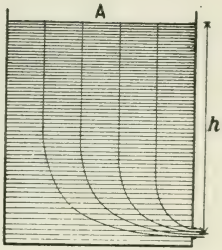

Fig. 243. – Calculation of velocity of outflow.

Suppose that in a very short time $t$ the level of the free surface falls a very small distance $x$ cm. Then the

$$\text{Volume of escaped liquid} = Ax \text{ c.c.}$$
$$\text{Its mass} = A\rho x \text{ grams.}$$
$$\text{And if its velocity} = v \text{ cm. per sec.,}$$
$$\text{Its kinetic energy} = \tfrac{1}{2}\rho A x v^2 \text{ ergs (see Sec. 101).}$$

This kinetic energy must be gained at the expense of the potential energy of the liquid. Now each layer has fallen through a height $x$ cm., or the entire volume has fallen through this distance. The mass is $hA\rho$ grams, and its weight is $hA\rho g$ dynes. Hence the

$$\text{Decrease in potential energy} = hA\rho g x \text{ ergs.}$$

Therefore,
$$\tfrac{1}{2}\rho A x v^2 = hA\rho g x,$$
$$\text{or } v^2 = 2gh,$$
$$\text{and } v = \sqrt{2gh};$$

that is, *the velocity is the same as that which would be acquired by falling through the distance of the opening below the free surface.*

This is known as *Torricelli's Law.* It was formulated by him in 1643, more than 200 years before the principle of the conservation of energy had been established.

This result was obtained on the assumption that the liquid was perfect, that there was no friction in the passage of one layer over another, or in other words, that it had no *viscosity.* As a matter of fact, water, ether, alcohol, mercury and such liquids possess very little viscosity, and the law is very nearly fulfilled by them. In the case of water the velocity is not quite that given by theory, a small amount of the energy being transformed into heat. The velocity is approximately $\frac{97}{100} \times \sqrt{2gh}$.

**210. The Contracted Vein.** The rate at which liquid is escaping, however, cannot be found from knowing the area of the opening and the velocity $v$ of the efflux. Just outside the opening the jet contracts somewhat, and we must take the area of a cross-section where it is least. The size and shape of the cross-section is modified by the shape of the opening, and the area in general can be determined only by experiment. When the opening is a sharp-edged round orifice in a plane surface the area of the jet is on the average $\frac{64}{100}$ of that of the opening, or the cross-section of the jet is about $\frac{5}{8}$ of the area of the opening.

*Experiment.*—Test the rate of flow from orifices of different shapes, circular, square, triangular. This can conveniently be done by making an opening of some size (say $1\frac{1}{2}$ in. in diameter) near the bottom of a tank, and then placing over this, plates with orifices of different shapes in them. The experiment in each case should continue only a short time so that the flow may be nearly uniform. The rate of flow can be determined by taking the time and observing the fall of the water in the tank, or better, by catching the water and measuring it.

Also compare the flow through a circular orifice in a thin plate with that through a short tubular orifice of the same internal diameter.

**211. Energy of a Liquid Under Pressure.** A liquid possesses potential energy by virtue of its being submitted to pressure, and the amount of this energy can be calculated in the following way.

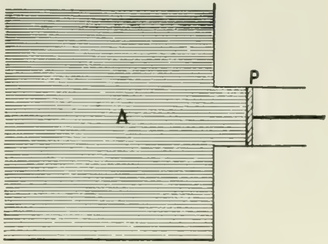

FIG. 244.—Energy due to pressure.

Let $A$ (Fig. 244) be a tank in which is water under a pressure of $p$ grams, or $pg$ dynes, per sq. cm., and let $P$ be the piston of a pump by which water is forced into the tank. Let $a$ sq. cm. be the area of the piston. The total pressure on the piston is $ap$ grams or $apg$ dynes, and when it moves inwards through a distance $x$ cm., it does $apx$ gm.-cm., or $apgx$ ergs, of work.

In doing so it forces $ax$ c.c. of water into the tank, which must possess as potential energy the energy expended in placing it where it is.

Hence, $ax$ c.c. have $apx$ gm.-cm., or $apgx$ ergs, of P.E., and 1 c.c. has $p$ gm.-cm., or $pg$ ergs, of P.E.; *i.e., the measure of the potential energy per unit of volume possessed by a liquid is the same as the measure of the pressure per unit of area to which it is subjected.*

Thus, if a liquid is under a pressure of 10,000 dynes per sq. cm., each c.c. of it possesses 10,000 ergs of potential energy. If the pressure is 60 pounds per sq. ft., each cu. ft. possesses 60 ft.-pd. of potential energy.

Examples of this effect are often seen. When water from the city waterworks, at a pressure of, say, 100 pounds per sq. inch, is admitted to the cylinder of an elevator in a high building, it performs work in raising the car of the elevator to the upper stories of the building. Or, when pumped into the

cylinder of a hydrostatic press, immense pressures are produced, which are used in compressing bales, etc.

**212. Energy of a Liquid in Motion.** Let the velocity be $v$ cm. per sec., and $m$ grams be the mass of 1 c.c. (*i.e.*, the density) of the liquid,

Then the kinetic energy per c.c. $= \frac{1}{2}mv^2$ ergs.

If the velocity is $v$ ft. per sec. and the density is $\rho$ lb. per cu. ft., then the kinetic energy per cu. ft. $= \frac{1}{2}\rho v^2$ ft.-poundals,

$$= \frac{1}{2}\frac{\rho v^2}{g} \text{ ft.-pounds,}$$

since 1 pound force $= g$ poundals.

**213. Rate of Flow of a Liquid.** First, consider steady flow in a tunnel or a pipe of uniform cross-section (Fig. 245). Let the area be $a$ sq. cm., and the velocity be $v$ cm. per sec.

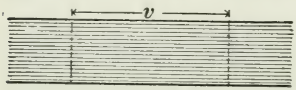


FIG. 245.—Rate of flow = area × velocity.

Then the amount which flows past any point in 1 sec. is $av$ c.c. In practical engineering work the rate of flow is usually stated in cu. ft. or cu. metres per sec.

Next, let the pipe have a contracted portion or throat, as in Fig. 246.

Let the area of the cross-section at $A_1$ be $a_1$ sq. cm., the velocity there be $v_1$ cm. per sec., and the pressure there $p_1$, dynes per sq. cm.

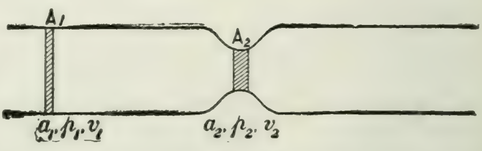


FIG. 246.—The velocity of a liquid is inversely proportional to its cross-section.

At $A_2$ let the corresponding values of these quantities be $a_2$, $v_2$, $p_2$.

Now the same quantity flows past $A_1$, and $A_2$ during 1 sec. Hence, $a_1v_1 = a_2v_2$.

But $a_1$ is greater than $a_2$; hence, $v_2$ is greater than $v_1$, and we have the law:—*The velocity of the liquid is inversely proportional to the area of the cross-section.*

**214. Relation between Pressure and Velocity.** Suppose the tube in which the liquid is flowing is horizontal, and consider the motion of 1 c.c. of the liquid along the axis, from the centre of the section at $A_1$ to the centre of that at $A_2$.

Its energy at $A_1 = p_1 + \frac{1}{2} \rho v_1^2$ ergs............(1)

Its energy at $A_2 = p_2 + \frac{1}{2} \rho v_2^2$ ergs............(2)

But these must be equal, and therefore

$$p_1 + \tfrac{1}{2} \rho v_1^2 = p_2 + \tfrac{1}{2} \rho v_2^2 \quad \text{.....................(3)}$$

= the corresponding expression for any section.

Hence, the quantity

$p + \frac{1}{2} \rho v^2$ is a constant for any section............(4)

The relation (3) can be written

$$p_1 - p_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2) \quad \text{.........................(5)}$$

But since the area at $A_2$ is smaller than that at $A_1$, the velocity $v_2$ is greater than the velocity $v_1$, and also $v_2^2$ is greater than $v_1^2$, Hence, $p_1$ is greater than $p_2$, and we obtain the law that *when the velocity increases the pressure diminishes.*

The pressure exerted by the liquid at a contracted portion of the pipe is less than where the pipe is larger. This is entirely contrary to the view commonly held. Most people think that when the liquid enters a contracted portion its particles are squeezed together and it exerts a greater pressure against the walls of the pipe. This view, however, is quite erroneous.

The relation between pressure and velocity given in (4) above is a simple case of a law of hydraulics known as BERNOULLI'S PRINCIPLE.

## 215. Experimental Illustrations of Bernoulli's Principle.

(**1**) Obtain a glass tube, blown as illustrated in Fig. 247, having

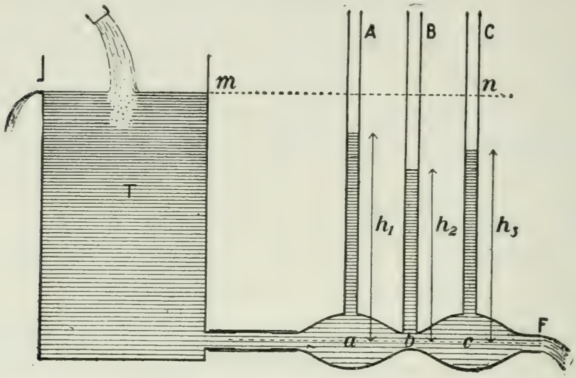


FIG. 247.—Apparatus to illustrate relation between pressure and velocity.

two larger portions separated by a smaller neck, with a small tube rising from each of these portions. The large portions should have a diameter as great as possible, and their lengths should be several times as great as their diameters. If the tube is too small, friction considerably affects the flow, and if the expanded portions of the tube are too "bunty" the water is thrown into eddies and the flow is far from being steady.

Connect to a tank *T* which is kept full of water, or attach directly to a water tap.

First, hold a finger over the end *F*. There will be no flow, and the water in the tubes *A*, *B*, *C* will rise to the line *mn*, assuming the same level as in *T*.

Next, let the water run freely. Now if the particles of water are crowded together as the sections of the cone get smaller, and are thus subjected to increased pressure, this would be shown in the water level in the tubes. We might expect that in *B* to be highest and that in *A* or *C* lowest; but such is not at all the case. They assume the levels shown in the figure. They are slightly lower than they would be if the water moved entirely without friction.

Observe that the pressures at *a*, *b*, *c*, etc., are those due to a 'head' $h_1$, $h_2$, $h_3$, etc. (cm.) respectively. Then if the corresponding pressures are $p_1, p_2, p_3$, etc. (dynes per sq. cm.), we have the relations

$$p_1 + \tfrac{1}{2}\rho v_1^2 = p_2 + \tfrac{1}{2}\rho v_2^2 = \text{(similarly for each section)} = \text{constant.}$$

In place of the glass tubes shown in Fig. 247, an apparatus such as illustrated in Fig. 248 may be used. This consists of two zinc or tin cones soldered together, 3 inches in diameter at the common base and tapering to $\frac{1}{2}$ inch at the ends. The shorter is 3 inches, and the other 12 inches long. Three openings are in the longer cone. In these can be inserted corks through which glass tubes pass.

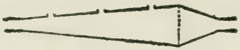

FIG. 248.—Simple apparatus to illustrate Bernoulli's principle.

A third convenient form of the apparatus is shown in Fig. 249. It is made of glass. One end of a U-tube is fused into the wide portion and the other end into the narrow portion of the tube. On causing the water to flow through, it rises in both arms of the U-tube, but higher in that portion joined to the wide part of the tube. It will be observed that the pressure of the air within the U-tube, exerted upon the surface of the water in two arms, is greater than an atmosphere, but it is the same in both arms.

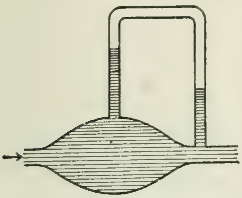

FIG. 249.—Another convenient form of apparatus.

(2) Another interesting experiment is illustrated in Fig. 250. *A* and *B* are two tanks, about 1 sq. ft. in horizontal section, provided with converging pipes as shown. They must be carefully placed so that the contracted openings are exactly opposite each other.

As water is supplied to *A* it spurts out of *c* into *d*, and continues to do so until the level in *B* almost reaches that in *A*. If the water were a perfect liquid the levels would be the same. In an actual experiment a level of 18 inches in *B* was maintained by a level of $20\frac{1}{2}$ inches in *A*, the $2\frac{1}{2}$ inches being lost by friction.

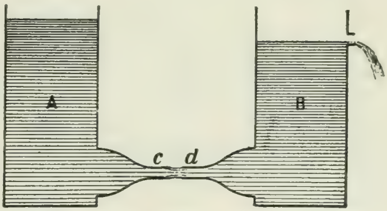


FIG. 250.—The water across from *c* to *d* without waste.

Notice that there is no waste in the water as it shoots across from *c* to *d*, except the small sprinkling caused by inexactness of aim and by want of exact circularity in the orifices. Also, in the space between *c* and *d* there is no pressure except the atmospheric pressure which acts uniformly throughout the system.

**216. Examples of the Flow of a Gas.** The laws according to which a compressible fluid, such as a gas, flows are much

more complicated; but when the variations in the pressure are not too great the relation between the pressure and the velocity still holds.

(1) Examine a Bunsen burner. The gas escapes from a small hole at the base of the burner with a high velocity. The pressure, consequently, is very low, and air rushes in through the opening in the tube, and the mixture of gas and air burns with a non-luminous flame at the top of the tube.

(2) In Fig. 251 a tube is fixed in a flat disc, the end of the tube being flush with the surface of the disc. A light disc of metal or cardboard is held near it by means of three metal pins which move freely through the lower disc. If a vigorous current of air is blown through the tube when it is held vertically, the lower disc will rise up to the other one. In this case the air spreads out in the space between the discs radially from the tube. As it spreads out its velocity is diminished and the pressure increased. Now at the rim the pressure is approximately that of the atmosphere, and so at the centre it must be less than one atmosphere. Hence, the atmospheric pressure on the lower side pushes the disc upward.

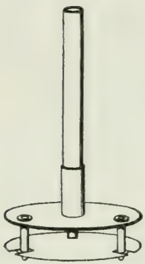

FIG. 251.—On blowing through the tube the lower disc rises.

A very simple form of the apparatus is shown in Fig. 252. A glass tube is pushed through a cork until its end is flush with the lower side. A thin layer of cork, with a pin through it to prevent it moving aside, will be drawn up to the thicker cork when a current of air is blown through the tube.

The above effect was first observed in some iron works in France, about 1826. One of the forge-bellows opened in a flat wall, and it was found that a board presented to the blast was sucked up against the wall.

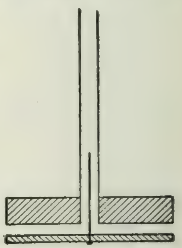

FIG. 252.—Simple form of apparatus.

(3) The simplest way to exhibit the effect, however, is due to Faraday. By means of the palm of the left hand hold snugly up against the palm of the right hand a piece of tissue paper 3 or 4 inches square, and then blow through the opening between the first and second fingers against the middle of the paper. Instead of being blown away, the paper will be sucked up to the hand. After a few trials the experiment will be easily performed.

(4) Another simple experiment is shown in Fig. 253, $T$ is a short, wide glass tube. Through a cork in one end is a glass tube $A$ drawn out to a small opening $a$. Through a cork in the other end a wider tube $B$ is inserted. At the bottom is a manometer $C$ filled with coloured water. On blowing through $A$ the liquid in $C$ rises. Explain this.

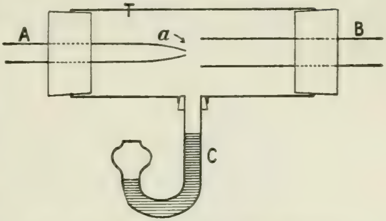


FIG. 253.—Experiment to test Bernoulli's principle.

*Some Practical Applications of Bernoulli's Theorem*

**217. The Venturi Water Meter.** The object of this instrument is to measure the rate of flow in a water-main. Its construction is shown in Fig. 254. Between points $A$ and $C$ a

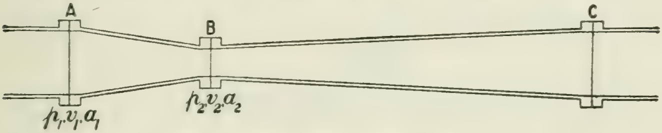


FIG. 254.—The Venturi water meter.

*throat* is inserted, the change in the area of the pipe being gradual in order to avoid eddies. The areas of the cross-sections at $A$ and $B$ are carefully measured and pressure gauges are inserted at these points. Now if we know the areas of these cross-sections and the difference between the pressures there, we can determine the flow in the pipe. To do so we must use the formula

$$p_1 - p_2 = \tfrac{1}{2}\rho(v_2^2 - v_1^2),$$

where $p_1$, $v_1$ are the pressure and velocity at $A$ and $p_2$, $v_2$ those at $B$. The calculation, however, is too difficult to give here.

This meter was invented in 1887 by Clemens Herschel, an American engineer, who named it after Venturi, an Italian, who first described an experiment illustrating the principle involved in it in 1797. There are other forms of water meters, but this one is especially convenient in the case of very large water-mains.

**218. The Jet Pump.** The principle of the jet pump is illustrated in Fig. 255. Water is led from a reservoir $A$ by a pipe $B$ which tapers at $C$. The velocity here is great and the

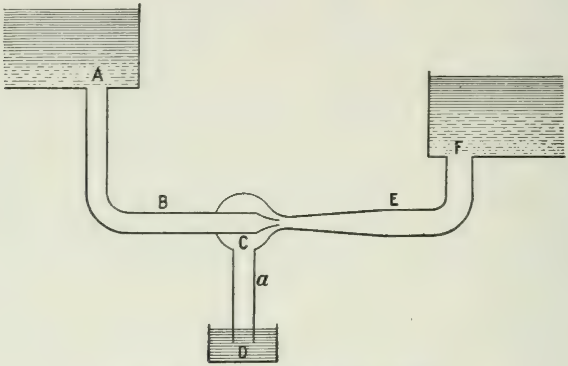


FIG. 255.—The principle of the jet pump.

pressure is reduced until below that of the atmosphere, which, acting upon the surface of the water in $D$, forces it up the pipe $a$. It mixes with the water flowing from $C$, and the combined stream flows on by the tube $E$ to the reservoir $F$, which, however, cannot be higher than $A$. Thus the water is pumped from $D$ up to $F$.

A simple apparatus for showing the action of this pump is illustrated in Fig. 256. The tube $B$ is attached to a water tap (or a supply of compressed air), and the tube $A$ is placed in the liquid to be pumped. To start the pump it may be necessary to fill it with water.

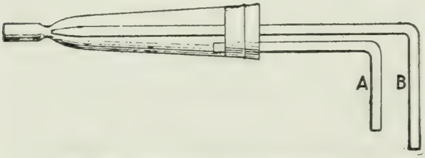


FIG. 256.—A simple form of jet pump.

In Fig. 257 is shown a practical form of the pump. The water which supplies the energy for pumping enters at $A$. It

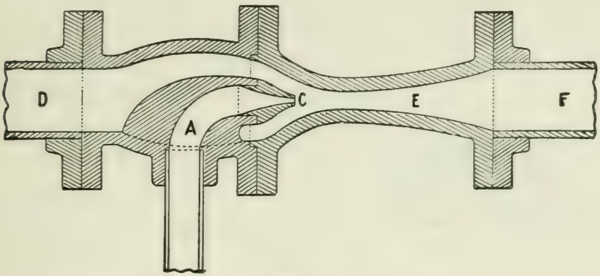


FIG. 257.—A practical form of jet pump.

discharges at $C$, and the water from $D$ is carried on by the pipe $E$ to the pipe $F$.

**219. The Bunsen Filter Pump.** Appliances for producing a suction current of air are known as aspirators. One of the best known of these is the Bunsen filter pump, a vertical section of which is shown in Fig. 258. Water is forced through the tube-nozzle $N$, which gradually tapers and then expands again. At the place where its section is least there is joined on an off-set tube $A$, which is connected to the vessel from which the air is to be removed. The usual explanation is that the water rushing with great velocity through the narrow passage reduces the pressure there, causing the air to flow in through $A$ and be carried off by the water. But a recent investigation * shows that Bernoulli's principle does not enter at all. The water simply drives before it the air as it expands into $A$.

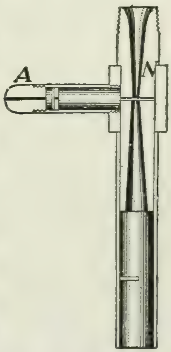


FIG. 258.—The Bunsen filter pump.

* By W. C. Baker, Queen's University, Kingston, in "Physical Review," September, 1919.

**220. The Atomizer.** The atomizer is an instrument for reducing a liquid to a fine spray. Its construction is shown in Fig. 259. On pressing the bulb $B$ an air-blast is forced in a jet from the fine opening $A$. It crosses the top of the tube $C$, and as its velocity is great the pressure just above the top of $C$ is much reduced. The pressure of the atmosphere on the surface of the liquid $D$ forces it up the tube, and as it escapes it is blown into a fine spray.

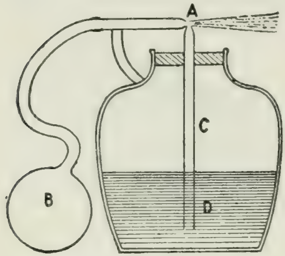

FIG. 259.– The atomizer.

The atomizer has many practical applications. It is used to obtain a fine shower of perfume, or a fine spray of oil in oil-burning engines. Artists render permanent drawings made with charcoal or crayon by spraying them with a solution of mastic in alcohol. The alcohol evaporates and leaves the picture covered with a thin transparent varnish of mastic. The atomizer is often used also in medical practice.

**221. The Steam Injector.** This is an appliance for supplying steam-boilers with water, especially used with locomotives but not exclusively so. It was invented in 1858 by Giffard, a French engineer. The steam and water within the boiler are under considerable pressure, but by means of the injector the steam from the boiler, or even steam at a lower pressure, is able to force water into the boiler.

In Fig. 260 is shown a longitudinal section of the injector. Steam enters at $A$ and blows through the round orifice $C$. Feed water flows in at $B$, and meeting the steam at $C$, causes it to condense. In this way a vacuum is produced at $C$, and the water rushes in with great velocity down into the cone $D$, its velocity being increased by the steam from $C$ striking it from behind. In the lower part of the nozzle $E$ the stream

expands; in doing so it loses velocity and gains pressure, and at the bottom the pressure is so great that it enters the boiler through a check valve which opens only in the direction of the stream. An overflow pipe $F$, by providing a channel through which steam and water may escape before the stream has acquired sufficient energy to force its way into the boiler, allows the injector to start into action. In the actual instrument there are certain valves for regulating the flow of the steam and the water which are not shown in the diagram.

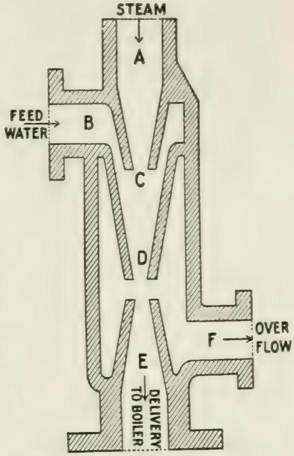

FIG. 260.—The steam injector.

The mechanical efficiency of the injector is much lower than that of the steam pump, but it has the advantage of working when the engine is still and of heating the feed water before delivering it to the boiler.

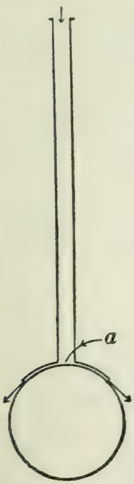

FIG. 261.—Ball nozzle.

**222. The Ball Nozzle.** This is illustrated in Fig. 261. At the end of a tube is a hollow cup in which a ball fits snugly. If a vigorous current of air or steam is forced through the pipe its velocity at $a$, where it leaves the pipe is greater than at the edge of the cup where it escapes into the atmosphere. Hence, the pressure at $a$ is less than at the edge of the cup, and the latter is the pressure of the atmosphere. Consequently the atmospheric pressure on the side of the ball opposite $a$ will prevent the ball from leaving the cup.

**223. Forced Draught.** In order to keep a locomotive moving steam must be generated rapidly, and to do this a fierce fire must be maintained. To secure this the exhaust steam from the cylinders of the engine is discharged through a contracted nozzle $A$, a little distance below the base of the smoke-stack $B$, which is usually flared out like an inverted funnel (Fig. 262). The steam escapes with high velocity. This reduces the pressure greatly and produces a powerful aspiratory effect, which draws in great quantities of air through the fire-box, thus keeping up an intense fire.

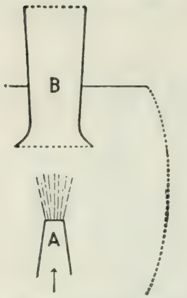

FIG. 262.—Forced draught in a locomotive.

*Other Illustrations of Bernoulli's Principle*

**224. Curve of a Ball.** The curve given to a ball by a 'cut' in tennis, by a 'slice' in golf or by a skilful pitcher in base-ball can also be accounted for by Bernoulli's Theorem.

In order to explain the effect it is more convenient to consider the ball as standing still while a current of air is forced past it, than to take the air as standing still and the ball to be rushing through it. From a mechanical point of view the conditions are equivalent, there is a motion of the ball relative to the air.

The essential requisite to produce a curve is to give the ball a spin as well as a motion forward. Let the ball be spinning about a horizontal axis in the direction shown by the two curved arrows (Fig. 263), and let the air current be in the direction from right to left. The ball in its spinning carries

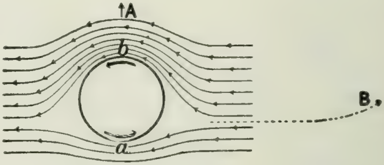

FIG. 263.—The curve of a ball.

around with it some of the air near its surface. At $b$ the air carried around by the ball will unite with the motion of the outer air current, while at $a$ it will oppose the outer air current. Consequently the velocity of the air current at $b$ will be greater than at $a$, and the pressure at $a$ will be greater than that at $b$. Hence, the ball will move across the air current in the direction from $a$ to $b$, as shown by the arrow $A$. If now we consider the air to be at rest and the ball to be moving from left to right and having the same spin as before, it will curve up, as shown by $B$.

A simple way to exhibit the curved path is as follows:—* Obtain a ping-pong or other light ball, varnish it, and while sticky roll it in sawdust, thus making its surface rough. After allowing to dry, place it in a paste-board mailing tube, somewhat larger in diameter than the ball, and by a quick side-wise motion, as indicated in Fig. 264, cause the ball to roll down the tube and dart out of the end. With a little practice a decided curve can be given to the path of the ball.

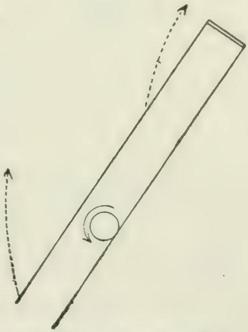

FIG. 264.—Simple method to make a ball curve.

That the pressure of the air on one side of the ball is greater than that on the opposite side can be shown by the following experiment, which is due to Sir J. J. Thomson† (Fig. 265).

Two golf-balls, one smooth, the other with the ordinary rough surface, are mounted on an axis and can be set in rapid rotation by an electric motor. An air-blast, produced by a fan, comes through a pipe $B$ and can be directed against the balls. By a suitable arrangement the axis can be slid up or down in its bearings so that either ball, at pleasure, may be

*Suggested by W. S. Franklin in "Science," Dec. 15, 1911.

† "Nature," vol. 85, p. 251, 1910. (Report of a lecture before the Royal Institution, London).

put in the air-blast. The pressure of the air is measured by the curved tubes $L$, $M$, connected with a pressure gauge $P$, $Q$. $L$ and $M$ are adjusted so that the ball just fits between them.

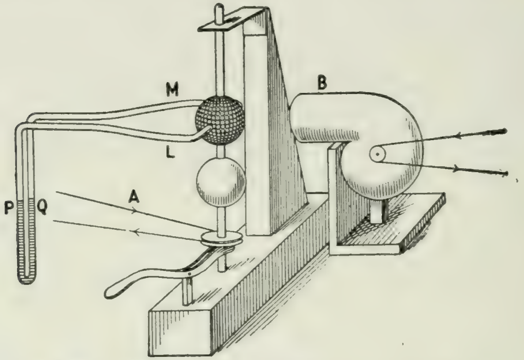


FIG. 265.—Apparatus to test the theory of a curving ball.

When the ball spins in the direction indicated by the belt $A$ the air current at $L$ is more rapid than that at $M$; it is seen that the column $Q$ is depressed, $P$ is raised. If the smooth ball is used the effect is similar but not so pronounced. The smooth surface does not carry so much air with it as does the rough one.


FIG. 266.—Ball held up by a jet of air.

**225. Light Ball in a Jet of Steam or Air.** A light ball (made of celluloid, or a tennis ball), may be held in equilibrium by a jet of air or steam as illustrated in Fig. 266. The ball is under the action of three forces:—Its own weight $W$; $I$, the force of impact of the fluid against the ball; and $P$, an excess of atmospheric pressure over the pressure on the other side of the ball, due to the high velocity of the escaping fluid. With a few trials a position can usually be found for the ball where the three forces are in equilibrium, and the ball remains there.

**226. Two Balls in a Current of Air.** If two light balls are suspended side by side in a current of air from an electric fan (Fig. 267) the wind-current between the balls is greater than that on the other side of them. The air-pressure on the outer sides is therefore greater than that in the space between, and the balls are consequently pushed toward each other.

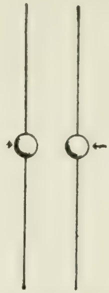

FIG. 267.—Balls pushed together when a current of air is directed upon them.

**227. Two Ships Steaming Side by Side.** If a ship is anchored in a river the water flows past it, the particles moving in definite stream lines. If the vessel is moving forward through still water, there is a similar relative motion between it and the water, and the resulting stream lines are similar to those in the other case.

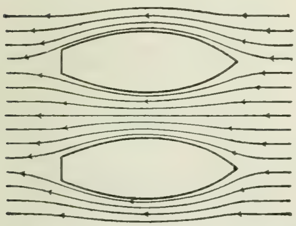

FIG. 268.—Ships drawn together when steaming side by side.

If two ships are steaming side by side (Fig. 268) the water streams past them more swiftly in the space between than on the outer sides. On account of this increased velocity the pressure exerted by the water against the inner sides of the ships is less than that against the outer sides, and the ships are pushed toward each other. One might expect the water between the ships to be heaped up, but such is not the case, its level is *below* the level at other places. Large ships should not manœuvre too close to each other; accidents have occurred through ships being apparently drawn together in the manner just described.

## PROBLEMS

1. In a water-works system the pressure is maintained by the water in a stand-pipe 100 feet high, situated on a hill, 50 feet above the valley. Find the pressure, in pounds per square inch, on the ground floor of a house in the valley.

2. If the stand-pipe is 30 metres high and the hill 20 metres above the valley, find the pressure in dynes per square cm., and also in kilograms per sq. cm.

3. A large tank, 3 metres high, is kept full by water continually running into it, and a small round opening, 1 cm. in diameter, is made at the base. At what rate will the water escape?

4. A can contains oil to a depth of 18 inches, and a small round hole $\frac{1}{16}$ inch in diameter is punched through it at the base. At what rate will the oil begin to run out? (Take velocity of efflux the same as that for water).

5. Find the work done in pumping 20 gallons of water into a boiler in which the pressure is 50 pounds per square inch. (1 gal. = 277.3 cu. in.).

6. At what velocity must the water flow in a canal 30 feet wide and 8 feet deep to discharge 1000 cu. ft. per second?

7. Water in a pipe is under a pressure of 60 pounds per square inch and is flowing at the rate of 5 feet per second. Find the energy per cubic inch. (Neglect potential energy due to gravity).

8. If the pressure is 5 kilos. per sq. cm., and the rate of flow is 2 metres per second, find the energy per c.c.

9. A large elevated tank supplies water to a house through a pipe of section 10 sq. cm.

(*a*) Find the pressure in the pipe 6 metres below the level of the water in the tank, when no water is flowing.

(*b*) Find the pressure at the same point (neglecting friction) when water is being drawn from the lower end of the pipe at the rate of 400 c.c. per second.

(*c*) Why does the rate of flow from one faucet diminish when a second one is opened?

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For further information on the subject of the flow of fluids consult:

*Encyclopedia Britannica*, 11th Edition. Art. "Hydraulics," Vol. XIV, p. 35.

EDWIN EDSER, *General Physics for Students*, Chapters XI to XV.

W. FROUDE, "Stream Lines in Relation to the Resistance of Ships," Brit. Assoc. Report, 1875; "Nature," Vol. XIII, p. 50, 1875.

## CHAPTER XXVI

### SOME TRANSFORMATIONS OF ENERGY

**228. Heat a Mode of Motion.** Until almost the middle of the last century it was the generally accepted belief that heat was a subtle fluid called *caloric,* which was distributed amongst the molecules of a body. When a piece of iron was hammered the caloric was driven out from its hiding place and revealed itself in a rise in the temperature of the iron.

An interesting investigation into the nature of heat was made in 1798 by Count Rumford.* While engaged in boring cannon at the arsenal in Munich he was surprised at the great amount of heat generated in the operation, and in order to make a thorough inquiry into the matter he prepared a hollow bronze cylinder which he mounted so that it could be rotated by horse-power while a blunt steel tool was pressed against the bottom inside. In one experiment the cylinder was immersed in about 20 pounds of water. The temperature steadily rose and in $2\frac{1}{2}$ hours the water actually boiled. Rumford found that as long as he kept the machine going the heat continued to be produced and he concluded that as the supply was inexhaustible, heat could not be a material substance but must be a *form of motion.*

**229. Relation between Heat and Mechanical Work.** Though heat could be obtained at the expense of mechanical work, the precise relation between these two was not determined until Joule published the results of experiments which he began in 1840. If the work is really all spent in

*Rumford's name was Benjamin Thompson. He was born in 1753 at Woburn, near Boston, Mass., went to England in 1775, and at the close of the Revolutionary War went to Bavaria in 1783. He was made Count by the Elector of Bavaria and chose his title from the name of a small town (now called Concord) in New Hampshire.

producing heat, then with every form of experiment one should obtain the same amount of heat for a given amount of work. The quantity of work which is required to create one unit of heat is called its *mechanical equivalent.*

The essential features of one method used by Joule to determine this mechanical equivalent is illustrated in Fig. 269.

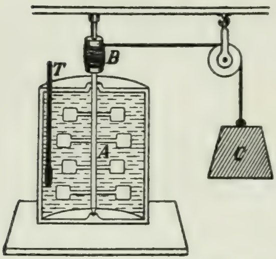

FIG. 269.—Principle of Joule's apparatus for determining the mechanical equivalent of heat.

A paddle-wheel is made to revolve in a vessel $A$, filled with water, by the descent of a weight $C$ on the end of a cord which is wound about $B$. A thermometer $T$ measures the rise in temperature. The heat generated is calculated from the mass of the water and its rise in temperature, and the amount of work which is equivalent to it is measured by the weight $C$ and the distance through which it falls.

As the result of many careful and tedious experiments Joule calculated that the mechanical equivalent of one British thermal unit (that is, the heat required to raise 1 pound of water through 1° F.) was 772 foot-pounds of work. Later investigations by Rowland and others give the value as

778 foot-pounds for 1 B.T.U.,

which is the same as

427 gram-metres for 1 calorie,

or, $4.187 \times 10^7$ ergs for 1 calorie.

One calorie is the amount of heat required to raise 1 gram of water through 1° C.

**230. Determination of the Mechanical Equivalent.** By means of the apparatus shown in Fig. 270 the mechanical equivalent may be determined rapidly and with considerable accuracy. $C$ is a drum made of thin brass which can be rotated

about the horizontal axis *B*. On the other end of this axis is the driving wheel *A* which can be turned by hand or

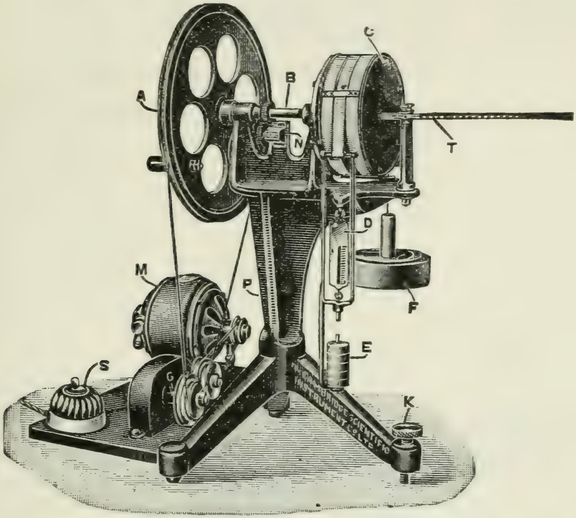


FIG. 270.—Callendar's apparatus for determining the mechanical equivalent of heat.

driven by a small electric motor *M* through reduction gearing *G*. The number of revolutions made by the drum is automatically recorded by the counter *N*. Around the drum is wound a silk belt, making one and one-half complete turns. Unequal adjustable weights *E* and *F* are suspended from the ends of this belt and a light spring balance *D* is added to secure stability of equilibrium.

Into the drum *C* pour through a pipette a measured quantity of water. A thermometer *T* is inserted through a hole in the centre of the front face of the drum. Its stem is bent almost at a right angle, so that the bulb may be fully immersed in the water, and any rise in the temperature of the water is shown by the reading on the thermometer. As the drum revolves the friction of the belt upon its circumference warms it and the heat is conducted to the water within.

If $W$ ergs be the work expended in rotating the drum and $H$ calories be the heat generated, then

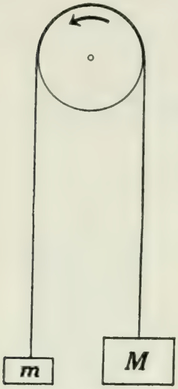


FIG. 271.—The drum rotates in the direction of the arrow and friction of the belt sustains the weight $M$.

Mechanical Equivalent of Heat $= W/H = J$ ergs per calorie.

First, calculate the work expended in rotating the drum. Consider a drum (Fig. 271) with a belt over it with weights $M$, $m$ grams on its ends, and suppose the drum to be rotated in the direction of the arrow so fast that the weights are kept in equlibrium. Then it is evident that the weight $m$ added to the friction of the belt balances the weight $M$.

Hence the friction $= M - m$ grams.

Then the work expended in rotating the drum is the same as if a cord, with one end attached to the drum, was wound about it, with a weight $M - m$ grams attached to the end, and the drum was made to rotate at the same rate as before. (Fig. 272). The cord would be wound around the drum and the work done would be determined from the height through which $M - m$ was raised.

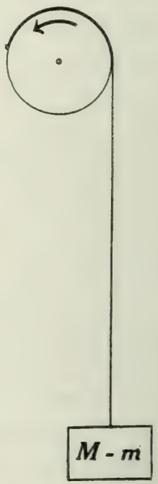


FIG. 272.—Calculation of the work done.

Let circumference of drum $= c$ cm.,

Time of rotation $= t$ minutes,

Rate of rotation $= n$ per minute.

Then total number of rotations $= nt$, and the length of cord wound on the cylinder $= ntc$ cm. The weight will be raised through this height.

Hence, the work done,

$$W = (M - m)\ ntc \text{ gram-cm.}$$
$$= (M - m)\ nctg \text{ ergs.}$$

Next, calculate the heat produced.

Let mass of drum $= w_2$ grams,
and its specific heat $= s$.
Then its water equivalent $= w_2 s$ grams.
Let mass of water $= w_1$ grams,
and rise of temperature $= T°$ C.

Then heat generated, $H = (w_1 + w_2 s)\, T$ calories.

$$\text{Hence, } J = \frac{(M - m)\, ntcg}{(w_1 + w_2 s)\, T} \text{ ergs per calorie.}$$

**Example** :—From an actual experiment.

$$\begin{aligned} M - m &= 1935 \text{ grams} \\ c &= 47.5 \text{ cm.} \\ nt &= 410 \text{ revolutions} \\ g &= 980 \\ w_1 &= 300 \text{ grams} \\ \left.\begin{aligned} w_2 &= 403.1 \text{ grams} \\ s &= .092 \end{aligned}\right\} & w_2 s = 37 \text{ grams} \\ T &= 2.6° \text{ C.} \end{aligned}$$

From which $J = 4.23 \times 10^7$ ergs per calorie.

QUESTION :—Why is a silk belt used rather than one of leather or of metal?

**231. Measurement of Electrical Energy.** (*a*) *Current Strength and Quantity.* The flow of electricity in a conductor may be compared to the flow of water in a pipe. The water flows from a place of high pressure to one of low pressure; electricity flows from a place of high potential to one of low potential. Difference of pressure in the pipe corresponds to difference of potential in the conductor. Indeed the words 'pressure' or 'tension' are often used by electricians when they mean potential.

Again, a pipe of large section corresponds to a conductor of large section. There is little *resistance* to the flow in either case. A pipe of small section or one filled with sand corresponds to a wire having a high resistance.

*Current Strength* is *rate of flow.* In the case of water, it depends on the difference in pressure at the ends of the pipe and on the resistance or friction in the pipe.

Quantity of water = rate of flow × time of flow.

Rate of flow may be measured in gallons-per-second, quantity in gallons.

Next, consider the quantity of electricity which passes a cross-section of a wire carrying a current in a given time, and as before we have

Quantity of electricity = current-strength × time of flow,

or Total amount = rate of flow × time of flow.

If the current is expressed in *amperes* and the time in *seconds*, the quantity will be given in *coulombs*; a coulomb being defined to be the amount of electricity which passes a point in an electrical circuit in one second when the strength of the current is one ampere.

The ampere corresponds to gallons-per-second, the coulomb to gallons.

Thus one coulomb = one ampere-second. The word coulomb is not often heard in ordinary electrical practice, but ampere-hour, a quite similar term, is commonly used in specifying the capacity of storage cells. A battery having a capacity of 100 ampere-hours is one which can deliver 100 amperes for 1 hour, or 50 for 2 hours, etc.

(*b*) *Work done by the Current.* The work which a stream can do depends upon the quantity of water which flows and the distance through which it falls. The work done in an electric circuit depends upon the quantity of electricity which passes through it and the P.D. (difference in potential) between the terminals of the circuit; 1 *coulomb* of electricity falling through 1 *volt* does 1 *joule* of work, and 1 joule = $10^7$ ergs. (See Sec. 98).

If $Q$ coulombs falls through $E$ volts, the work done $= EQ$ joules.

Suppose $A$ and $B$ (Fig. 273) are the brushes of a dynamo (driven by a steam engine) which maintains a P.D. of 1 volt between $A$ and $B$ and causes a current of 1 ampere to flow through the circuit $D$. Then in 1 second 1 coulomb will flow from $A$ to $B$ and 1 joule of work will be done. The *power* required to perform 1 joule of work per second is 1 *watt.*

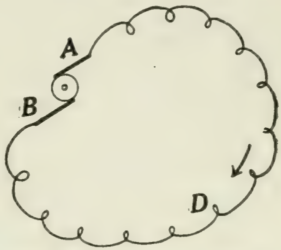

FIG. 273.—A generator driving electricity through a circuit $D$.

If the dynamo maintained a P.D. of 110 volts and supplied a current of 20 amperes, the power required (or the output of the dynamo) $= 110 \times 20 = 2200$ watts.

Hence power (in watts) = P.D. (in volts) × current (in amperes)

1000 watts = 1 kilowatt (k.w.)

Also, 1 h.p. = 746 watts = $\frac{3}{4}$ k.w. (nearly, see Sec. 106),

$$\text{and power (in h.p.)} = \frac{\text{P.D. (in volts)} \times \text{current (in amperes)}}{746}.$$

Again, by Ohm's law $C = E/R$ or $E = CR$.

Hence, power $= EC = C^2R$ watts, where $R$ is the resistance of the circuit in ohms.

**232. Work done in an Electric Lamp.** The efficiency of a lamp is usually determined in watts per candle-power.

The old 16-c.p. carbon lamp consumed $\frac{1}{2}$ ampere at 110 volts, which $= \frac{1}{2} \times 110 = 55$ watts. The efficiency of such a lamp is $55 \div 16 = 3.4$ watts per c.p.

The ordinary 50-watt tungsten lamp gives about 32 c.p., which $= 50 \div 32 = 1.6$ watts per c.p.

It is easy to understand why the tungsten lamps have almost entirely superseded those with carbon filaments. If

the lamp is filled with an inert gas, such as nitrogen, the efficiency is still higher, namely about 1 c.p. per watt. In these lamps the filament can be raised to a higher temperature and a slight rise in temperature causes a large increase in radiating power.

**233. Electrical Equivalent of Heat.** It is possible to determine the mechanical equivalent of heat by using an electric current. If the current is not effecting chemical change or doing mechanical work (such as driving an electric motor), it is simply heating the conductor through which it flows. If we measure the work required to cause the current to flow and at the same time measure the heat arising from it, we can deduce the mechanical work which is equivalent to one unit of heat.

A suitable apparatus is shown in Fig. 274. In the long

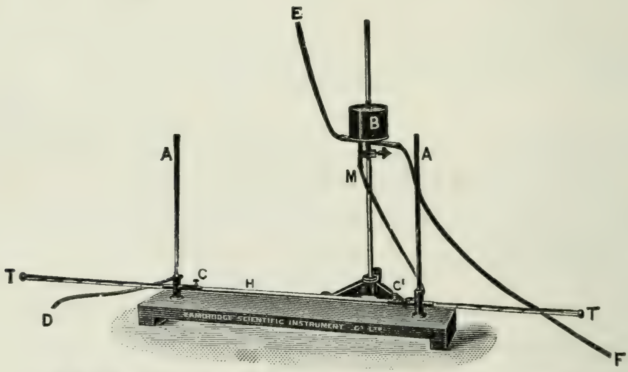


FIG. 274.—Callendar and Barnes's continuous flow calorimeter.

glass tube $H$ a helix of resistance wire (manganin) is fixed with its ends connected to binding-posts, $C$, $C'$. Into the ends of the tube, thermometers $T$, $T'$ are inserted. The aim is to have a constant current of water flowing through $H$ and heated by a constant current of electricity passing through the helix. The constant current of water is obtained

from a cistern $B$, mounted on a stand. This contains two concentric chambers (Fig. 275). The tube $E$ carries water from the tap (or other source) into the outer chamber, while the tube $M$ runs from the outer chamber to one end of the tube $H$. A third tube $F$ leads from the inner chamber to the sink. The supply of water through $E$ is adjusted so that the outer chamber is always full and a small amount flows over into the inner chamber and runs away through $F$. In this way a constant head of water is maintained and the flow through $H$ is steady.

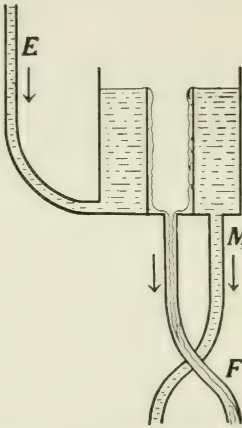

Fig. 275.—Arrangement to secure a constant head of water.

The cistern can be raised to increase the current through $H$, but it should not be raised high enough to cause water to overflow out of the vertical tubes $A$, $A$, which are provided to allow the escape of any air-bubbles which may be entrapped in the stream of water. With this apparatus the usual rate of flow is about 60 c.c. per minute, and it is desirable that the initial temperature of the water should be about as much below the temperature of the room as the final temperature is above it. If such is the case no correction for radiation need be made.

The strength of the electric current is measured by an ammeter placed in series with the helix, and the current may be obtained from a commercial source or from storage batteries, suitable resistance being inserted to reduce the current sufficiently. A voltmeter, with terminals joined to $C$, $C'$ gives the P.D. between the ends of the helix.

The water runs off through the tube $D$ and is collected in a graduated glass, from which the amount flowing in any interval may be read.

Let the P.D. $= E$ volts, the current $= C$ amperes, and the duration of an experiment $= t$ seconds.

Then the work done $= ECt$ joules,

$$= ECt \times 10^7 \text{ ergs.}$$

In the $t$ seconds suppose $M$ grams of water have been caught and let the rise in its temperature be $T^\circ$ C.

The heat evolved, $H = MT$ calories, and if 1 calorie is equivalent to $J$ ergs

$$H = MTJ \text{ ergs.}$$

Hence, $MTJ = ECt \times 10^7$

$$\text{or} \quad J = \frac{ECt \times 10^7}{MT} \text{ ergs per calorie.}$$

**Example**:—In the following table are given the readings in an experiment.

| Ammeter Reading *C.* | Voltmeter Reading *E.* | Flow of Water c.c. per min. | Temp. of Inflowing Water. | Temp. of Outflowing Water. | Rise in Temp. | Temp. of Room. |
|---|---|---|---|---|---|---|
| 2.12 | 21.2 | 63.8 | 13.25° C. | 23.70° C. | 10.45° C. | 18.6° C. |
| 2.12 | 21.2 | 61.0 | 12.70 | 22.90 | 10.20 | 18.7 |
| 2.12 | 21.2 | 61.0 | 12.15 | 22.50 | 10.35 | 18.7 |
| Av. 2.12 | 21.2 | 61.9 | 12.70 | 23.00 | 10.33 | 18.7 |

$$\text{Hence, } J = \frac{2.12 \times 21.2 \times 60 \times 10^7}{61.9 \times 10.33} = 4.23 \times 10^7 \text{ ergs per calorie.}$$

### PROBLEMS

(Take $J = 778$ ft.-pd. per B.T.U. or $4.19 \times 10^7$ ergs per calorie)

1. The Falls of Niagara are 166 ft. high. Find the rise in temperature due to the impact of the water on the rocks below, assuming that all its mechanical energy is changed into heat energy.

2. A jet of water is driven with a velocity of 150 metres per sec. against a wall. If the mechanical energy is used up in heating the water, find its rise in temperature.

3. How much heat is generated when a train of 200 tons moving with a velocity of 60 ml. per hr. is brought to rest by the brakes?

4. In an experiment a 5 h.p. motor in 1 min. raised the temperature of 1 gal. of water through 22° F. Calculate the ft.-pd. expended per B.T.U. What per cent. is the result in error?

5. If 1 lb. of good coal can raise the temperature of 60 lb. of water from 0° to 100° C. find the energy in ft.-pd. in 1 lb. of coal. If the efficiency of a steam engine is 8 per cent., what must be the consumption of coal per hr. to produce 150 h.p. ?

6. If the atmosphere were removed and sunlight fell perpendicularly upon 1 sq. cm. of the earth's surface, it would receive in 1 min. approximately 2 calories of heat. Assuming that 40 per cent. of the sun's radiation is lost by absorption in the atmosphere, find in k.w. the energy falling upon 1 sq. metre of the deck of a steamer when the sun is directly overhead.

7. In an electric furnace a current of 8000 amperes at 50 volts is used ; find the calories generated per second.

8. Water flows steadily at the rate of 30 c.c. per min. through a glass tube in which is a wire coil. The temp. of the water on entrance is 13.25° C., and on exit 20.65° C., and the p.d. at the ends of the wire is 25 volts. Find the resistance of the wire.

9. An electric tea-kettle consumes 8.6 amperes at 110 volts and in 10 min. can raise 1.5 litres of water from 15° to 100° C. What percentage of the energy supplied is used in actually boiling the water ?

10. In a power station 4 engines, each of 150 h.p., drive 2 dynamos, each of which delivers 150 amperes at 540 volts, and 2 others, each of which delivers 225 amperes at 270 volts. Calculate the efficiency of the arrangement.

**234. The Buying and Selling of Energy.** We are accustomed to dealing in flour, sugar, lumber, and other things which we can see and handle, but energy, though invisible and intangible, is quite as real a *thing* and can equally well be bought and sold. Energy is *ability to do work*, and it is as reasonable that we should pay for any energy which is supplied to us as for the objects produced thereby.

Further, it is clear that the charge for energy should depend upon

(i) the rate at which it is supplied; and

(ii) the length of time it is supplied.

A man is a source of energy and the pay for his services should be according to his ability and the length

of time he works, that is, per man-power–hour, or per man-power–day.

At Niagara the falling water turns the great turbine water-wheels; these drive electric dynamos which generate electricity and supply it in a continuous stream. The *activity* or *power* of this current depends upon its strength or magnitude and its potential, that is, upon its amperes and its volts. As we have seen,

$$1 \text{ watt} = 1 \text{ volt} \times 1 \text{ ampere}.$$

If a dynamo can deliver 500 amperes at 12,000 volts its output $= 500 \times 12{,}000 = 6{,}000{,}000$ watts,

$$= 6{,}000 \text{ kilowatts},$$

$$= 8{,}043 \text{ horse-power}.$$

The charge for the use of this power should evidently depend upon the time it is supplied. Power when used for a time does work. The unit of electrical work or energy is the *watt-hour* or (since that is rather small) the *kilowatt-hour*. Of course this can be easily transformed into *horse-power–hour*.

The buying and selling of energy is well illustrated in the operations of the Hydro-Electric Power Commission of Ontario which during recent years has extended its distribution lines to many parts of the province. This great public utilities organization is managed for the benefit of the people, and is rapidly providing electric energy to the cities, towns, villages and farms of the country.

The Commission owns a number of power stations and transmission systems throughout the Province and controls others, and in some cases purchases power for distribution from independent supply corporations. Its largest source of energy is, of course, the Niagara River. At first the Commission secured from one of the companies on the Canadian side a large amount of power for its Niagara

transmission system which extends from Toronto in the east to Windsor in the west, but this soon became insufficient to supply the demands. At present (1919) the Commission controls the largest plant at the Falls and purchases, directly or indirectly, a large portion of the output of other plants there, the price paid averaging about $9 per horse-power per year. In addition, the Commission is vigorously proceeding with the construction of a canal to convey water from the Niagara River, near Chippawa, to the neighbourhood of Queenston, where, to reach the Niagara River again, it will fall over 300 feet and will develop over 300,000 h.p.

From the generating and transforming stations the energy is sent over the transmission or distribution lines built by the Commission, and is sold to the various municipalities at rates depending on the distance from the station and on the amount taken. Thus, Toronto pays $14.50, London $21.00, Guelph $20.00, Owen Sound $31.00, Clinton $42.00, per horse-power per year.

These places then sell the energy to the citizens for lighting, for heating (as in stoves, toasters, flat-irons, etc.), and for driving motors (as in manufacturing, operating elevators, etc.), the rate being adjusted to the service and the place. The charges are practically always per kilowatt-hour, and some typical accounts are given in the problems below.

Near the place where the electric current enters the house a *watt-meter* is placed, and as the current passes through this it makes a light disc rotate. The number of rotations (which are recorded on small dials) depends upon the voltage, the magnitude of the current and the length of time it is flowing, and it consequently indicates the kilowatt-hours of energy supplied. An officer reads the dials periodically and the account for the energy used is then sent to the customer.

## PROBLEMS

The manner of charging for electrical energy varies somewhat with the companies supplying it. The Hydro-Electric Power Commission divides its rates under three headings :

(*a*) *Domestic service*, which includes all energy used for domestic or household purposes.

(*b*) *Commercial service*, for stores, churches, hotels, etc.

(*c*) *Power service*, which includes all other services.

In each case there are certain fixed charges depending on the size of the installation or the amount of energy used. The following examples taken from the accounts of an Ontario town, will illustrate the methods used.

Domestic Service—

1. Name, *First Citizen*. Date, June 1, 1919.

| | | |
|---|---|---|
| Meter reading, June 1, 1919 | 5601 | k.w.h. |
| Previous reading, May 1, 1919 | 5314 | " |
| Consumption | 287 | " |

Consumption charge

| | |
|---|---|
| 69 k.w.h. at 4.5c. per k.w.h. | $3.10 |
| 218 " " 2.25c. " " | 4.90 |

Service charge

| | |
|---|---|
| 2300 sq. ft. at 3c. per 100 sq. ft. | .69 |
| Total bill | 8.69 |
| 10% discount if paid within 10 days | .87 |
| Net bill | $7.82 |

*Explanation* :—The floor space in this house, measured in a specified manner, was 2300 sq. ft. There is a fixed charge of 3c. per 100 sq. ft. per month. The charge for consumption is $4\frac{1}{2}$c. per k.w.h. for all up to 3 k.w.h. per 100 sq. ft. (or 69 k.w.h.) and $2\frac{1}{4}$c. per k.w.h. for all above this.

2. Name, *Second Citizen*. Date, June 1, 1919.

| | | |
|---|---|---|
| Meter reading, June 1, 1919 | 588 | k.w.h. |
| Previous reading, May 1, 1919 | 576 | " |

Make out the bill. Floor space, 1800 sq. ft.

3. Make out the bills for the above persons at the Toronto rates, which are as follows :

| | |
|---|---|
| For first 3 k.w.h. per 100 sq. ft. . . . . . . . . . . . . . . . . . . | 2c. |
| For all additional k.w.h. . . . . . . . . . . . . . . . . . . . . . . . . . . . | 1c. |

Service rate 3c. per 100 sq. ft. ; discount 10%.

COMMERCIAL SERVICE—

4. Name, *Third Citizen.* Date, June 1, 1919.

Installed load, 2970 watts.

| | | |
|---|---|---|
| Meter reading, June 1, 1919. . . . . . . . . . . . . . . . | 8007 | k.w.h. |
| Previous reading, May 1, 1919. . . . . . . . . . . . . . | 7596 | " |
| Consumption . . . . | 411 | " |

Consumption charge

| | |
|---|---|
| 89 k.w.h. at 9c. per k.w.h. . . . . . . . . . . . . . . . . . . . . . . | $8.01 |
| 208 " " 4.5 " " . . . . . . . . . . . . . . . . . . . . . . | 9.36 |
| 114 " " .09 " " . . . . . . . . . . . . . . . . . . . . . . | .10 |
| Total bill. . . . . | 17.47 |
| 10% discount if paid within 10 days. . . . . . . . . . . . . . . . | 1.75 |
| Net bill. . . . | 15.72 |

*Explanation* :—The "installed load" is the capacity of all the lights installed. A charge is made for the entire installed load for the first 30 hours ($30 \times 2.970 = 89$ k.w.h.) at 9c. per k.w.h., and for the next 70 hours at $4\frac{1}{2}$c. per k.w.h. For all additional consumption the charge is .09c. per k.w.h.

5. Name, *Fourth Citizen.* Date, June 1, 1919.

Installed load, 1042 watts.

| | | |
|---|---|---|
| Meter reading, June 1, 1919. . . . . . . . . . . . . . . . | 2384 | k.w.h. |
| Previous reading, May 1, 1919. . . . . . . . . . . . . . | 2359 | " |

Make out the bill.

6. Name, *Fifth Citizen.* Date, June 1, 1919.

Installed load, 745 watts.

| | | |
|---|---|---|
| Meter reading, June 1, 1919. . . . . . . . . . . . . . . . | 2924 | k.w.h. |
| Previous reading, May 1, 1919. . . . . . . . . . . . . . | 2890 | " |

Make out the bill.

7. Make out bills for the above three persons at the Toronto rates, as follows:

| | |
|---|---|
| For first 30 hours of installed load........ | 5c. per k.w.h. |
| For next 70 " " " " ........ | 2.5c. " |
| For all additional consumption........... | .5c. " |
| Discount, 10%. | |

Power Service—

8. Name, *Sixth Citizen.* Date, June 1, 1919.

| | |
|---|---|
| Connected load, 50 h.p. | |
| Meter reading, June 1, 1919................ | 25,040 k.w.h. |
| Previous reading, May 1, 1919............. | 14,100 " |
| Consumption.... | 10,940 " |

Consumption charge

| | |
|---|---|
| First 50 hours use, 1865 k.w.h. at 4.7c. per k.w.h .. | $87.66 |
| Second 50 " " 1865 " " 3.1 " " .. | 57.82 |
| Remaining consumption, 7210 k.w.h. at .15 per k.w.h. | 10.81 |

Service charge

| | |
|---|---|
| 50 h.p. at $1 per h.p. per month................. | 50.00 |
| | 206.29 |
| 25% class discount............................ | 51.57 |
| Total bill.... | 154.72 |
| 10% discount if paid in 10 days.................. | 15.47 |
| Net bill.... | $139.25 |

*Explanation*:—In this case the "connected load or maximum demand" is 50 h.p. = 37.3 k.w. and the customer is charged for the full load for 50 hours (37.3 × 50 = 1865) at 4.7c. per k.w.h. The charge for the full load for the second 50 hours is at 3.1c. per k.w.h. The charge for the remainder is .15c. per k.w.h.

In addition he pays $1 per h.p. of "connected load" per month.

The "class discount" is a discount depending on the number of hours per day which the power may be used, in this case, 18 out of the 24 hours.

9. Name, *Seventh Citizen.* Date, June 1, 1919.

Meter reading, June 1, 1919 . . . . . . . . . . . . . . 79,920 k.w.h.

Previous reading, May 1, 1919. . . . . . . . . . . . . 77,450 „

Connected load, $19\frac{1}{2}$ h.p.

Class discount, 10%

Make out the bill.

10. Make out bills for the above two persons at Toronto rates, as follows:

| | | | | | |
|---|---|---|---|---|---|
| First 50 hours use per month, | | | 1.5c. | per | k.w.h. |
| Second „ | „ | „ | .5c. | „ | „ |
| All addditional | | | .15c. | „ | „ |

Service charge, \$1.25 for first 10 h.p., \$1 for all additional.

Class discount the same.

Discount for prompt payment, 20%.

11. In an advertisement for electric heaters the following phrases are found :—"615 watts per hour," "consumes only 960 watts per hour," "consumes only 1250 watts per hour."

Criticise these statements.

# ANSWERS

**Page 9.** **1.** 2,500,000 mm. **2.** 299,804.97 km. **3.** 29.921 in. **4.** 183.49 m. **5.** 4.79 mm. **6.** 16.535. **7.** 535.797.

**Page 14.** **1.** 88. **2.** 108. **3.** (1) $27\frac{3}{11}$, (2) $3\frac{9}{22}$. **4.** $48\frac{8}{9}$. **5.** (1) 2 : 1, (2) 11 : 6. **6.** 5 : 56. **7.** $\frac{44}{45}$. **8.** 60 miles. **9.** 7200. **10.** $\frac{15}{22}\,ab$. **11.** $36\,\frac{ch}{8}$ m. **12.** (1) $\frac{5}{44}$, (2) $1\frac{3}{32}$. **13.** $\frac{22}{15}\,\frac{bc}{a}$. **14.** **11 miles per day.** **15.** **$2\frac{9}{11}$ miles per day.** **16.** (1) 10.5 cm. per sec.; (2) 10 cm. per sec.; (3) 1 cm. per sec.; (4) 11 cm. per sec. **17.** (1) 1 cm. per sec.; (2) 3 cm. per sec.; (3) 1 cm. per sec.; (4) 1 cm. per sec.

**Page 32.** **1.** $2\frac{17}{54}$ cm. per sec. per sec. **2.** $-\frac{1}{12}$ ft. per sec. per sec. **3.** 600 ft. per sec.; 600. **4.** (1) 300 cm. per sec., (2) 18,000 cm. per sec. **5.** (1) $\frac{1}{5}$ ft. per sec., (2) $\frac{1}{300}$. **6.** (1) 0.5 ft. per sec., (2) $\frac{1}{120}$. **7.** 10 minutes. **8.** 1 sec. **9.** (1) 6, (2) 2, (3) $\frac{1}{10}$, (4) $\frac{1}{30}$. **10.** (1) 50, (2) 5000, (3) $\frac{5}{6}$, (4) $83\frac{1}{3}$. **11.** (1) 1, (2) 3600, (3) 3600, (4) 60. **12.** (1) 1200, (2) 72,000, (3) 720, (4) 12, (5) $\frac{1}{5}$. **13.** (1) 30, (2) 108,000.

**Page 37.** **1.** 100 cm. per sec. **2.** 20. **3.** $-185$ cm. per sec. **4.** (1) 5, (2) 165 cm. per sec., (3) 20 sec. before its velocity was 100 cm. per sec. **5.** (1) 10 sec., (2) $3\frac{1}{8}$ sec. **6.** (1) 550 cm., (2) 1 sec. **7.** (1) 1.5 sec., (2) 11.25 cm. from starting point. **8.** (1) 160 ft., (2) 250 ft., (3) 90 ft. **9.** 156 ft. **10.** 20 ft. per sec. per sec. **11.** (1) 12, (2) 78 ft. **12.** 6 ft. per sec. per sec. **13.** $-32$ ft. per sec. per sec. **14.** 2 sec.; $\frac{7}{8}$ sec.

**Page 41.** **1.** 4 ft. per sec. **2.** 1000 cm. per sec. **3.** 192 ft. per sec.; 576 ft. **4.** 190 cm. per sec. **5.** 128 ft. per sec. **6.** 7.82 sec.; 4.37 sec. **7.** $759\frac{3}{8}$ ft.; $33\frac{3}{4}$ sec. **8.** $\frac{1}{2}$ ft. per sec. per sec. **9.** 4 sec.; 1 sec.; 78.4 m. **10.** 144 ft. or 44.1 m. **11.** 15 sec. **12.** Yes; $29\frac{1}{3}$ ft. to spare. **13.** (1) 160 ft. per sec., (2) 320 ft. per sec. **14.** (1) 420 ft. per sec., (2) 260 ft. per sec. **15.** (1) 1960 cm. per sec., (2) 980 cm. per sec. **16.** (1) 256 ft., (2) 112 ft., (3) $156\frac{1}{4}$ ft. **17.** 49 m. **18.** $156\frac{1}{4}$ ft. **19.** (1) 400 ft., (2) 16 ft. **20.** 25 ft. **21.** 100 m. **22.** (1) $1\frac{1}{2}$ sec. and $4\frac{1}{2}$ sec., (2) 3 sec. **23.** (1) $2\frac{1}{2}$ sec., (2) $4\frac{1}{2}$ sec. **24.** (1) 6 sec., (2) 5 sec. **25.** (1) 96 feet per sec., (2) 126 ft. per sec., (3) 80 ft. per sec. **26.** (1) 36 ft. per sec., (2) 20 ft. per sec. **27.** (1) $39\frac{1}{16}$ ft., (2) 116.49 ft. per sec.

**Page 48.** **1.** 7 : 15. **2.** (1) $a:1$, (2) $1:a$. **3.** $a:1$. **4.** 625. **5.** 15 lb. **6.** (1) 200 dynes, (2) 25,000 dynes, (3) 30,000 dynes, (4) $55\frac{5}{9}$ dynes, (5) 30 dynes, (6) $\frac{av}{t}$ dynes. **7.** (1) 1 cm. per sec. per sec., (2) $\frac{3}{1000}$ cm. per sec. per sec., (3) 1960 cm. per sec. per sec. **8.** (1) $\frac{1}{2}$ gm., (2) $2\frac{1}{2}$ gm., (3) 216 gm., (4) 3920 kg.

9. 5 cm. per sec. per sec.; 25 cm. per sec.; 10,000 units. 10. 5 gm.; 2 cm. per sec. per sec. 11. 200 dynes. 12. 75 ft. per sec.; 5 ft. per sec. per sec.; 750 units.

**Page 52.** 1. (1) 9,800,000 dynes, (2) $\frac{1}{98}$ dynes, (3) 384, (4) 10. 2. (*a*) 25, (*b*) 800. 3. (*a*) 500,000 dynes, (*b*) 510.2 gm. 4. 3750 : 49. 5. $\frac{am}{980}$ gm. 6. 10 min. 7. 37.5 dynes. 8. 785 cm. per sec. per sec. 9. 7,350,000 units. 10. 80 cm. per sec. per sec.; 144,000 dynes. 11. 2520 cm. 12. $\frac{4}{7}$ sec.; 56 cm. per sec. 13. (1) 1750 cm., (2) 1050 cm., (3) 700 cm. per sec. 14. 709.1 cm. per sec. 15. 8960 cm.; 1120 cm. 16. (1) 7750 cm., (2) 2790 cm. 17. 985 gm. 18. 5 : 3.

**Page 54.** 1. 50 ft. from the house. 2. 1250 m. per sec. 3. 12 sec.; 480 ft. from point on earth directly below the balloon.

**Page 58.** 1. $4 \times 10^6$ dynes. 2. 8.6 pd.; 1548.8 pd. 3. $10\frac{5}{12}$ pd.

**Page 63.** 3. 11.72 ft. per sec.

**Page 66.** 1040.5, 23011 ml. per hr.

**Page 70.** 1. 8224.5. 2. 7254. 3. $2\frac{1}{4}$ ft. per sec. 4. 1900 ft. per sec. 5. 5 ft. per sec. 6. 31.25 F.P.S. units; 24.4 pd.

**Page 73.** 8. 35 gm.; 5 gm. 9. 2 $P$; 2 $Q$. 10. 39 pd. 11. 37 kg. 12. 18 pd. 13. 12 $P$. 14. 13 pd. 15. 15 pd. 16. 400 pd.

**Page 75.** 4. 120° between any two forces. 6. (1) 120°, (2) 90°.

**Page 79.** 1. (1) 84 pd.; (2) 18.477 pd.; (3) 5.176 pd.; (4) 70 pd.; (5) 8.789 pd.; (6) 2.125 pd.; (7) 18.915 pd.; (8) 12.64 pd.; (9) $P$ pd. north. 3. $\frac{1}{2}\sqrt{7}$ times side of triangle. 4. 50 pd. acting towards centre. 5. $\sqrt{6}$ pd. 10. 8 grams. 11. 12 pd. 12. $5\sqrt{2}$ kg. at 135° with first force. 13. (1) $5\sqrt{3}$ pd., (2) $5\sqrt{2}$ pd., (3) 2.58 pd. 14. $10\sqrt{3}$ and 10 pd. 15. $6\sqrt{2}$ pd. 16. $50\sqrt{2}$ pd. 17. $8\sqrt{3}$ and 8 pd. 18. $\frac{4}{3}\sqrt{3}$ pd. 19. 199.23 pd.

**Page 85.** 1. (1) 0, − 6; (2) 18, 18; (3) 0, 0; (4) 0, − 11 $\sqrt{2}$; (5) $\sqrt{2}$, 0; (6) 40, 0. 2. 0; 108; − 108. 3. 30 $\sqrt{3}$. 4. (1) 201.47; (2) $62\frac{1}{2}$. 5. 0; 160; 0; − 160. 6. 25 $\sqrt{2}$ ft. from ground. 7. (1) 66.352, (2) 79.672. 8. 1 : 2.

**Page 89.** 1. 5 dynes acting 3 metres from smaller force. 2. 8 dynes acting 25 metres from smaller force. 3. (1) 8 pd. 7.5 ft. from smaller force; (2) 22 pd. $2\frac{8}{11}$ ft. from 7 pound force. 4. $70\frac{7}{12}$ pd.; $50\frac{5}{12}$ pd. 5. $37\frac{1}{2}$ pd.; $74\frac{3}{8}$ pd. 6. 24 and 16 pd. 7. 2ft. from stronger man. 8. $6\frac{1}{4}$ pd.; $3\frac{3}{4}$ pd. 9. 35 and 40 pd. 10. 42 and 21 pd.

**Page 96.** 1. 6.6 metres from 20 kg. mass. 2. $1\frac{1}{2}$ ft. from fulcrum. 3. $1\frac{3}{8}$ ft. from 7 lb. mass. 4. 2 gm. 5. $267\frac{3}{8}$ pd.; $624\frac{1}{3}$ pd. 6. 27 dynes at a point $1\frac{2}{2}\frac{2}{7}$ metres from end. 7. 6 dynes. 8. $11\frac{1}{17}$ cm. from 3 dyne force. 9. 5 lb. 10. 35 lb.; 40 lb. 11. One-quarter of the length of the beam.

**12.** 11 ft. from smaller end. **13.** $1\frac{1}{2}$ in. **14.** 30 pd.; 60° with rod. **15.** $\frac{1}{2}$ kg.; $\frac{\sqrt{3}}{2}$ kg. **16.** $10\sqrt{3}$ pd. **17.** 120 pd. **18.** $\frac{\sqrt{13}\,W}{2\sqrt{3}}$; $\frac{W}{2\sqrt{3}}$. **19.** 12 pd.; $6\sqrt{3}$ pd. **20.** (1) $\frac{100}{\sqrt{3}}$ pd., (2) $\frac{100}{\sqrt{3}}$ pd., (3) 200 pd. **21.** $10\sqrt{3}$ pd. **22.** $30\sqrt{3}$ pd.; $30\sqrt{39}$ pd. **23.** (1) $22\frac{1}{2}$ pd., (2) 54.8 pd. **24.** $46\frac{2}{3}$ pd.; 68.4 pd. **25.** *W.*; *P.* **27.** 10 and 20 pd. **28.** 2 : 1. **29.** $\frac{10\sqrt{2}}{1+\sqrt{3}}$ and $\frac{20}{1+\sqrt{3}}$ pd. **30.** $\frac{2}{\sqrt{3}}$. **31.** $4\sqrt{3}$ pd. **32.** $3\sqrt{2}$ pd. **33.** $3\sqrt{3}$ pd. **34.** $W.\ (\sqrt{2}-1)$. **35.** 100 pd. **36.** $10\sqrt{3}$ and 10 pd. **37.** (1) $20\sqrt{3}$ pd., (2) 40 pd. **38.** $\frac{100}{\sqrt{3}}$ pd.; $\frac{100}{\sqrt{3}}$ or $\frac{200}{\sqrt{3}}$ pd. **39.** (1) $\sqrt{3}$ pd.; (2) $\sqrt{3}$ pd.; (3) 45°.

**Page 109.** 6. 2 pd.; 10.198 pd. **7.** $\frac{1}{15}$. **8.** 4.714. **9.** $\sqrt{3}$. **10.** $\sqrt{3}$; 1; $\frac{\sqrt{3}}{3}$. **11.** $\frac{1}{\sqrt{3}}$. **12.** 11.732 pd. **13.** 36 pd. **14.** 10 tons; $42\frac{2}{3}$ ft. per sec. **15.** 0.732. **16.** $24\frac{4}{9}$ pd. **17.** $\frac{4}{3}\sqrt{3}$ pd. **18.** .268. **19.** 30°.

**Page 117.** **1.** Doubled. **2.** $44\frac{4}{9}$, 25, 16 kg. **3.** 0.37 pd.

**Page 122.** **1.** $2\frac{2}{3}$ cm.; 3.283 cm. **3.** 2 ft.; $3\frac{1}{5}$ ft.; $3\frac{1}{5}$ ft. **4.** $\frac{4}{3}$ ft.; 1 ft.; $\frac{4}{5}$ ft. **5.** $3\frac{1}{3}$ ft.; 5.696 ft.; 4.807 ft. **6.** 10 in. **7.** 18.04 in. **8.** At the centre of the base of the triangle. **9.** $7\frac{3}{5}$ in. **10.** $\frac{11}{30}$ of the side of the square from the middle point of the base. **11.** $\sqrt{3} : 1$. **12.** $\frac{a}{9}$ from *E.* **13.** $\frac{2}{9}$ height from base. **14.** $OG = \frac{2}{21}\,OC$. **15.** In the straight line drawn parallel to *BC* from the middle point of *AB* and at a distance $\frac{26}{45}$ of the side of the square from this point. **16.** Distances from *AD* and *AB* are $\frac{11}{18}$ *AB* and $\frac{4}{9}$ *AD*.

**Page 124.** **1.** 6 in. **2.** 10 in. from the 12 lb. mass. **3.** $4\frac{2}{3}$ in. from the end. **4.** $8\frac{1}{2}$ in. from the 7 lb. mass. **5.** 15 in. from end. **6.** $28\frac{4}{5}$ ft. from first man. **7.** $6\frac{2}{5}$ ft. from 12 lb. mass. **8.** $3\frac{1}{4}$ ft. from 1 lb. mass. **9.** 3.26 in. from the top. **10.** 3.3 in. from the base.

**Page 126.** **1.** $\frac{2}{5}$ of diagonal from 2 lb. mass. **2.** $OG = \frac{1}{4}\,OD$. **3.** 4.34 in. **4.** $\frac{1}{4}$ of the side of the square. **5.** 3.6 ft. (nearly). **6.** 7.8 in. (nearly). **7.** $8\frac{1}{3}$ in.; $11\frac{1}{3}$ in.

**Page 129.** **5.** 5 ft. **9.** $3\frac{11}{13}$ ft. **10.** 120. **13.** 10 kg. **14.** 50 lb.

**Page 135.** **1.** (*a*) 100 ft.-pd.; (*b*) 3200 ft.-pd. **2.** (*a*) 640 ft.-pd.; (*b*) 20, 480 ft.-pd. **3.** 100,000 ergs. **4.** 1800 ft.-pd. **5.** 50,000 ft.-pd. **6.** $\frac{1}{245}$ kg.-m. **7.** 150,000 ft.-pd. **8.** 528,000 ft.-pd. **9.** 98,000 joules. **10.** 98,000 joules. **11.** 3,920 joules. **12.** 144 joules. **13.** 1,886,500 joules. **14.** 1509.2 joules.

**Page 142.** 1. (1) 3200 ft.-pdl. ; (2) 100 ft.-pd. 2. (1) 4,802,000 ergs, (2) 1,200,500 ergs, (3) 0, (4) 4,802,000 ergs. 3. 112,500 ft.-pdl. 4. 7,203 joules. 5. 8000 joules. 6. 4000 ft.-pd.; 4000 ft.-pd. 7. 200 ft.-pdl.; 50 pd. 8. 20 joules. 9. 9.8 joules. 10. 9.8 joules. 11. (1) 25600 ft.-pd. ; (2) 14400 ft.-pd. 12. (1) 10240 ft.-pdl.; (2) 32 ft. per sec.

**Page 144.** 1. 200. 2. 400. 3. 187.5 4. $678787.\dot{8}\dot{7}$. 5. 1600. 6. 60 erg-sec. 7. 100 erg-sec. 8. 10,000 erg sec. 9. 100. 10. 1,000. 11. 20. 12. 39.2. 13. 0.28 h.p. 14. $65\frac{1}{3}$. 15. 784 h.p. 16. 7,800,000 ft.-pd.; 2.44 h.p. 17. 9.6 18. 980. 19. 70 watts. 20. 735. 21. 196. 22. 990. 23. 600 litres. 24. 80.

**Page 151.** 1. $1\frac{1}{4}$ lb. 2. 90 pd., 120 pd. 3. $37\frac{1}{2}$ pd. 4. 225 pd.

**Page 155.** 1. Twice as great; twice as long. 2. 4. 3. $\frac{1}{3}$. 4. 60 pd.

**Page 158.** 1. $26\frac{2}{3}$ pd. ; 2. 6,400 pd.

**Page 161.** 2. $20\frac{5}{9}$ pd. 3. $4\frac{17}{27}$ lb. 4. (*a*) $73\frac{1}{2}$, 96. (*b*) $8\frac{3}{4}$ pd., 7.57 pd. (nearly). 5. $\frac{1}{126}$ pd. ; $\frac{1}{252}$ pd.

**Page 172.** 1. 11 : 40. 2. 1221.82 (nearly).

**Page 178.** 1. (1) 4 ; (2) 576. 2. (1) 6 ; (2) 54. 3. (1) 50 ; (2) 500,000. 4. (1) 0.2 ; (2) 0.8. 5. 80 pd. 6. $312\frac{1}{2}$ gm. 7. $\frac{9}{49}a$. 8. 800 kg. 9. 11550 pd. 10. $30\frac{1}{4}$ pd. 11. 20 kg. 12. $31\frac{1}{14}$ gm.

**Page 185.** 1. 9.122 pd. 2. 0.0375 gm. 3. 11.5 pd. 4. 10,000. 5. 36 kg. 6. 184.87 (nearly). 7. $31\frac{1}{4}$. 8. 9 kg. 9. $230\frac{2}{5}$ ft. 10. 37,500 pd. 11. (1) 2.4 gm., (2) 0.64 gm., (3) 0.48 gm. 12. (1) 416 kg., (2) 1104 kg., (3) 276 kg., (4) 319.8 kg., (5) 96.2 kg.

**Page 190.** 1. 62.5 pd.; 97.5 pd. 2. 4.57 pd. 3. 2.5 kg. 4. 4.9 kg. 5. 295 pd. 6. 7 kg. 7. 600 gm. 8. $\frac{1}{4}$. 9. 0.413 oz. 10. $\frac{n-m}{n}$ gm. 11. 0.881. 12. 5 c.dm. 13. $4\frac{4}{5}$ cu. ft. 14. $32\frac{1}{2}$ pd. 15. 520 gm. 16. 42 gm. 17. 9 pd. 18. 1562.5 lb. 19. $133\frac{1}{3}$ c.c.

**Page 194.** 1. 1.47 kg. 2. 54.05 c.c. 3. 2.7 gm. per c.c. 4. 12 kg. 5. 0.77 gm. per c.c.

**Page 199.** 1. $\frac{3}{7}$ gm. per c.c. 2. $\frac{1}{4}$ gm. per c.c. 3. 1.2 gm. per c.c. 4. $\frac{2}{3}$ gm. per c.c. 5. 20 c.c.; 6 gm. per c.c.; 0.8 gm. per c.c. 6. $\frac{8}{9}$ gm. per c.c. 7. S.g. = $\frac{5}{8}$; s.g. = $\frac{5}{6}$; $6\frac{1}{4}$ inches. 8. 1.072 (nearly) gm. per c.c. 9. Gold, 386.4 gm.; silver, 21.04 gm. 10. 159.14 gm. 11. 311.9 gm. 12. 28.5 gm. 13. $40\frac{5}{8}$ lb. 14. $13\frac{1}{3}$ lb. 15. (1) 30 gm.; (2) 20 gm. 16. 1000. 17. 0.514. 18. 15 oz. 19. 0.64 in. 20. 6 gm. 21. (1) None, (2) 30 gm. increase, (3) None.

**Page 202.** 1. 10 in. 2. 68 cm. ; 170 cm. ; 255 cm. 3. 2.427 ft. 4. 13.619. 5. 11 : 7. 6. 15 cm.

**Page 206.** 6. 1.291 gm.

**Page 213.** 1. 14.756 pd. 2. 1033.6 gm. 3. (1) 15 pd., (2) $14\frac{1}{2}$ pd., (3) 438.48 gm. 4. (1) 952 gm., (2) 1034 gm., (3) 1030 gm. 5. $7066\frac{2}{3}$ pd. 6. 999,600.

**Page 216.** 4. 2908.75 kg.

**Page 221.** 1. $6\frac{2}{3}$ c. ft. 2. 22.85 litres. 3. 75,314.7 c. in. 4. $483\frac{1}{3}$ in. of mercury. 5. $562\frac{1}{2}$ mm. 6. 174 in. of mercury. 7. 0.000125 gm. (nearly) per c.c. 8. 101.34 gm. 9. $3.60.

**Page 231.** 1. (*a*) $\frac{2}{3}$, (*b*) $\frac{8}{27}$. 2. $\frac{4}{5}$. 3. $\frac{3}{2}$. 4. $\frac{16}{25}$. 5. 2 : 1.

**Page 233.** 1. 10.336 m. 2. 17 ft. 3. 12.92 m.

**Page 237.** 2. (*b*) 13.6 times height of mercury barometer. 3. $219\frac{1}{9}$ inches. 9. $h\,\dfrac{\rho}{\rho_1}$.

**Page 261.** 3. $50{,}914\frac{2}{7}$ ergs ($\pi = 22/7$). 4. $2 : 1 : \dfrac{1}{1.04}$, or $1 : \frac{1}{2} : \dfrac{1}{2.08}$. 6. 3.306, 1.343, 1.376 cm., respectively. 8. Height in tube twice distance between plates.

**Page 290.** 1. 65.1 pd. per sq. in. 2. 4,900,000 dynes per sq. cm.; 5 kg. per sq. cm. 3. 374.01 c.c. per sec. 4. .224 cu. in. per sec. 5. $23{,}108\frac{1}{3}$ ft.-pd. 6. $4\frac{1}{6}$ ft. per sec. 7. 5.014 ft.-pd. (nearly). 8. 4,920,000 ergs. 9. (*a*) 600 gm. per sq. cm.; (*b*) 587,200 dynes = 599.2 gm. per sq. cm. ($g = 980$).

**Page 300.** 1. 0.213° F. 2. 2.68° C. 3. 62,211 B.T.U. (nearly). 4. 750; 3.6% low. 5. 8,402,400 ft.-pd.; 441.8 lb. 6. 0.838 k.w. 7. 95,465 cal. 8. 40.3 ohms. 9. 94.1%. 10. 63.36%.

**Page 304.** 2. 97 c. 3. $3.83; 70 c. 5. $2.03. 6. $2.27. 7. $10.31; $1.13; $1.26. 9. $62.97. 10. $60.37; $26.68.

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